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Qualitative analysis of periodically forced nonlinear oscillators responses and stability areas in the vicinity of bifurcation cascade

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Abstract

Bifurcation theory is the mathematical investigation of changes in the qualitative or topological structure of a studied family. In this paper, we numerically investigate the qualitative behavior of nonlinear RLC circuit excited by sinusoidal voltage source based on the bifurcation analysis. Poincare mapping and bifurcation methods are applied to study both dynamics and qualitative properties of the periodic responses of such oscillator. As numerically illustrated here, a small variation of amplitude or frequency of the driver sinusoidal voltage may involve qualitative changes for witch the system exhibits fold, period doubling and pitchfork bifurcations. In fact, the presence of these kinds of bifurcation necessitates an examination of the role of these singularities in the dynamical behavior of circuit. Particularly, we numerically study the qualitative changes may affect number and stability of the periodic solutions and the shapes of its basins of attraction associated while approaching the neighborhood of a particular bifurcation structure known as isoordinal lips cascade. Using a numerical scanning technique, higher harmonic domains which can prove the existence of such cascade of bifurcation are numerically computed. Furthermore, we report on some numerical simulations of bifurcation singularity and basins attractor which are useful tools for understanding and illustrating these effects.

Keywords: **Qualitative behavior, bifurcations cascade, fold, flip, pitchfork, *higher harmonic*, Attraction basins**

1. Introduction

Nonlinear dynamical systems undergo abrupt qualitative changes when crossing bifurcation points. Multistability is one of the most important properties of nonlinear systems. One can have two or more stable states for the same system parameters and for different initial conditions set. For a more exhaustive study of nonlinear system responses, it is compulsory to identify the singularities of the parameter plane (bifurcations, chaos, ...) and the singularities of the phase plane (fixed point, cycles, invariant closed curve, attraction basins, ...). The singularity considered here is the attraction basins associated to the attractors which coexist for same parameters of the RLC circuit. The influence domain or stability domain or basin of an attractor Λ is the open set (D) of the points X_n such that the consequent of all X_n approach asymptotically Λ as $n \to \infty$. The influence domain (or basin) of X^* is the set of points X_0 giving the convergence of X_n towards X^* . The attraction basin may be either all in one block, or made up of finite or infinite number of disjointed parts with only one accumulation point [1]. The structure of a stability domain can undergo a global bifurcation changing the connexity property of this area to non-connexity or vice versa. Previous studies have investigated global bifurcations that change the structure and properties of attraction basins and their boundaries for both two-dimensional diffeomorphisms [1], [2] and endomorphisms [3]. When a cycle is locally asymptotically stable, it is possible to enquire about its influence domain, i.e. about the largest admissible initial perturbation δ or more exhaustively, about the set X_i of points X_n such that the consequents of all X_n approach asymptotically and successively the k points of the cycle as $n \to \infty$.

The forced RLC circuit with nonlinear inductor exhibits a wide variety of nonlinear phenomena, such as the jump and hysteresis, bifurcation and chaotic states, the frequency entrainment, harmonic and subharmonic oscillations, quasi-periodic behavior [4],[5]. A nonlinear forced oscillator containing a ferromagnetic core with saturation and hysteresis or an other Hard Characteristics exhibits a complex bifurcation phenomena near points of resonance [6], [7]. The Duffing Van der Pol oscillator of [8] shows a broad spectrum of dynamic behaviors, both chaotic as well as periodic. Such a considered circuit composed of a resistor, an inductor and a capacitor is described by two dimensional dynamical system modeled by the following second-order nonlinear ordinary differential equation:

$$\ddot{x} + \xi(x, \dot{x}) + f(x) = h(t) \qquad (1)$$

A particular bifurcation structure namely an isoordinal cascade of bifurcations, studied in former works [9-11], include local bifurcations of codimension one such as fold, flip and pitchfork and bifurcations of codimension 2 such as cuspidal points which correspond to the intersection of two fold curves. The symmetry property of the circuit is introduced by the Pitchfork bifurcation, it was stated that a lip structure which is a combination of two fold curves related in the edges by cusp points are surrounded by pitchfork bifurcation curve and is associated to an even higher harmonic predominance. The aim of this work is to study the qualitative changes of the attraction basins of symmetrical attractors in proximity of certain bifurcation points. The rest of the paper is organized as follows. In section 2, we present an overview of the governing differential equations of a nonlinear RLC circuit excited by sinusoidal voltage source. Section 3 is devoted to a reminder of some basic results on singularities in a phase plane and bifurcation sets in parameter plane. In section 4, the analysis of higher harmonic of 2π - periodic solutions is examined. Finally, section 5 presents the main results of the paper. We numerically compute bifurcation diagrams and we report the effects of the structure of singularities on the attraction basins of stable attractors.

2. RLC circuit equations

Fig. 1 shows typical RLC circuit modeled as a series combination of a resistor R, inductor L and capacitor C. Such inductor is characterized by a single valued characteristic (i.e. without hysteresis). The current i is approximated by a cubic polynomial $i = a_1\phi + a_3\phi^3$, $a_1 > 0, a_3 > 0$, where ϕ is the magnetic flux and a_1, a_3 are constants.

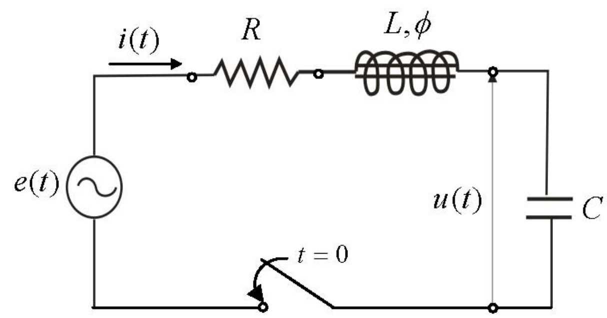


Fig. 1 Typical series RLC circuit.

The system is driven by a sinusoidal voltage source. Using the notation in Fig. 1, the fundamental equation for the circuit is described by:

$$R.i(t) + \frac{d\phi}{dt} + \frac{1}{C}\int i(t).dt = e(t) \qquad (2)$$

* $e(t) = e_m \sin(\omega t)$ is a sinusoidal voltage source, were ω is the excitation frequency and e_m is the amplitude.

* $u(t) = \frac{1}{C}\int i(t).dt$ is the voltage across the capacitor.

We normalize the state variables and the time variable as: $x = \phi(t)$, $y = u(t)$ and $\tau = \omega t$. Rewrite equation (2) as follows:

$$\begin{cases} \frac{dx}{d\tau} = \frac{1}{\omega}(e_m.\sin\tau - R(a_1.x + a_3.x^3) - y) \\ \frac{dy}{d\tau} = \frac{1}{C\omega}(a_1.x + a_3.x^3) \end{cases} \qquad (3)$$

We can easily verify that (3) is invariant to the transformation $(\tau, x, y) \to (\tau + \pi, -x, -y)$.

3. Reminder of some basic results

This section summarizes some basic results about the singularities of nonlinear systems described by second order nonlinear differential equations. These results will be useful for the analysis of the temporal behavior of a Duffing type equation modeling an RLC circuit with nonlinear core inductor. The Poincare map is usually studied to understand the nature of non linear oscillatory systems responses in the phase space and their bifurcations in the parameter space. A complete treatment of the bifurcation types and their computation methods may be found in [12].

3.1 Phase plane singularities

The two-dimensinal differential equations system (3) can be rewritten as the following general expression:

$$\frac{dX}{d\tau} = f(X,\tau,\lambda);\ \tau \in \Re,\ X \in \Re^2,\ \lambda \in \Re^2 \qquad (4)$$

Where $X = (x, y)^T$ denotes the state vector, $\lambda = (\omega, e_m)$ is the parameters vector and f is supposed to be C^∞ and periodic of period 2π.

A classical technique for qualitatively investigating the system dynamics controlled by the parameters vector (ω, e_m) is based on the Poincare map T.

This map, denoted T_λ, is derived from equation (4) by merely sampling the continuous phase trajectories at $t = 2\pi$. This geometrical method, called Poincare section, permits to give rise to a discrete trajectory computed implicitly through numerical integration of the differential equations system.

By using the solution $\psi(t, U_0, \lambda)$ of (4), with an initial condition given by $X(t_0) = U_0$, the Poincare mapping is defined as:

$$T_\lambda : \Re^2 \to \Re^2;\ U_0 \mapsto \psi(t_0 + 2\pi, U_0, \lambda) \qquad (5)$$

Where $\lambda = (\omega, e_m) \in \Re^2$ denotes the system parameters. Thus the analysis of the discrete dynamical system properties defined by the relation (5) via studying the singularities of T_λ enables to perform a more complete description of the original system behavior defined by the relation (4). Indeed, a periodic solution of (4) of period 2π is associated to a periodic point namely a fixed point of T_λ. While a k-order cycle (made up of k points) will correspond to a $2k\pi$ periodic solution of (4), then U satisfies the following equation:

$$T_\lambda^k(U) - U = 0 \qquad (6)$$

In this paper only fixed point singularities type of T_λ and their bifurcations will be considered.

3.2 Parameter plane singularities

Stability of periodic solutions is obtained by examination of Jacobian of the system at these solutions. Therefore it is possible to show the dynamical behavior around these points and make qualitative studies without having to solve the system equations. Thus the stability nature of a periodic point U is known from the roots S of the following characteristic equation:

$$\left| \frac{dT_\lambda^k(U)}{dU} - S.I \right| = 0 \qquad (7)$$

These roots, also called the multipliers, are denoted by S_1 and S_2 ($|S_1| \geq |S_2|$). Three topologically points are defined as follow: if $|S_1| < 1$ the point is asymptotically stable, if $|S_1| \geq 1 > |S_2|$ the point is unstable and is called saddle and if $|S_2| \geq 1$ the point is completely unstable.

At the particular value $|S| = 1$, we have a critical case of lyapunov, a bifurcation may occur. The following local bifurcations are to be identified in the equation (3).

* ***The tangent bifurcation (or fold):***

This type of bifurcation occurs when one of the multipliers of a fixed point (or a cycle) $S_p = +1$, (p = 1 or 2), this bifurcation is schemed in this paper as follows:

$$\Phi \leftrightarrow cycle(k, j)\,a + cycle(k, j)\,r$$

Where Φ is used to indicate the non existence of the two cycles before the bifurcation point. Whereas cycle (k, j) denotes a k-order cycle, j characterizes the order of iterations of the points of the cycle. Finally, we note that “a” (resp. “b”) is attributed to attractive cycle (resp. repulsive cycle). In the following discussion the curve associated to this type of bifurcation is denoted by $\Lambda_{(k)_0}^j$.

* ***The doubling period bifurcation (or flip):***

This type of bifurcation happens when $S_p = -1$, (p = 1 or 2), this bifurcation is schemed in this paper as follows:

$$cycle(k, j)\,a \leftrightarrow cycle(k, j)\,r + cycle(k.2, j)\,a$$
$$cycle(k, j)\,r \leftrightarrow cycle(k, j)\,a + cycle(k.2, j)\,r$$

In a parameter plane the curve of bifurcation flip is denoted by Λ_k^j

* ***Pitchfork bifurcation:***

This type of bifurcation occurs when $S_p = +1$, (p = 1 or 2) after a k-order cycle crosses a Pitchfork bifurcation, the stability of such a cycle is changed and two other k-order cycles with different stability occur. This bifurcation is presented here as follows:

$$cycle\,(k,\,j)\,r \leftrightarrow cycle\,(k,\,j)\,a\; + cycle\,(k,\,j')\,r + cycle\,(k,\,j'')\,r$$

$$cycle\,(k,\,j)\,a \leftrightarrow cycle\,(k,\,j)\,r\; + cycle\,(k,\,j')\,a + cycle\,(k,\,j'')\,a$$

3.3 Attraction basins properties

The trajectory of a given system, in state space will head for some final attracting region, or regions, which might be a point, curve, area, and so on. Such an object is called an attractor for the system, since a number of distinct trajectories will be attracted to this set of points in the state space. Indeed the non-unicity of these attractors led mainly to characterize each stable state by its associate stability domain (or attraction basin).
These domains include the open sets of the points in the initial conditions space for which the solutions of the differential equation converges towards this solution this stable state. Thus, an attraction basin (D) is a stability domain of an attractive set (or attractor) having a border (F) see Fig. 2). The analysis of stability domains (D) properties of these attractors and their borders (F) (connexity, complex shape, fractal,) for two dimensional maps was undertaken in several works [1], [13], [14].

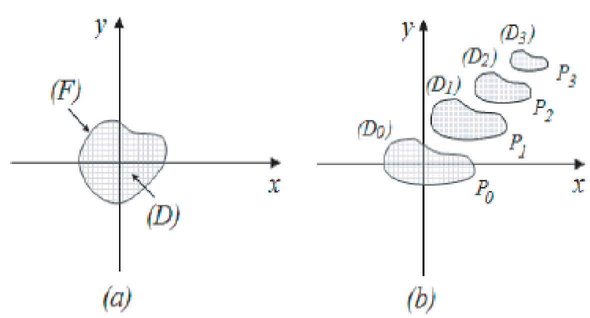


Fig. 2 Connectedness of stability domains of fixed point
a) connected attraction basin b) disconnected basins

A Poincare geometrical transformation T associated to a continuous dynamical system can be a diffeomorphism (invertible) or an endomorphism (non invertible, non unicity of T^{-1}). An attraction basin is connected (see Fig. 2) if the punctual transformation is invertible otherwise it is disconnected and made of a finite or infinite domains [14], the attraction basin can also be connected but including holes, it is the case of multiply connected basins [1]. In the case of our studied circuit shown in the section 2, we analyze in particular the multistability of periodic attractors and the basin of attraction structure in phase space and its dependence with the bifurcation points.

4. Higher harmonic spectral analysis

In former studies [9], [10], [15] it had been shown that the $2\pi-$ periodic solutions of a nonlinear differential equation governing the behavior of the considered RLC circuit with core inductor can be classified according to their Fourier spectra. In an ordering based on line amplitudes of a frequency spectrum in descending order, this means that a rank-m harmonic (m > 1) has the second place, and the first (i.e. the greatest amplitude) in the case of full predominance. It is shown that the study of the higher harmonic predominance in a parameter plane leads to conclude about the existence of a certain bifurcation structure namely isoordinal cascade. Such a bifurcation structure is established in one cell of parameter plane (ω, e_m).

Numerically, we consider a (ω, e_m) parameter plane, the spectral 'scanning' consists in dividing the plane in small pixels having the same width $\Delta\omega$ and the same height Δe_m, then compute the Fourier expansion of $2\pi-$ period solution to identify the corresponding order of the predominant higher harmonic. And finally the pixel takes the color assigned to the predominant higher harmonic of the oscillatory attractor (or fixed point). The numerical computed domains of the higher harmonic predominance are shown in Fig. 3.

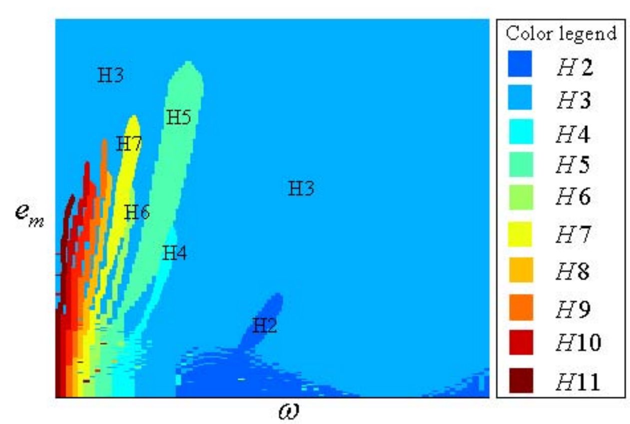


Fig. 3 Higher harmonic domains in (ω, e_m) - plane

As previously reported in [15], the domains shown in Fig.3 can be used to proof the identification of an isoordinal lips cascade embedded in the different colored cells.

5. Numerical simulations of the qualitative behavior of RLC circuit

From the whole structure of isoordinal cascade of lips extracted from [9], and numerically illustrated in Fig. 4, we consider a lip structure associated to the fourth higher harmonic predominance Fig. 4. This lip structure, made of two fold bifurcation curves ${}^4\Lambda^1_{(1)_0}$ and ${}^4\Lambda^{1'}_{(1)_0}$ joined at their extremities in two cuspidal points ${}^4C^1_1$ and ${}^4C^{1'}_1$, is surrounded by a Pitchfork bifurcation ${}^4\overline{\Lambda}^1_{(1)_0}$. This means that we have two symmetric lips sketched out in the foliation structure of Fig. 7. A flip curve ${}^4\Lambda^1_1$ is also related to such a symmetric fold structure.

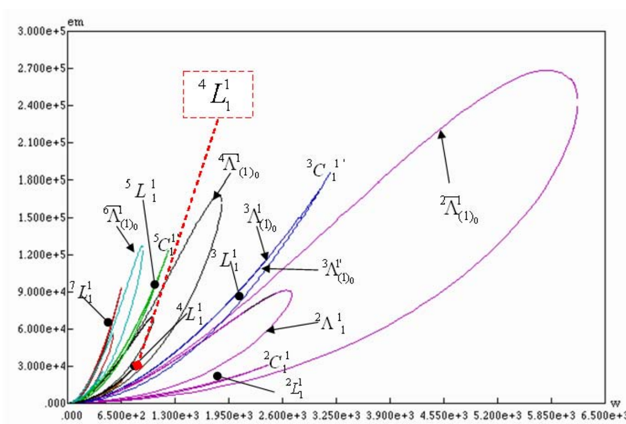


Fig. 4 Isoordinal lips cascade and associated flip and pitchfork curves

Let us consider a point chosen between two fold bifurcation points A1 and A1' see Fig. 8. Actually the vertical cross-section revealed 7 attractors of which three are unstable (M2, M4 and M6) and four are stables (M1, M3, M5 and M5). We are concerned with attractive periodic solutions. In Fig. 9 the time series, phase trajectories and spectra of these attractors are presented. We note that in the phase trajectories a small red point is used to identify a fixed point which is the accumulation point in attraction basin of this attractor. The immediate basins of stable attractors M1, M3, M5 and M7 will be numerically illustrated in this section. The lip structure related to an even higher harmonic predominance chosen below is aiming to have four different stable attractors. Each of these attractors has its stability domain which will be estimated in the phase plane (x, y).

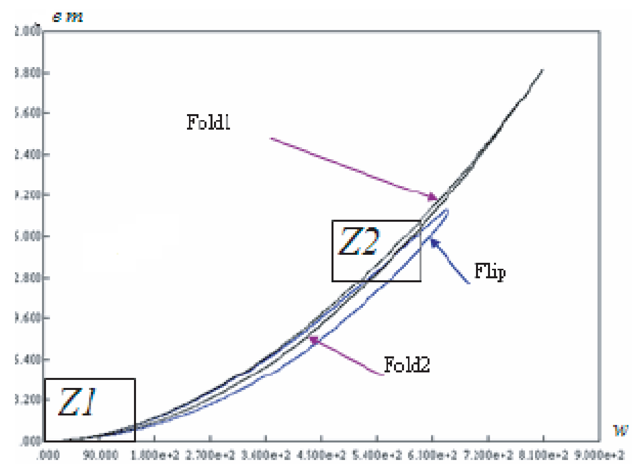


Fig. 5 The lip bifurcation structure (L^4).

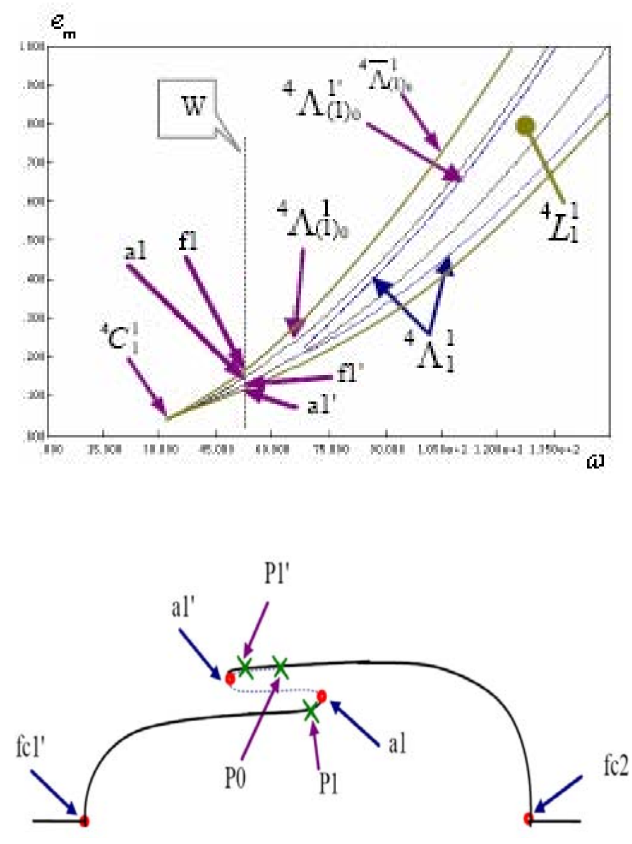
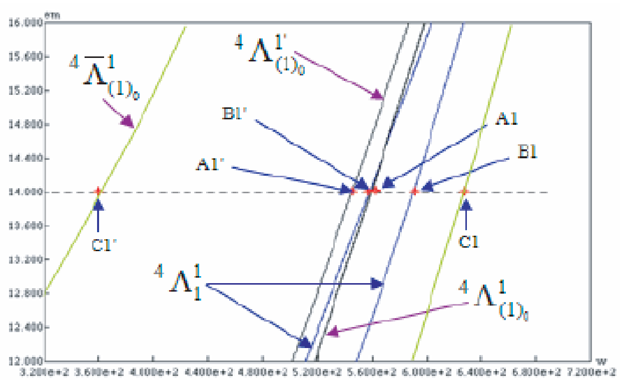


Fig. 6 Detailed bifurcation diagram of W-section

We shall consider in this section two cross-sections: (W)-section, for ω = constant and (E)-section, for e_m = constant in two different regions (Z1) and (Z2) respectively as in Fig. 5. Detailed diagrams of such cross-sections are given in Fig. 6 and Fig. 7. (W)-section corresponds to relatively small values of ω and e_m, and includes a set of points (ω, e_m) bounded by two fold bifurcation points. This section is relatively far from flip bifurcation curve and includes only fold bifurcations.

The second cross-section namely (E)-section is chosen in order to analyze the effect of both of fold and flip bifurcation on attraction basins shapes and sizes, this section contains two tangent bifurcations and a flip bifurcation. Also, we recall from the section 2, that the axis of our phase plane are defined by the variable states $\phi(t)$ and $v(t)$ in the x-axis and in the y-axis respectively.



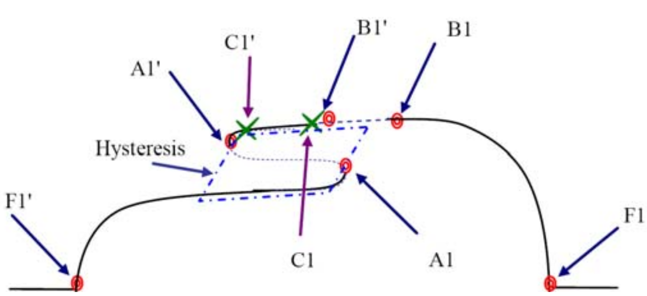


Fig. 7 Detailed bifurcation diagram of E-section.

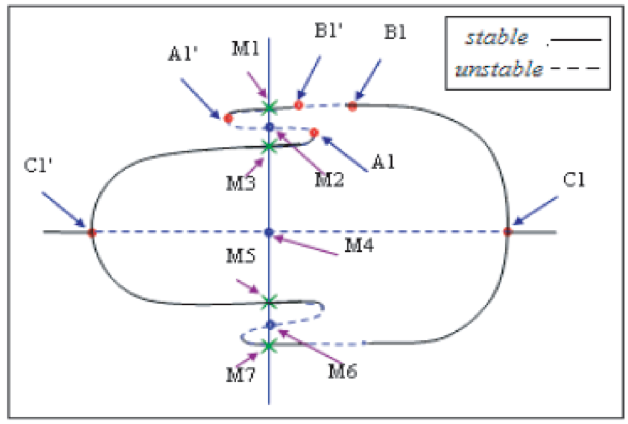
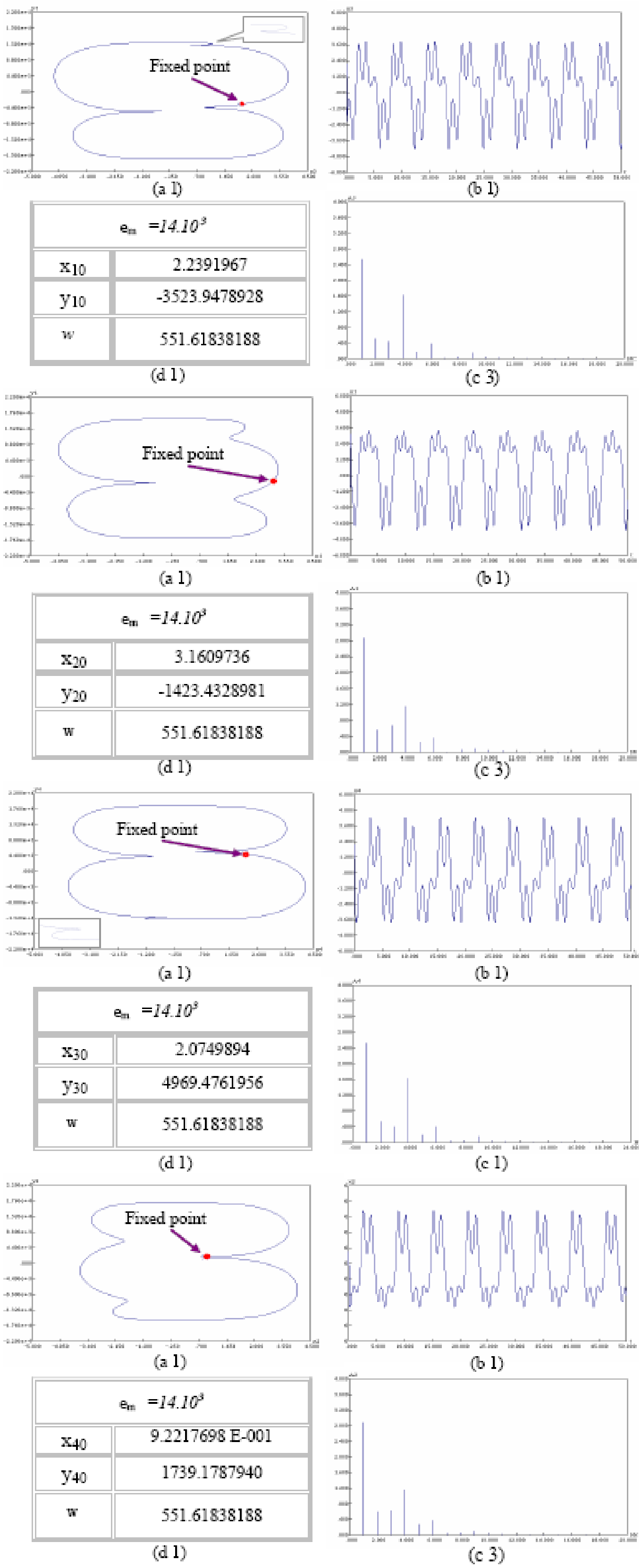


Fig. 8 Foliation of the bifurcation diagram.

5.1 W-section attraction basins

As mentioned above this section intersects the lip structure L^4 in two fold bifurcation points ($\omega = 50, e_m = 112.959$) and ($\omega = 50, e_m = 128.256$). Choosing two points from W-section, for a given fixed value of $\omega = 50$ and for a couple of values $e_m = 113.261$ and $e_m = 116.193$, the attraction basins are numerically computed by using the phase plane 'scanning' technique. The proposed method consists in dividing a phase plane cell $[-\omega_{Min}, +\omega_{Max}]$ $\times[-e_{mMin}, +e_{mMax}]$ in small pixels having the same dimensions, width Δx and height Δy . The basin is computed in the obvious way by numerically integrating the differential equation starting from the set of initial conditions on the obtained 400*400 size grid, and in each case, after allowing the transient to decay sufficiently, deciding which solution has been reached. And finally each one of the 16.10^4 pixels in the figure takes the color assigned to the attractive periodic solution (or attractor) given by considering the initial point in it.

The details of the intersection of W-section with the lip structure is given in Fig. 5, we have two different fold bifurcation points: f1, a1. Since the parameter space is foliated [16] the two fold curves including f1 and a1 respectively are the boundaries of three different sheets , two stable sheets related through a third unstable one. This kind of bifurcation feature exhibits phenomena of jump and hysteresis. At point fc2 ($\omega = 50, e_m = 97.785$) and for increasing values of e_m, a stable fixed point undergoes a pitchfork bifurcation becoming unstable and generating two symmetric stable fixed points.



e_m =14.10³	
x_{10}	2.2391967
y_{10}	-3523.9478928
w	551.61838188

e_m =14.10³	
x_{20}	3.1609736
y_{20}	-1423.4328981
w	551.61838188

e_m =14.10³	
x_{30}	2.0749894
y_{30}	4969.4761956
w	551.61838188

e_m =14.10³	
x_{40}	9.2217698 E-001
y_{40}	1739.1787940
w	551.61838188

Fig. 9 Spectra and phase trajectories of stable attractors.

The phase trajectories of stable attractors, which are periodic solutions of original differential system having the period of the forcing sinusoidal input (normalized to 2π), are given in Fig.10.

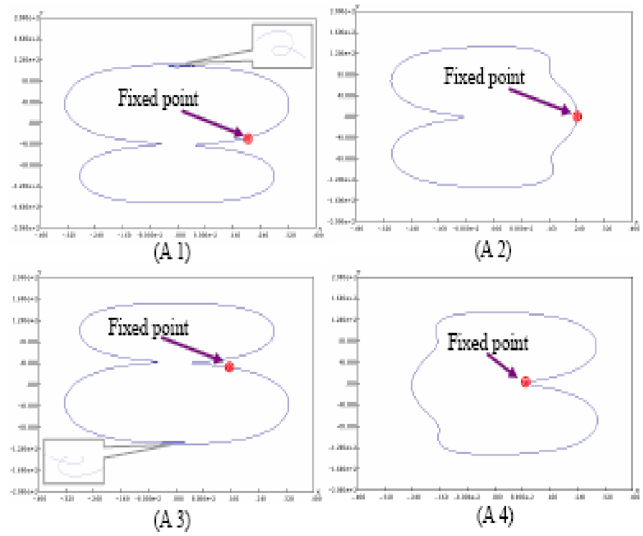


Fig.10. Phase trajectories of stable attractors.

The attraction basins of four stable attractors which corresponding to the same system parameter ($\omega = 50, e_m = 113.261$) is given in Fig. 11, A1, A2, A3 and A4 are the accumulation points of such stability domains.

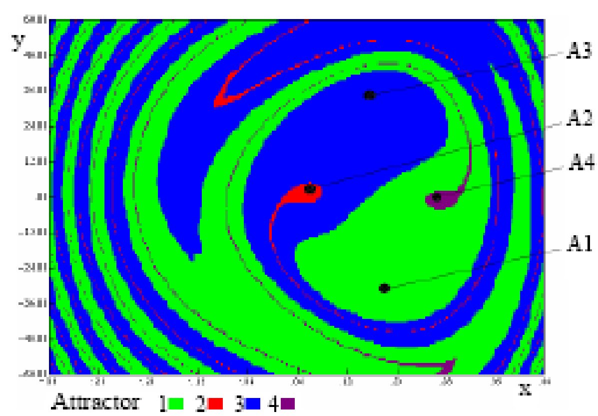


Fig.11 Attraction basins in proximity of fold bifurcation
(w= 50, em = 113.261199).

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It is obvious to remark that the attraction basins of A2 and A4 are smaller compared to those of A1 and A3, this is due to the fact that the symmetric attractors A2 and A4 are very close to a fold bifurcation. Picking another point of W-section, relatively far from the two bifurcation points ($\omega = 50, e_m = 116.193$), the obtained basin is shown in

Fig. 12. It is worth noting that the stability domains sizes of A2 and A4 increase, thus these domains are vanishing in vicinity of bifurcation points. The attraction basins are seemingly scrolled around a central part including accumulation points.

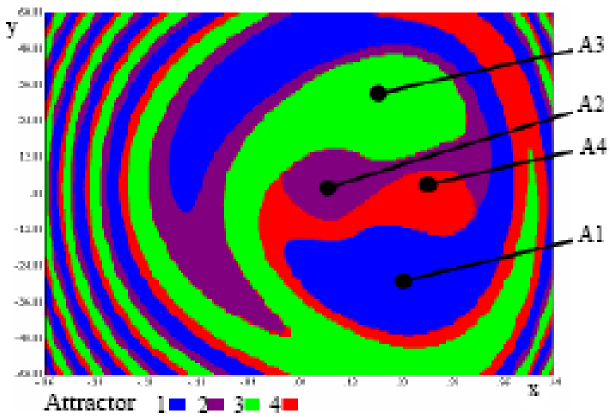

Fig.12 Attraction basins of attractors relatively far from fold bifurcation (w = 50, em = 116.193).

5.2 E-section attraction basins

The E-section includes 2 folds and a flip bifurcation, in this particular case we choose two particular points ($\omega = 545.989,\ e_m = 14.10^3$) nearby a fold bifurcation and ($\omega = 557.629,\ e_m = 14.10^3$) close to both a flip and fold bifurcation, in the latter case there is no contact between the two bifurcation points because they are not in the same sheet. For the first case, two attractors M1' and M7' are in proximity of fold bifurcation Fig. 13, that is why their stability domains are greatly reduced.

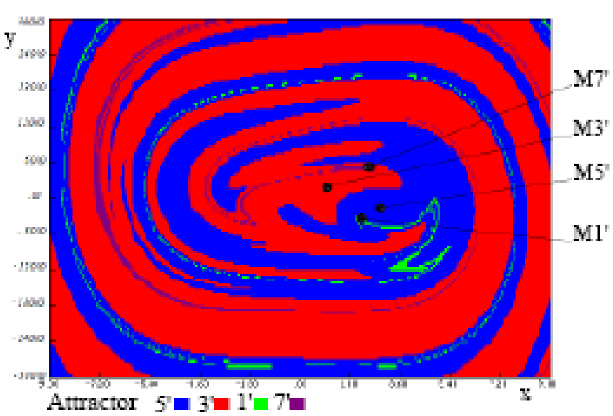

Fig. 13 Attraction basins in proximity of fold bifurcation (w= 545.989, em = 14000).

Whereas, when the four attractors are altogether closer to bifurcation points (two attractors close to fold bifurcation and two others are close to flip bifurcation) their attraction basins have relatively important sizes Fig. 14.

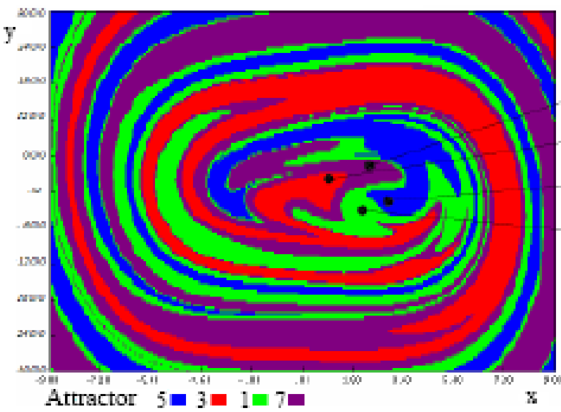

Fig.14 Attraction basins in proximity of 2 bifurcations points ($\omega = 557.629,\ e_m = 14000$).

6. Conclusion

We have presented a combined qualitative and numerical analysis of the global behavior of a nonlinear RLC circuit by investigating both the dynamic responses of nonlinear model and the bifurcation structure in the amplitude-pulsation parameter plane. An analysis of particular bifurcation structure known as isoordinal lips cascade is treated. Especially, we have numerically illustrated the effect of a parametric singularities such as fold, flip and cuspidal bifurcation on a phase plane singularity namely attraction basins of stable attractors. Some properties of these stability areas, however, began to change while approaching the neighborhood of these kinds of bifurcation points. In addition, several basic properties such as multistability and symmetry of the proposed oscillator are carried out.

Appendix

Using our bifurcation computing algorithms developed in FORTRAN, the numerical results in this work are obtained with respect to the following values of RLC circuit parameters.

Table 1: RLC circuit parameters

R [Ω]	20
C [μF]	1
a_1	0.015

a_3	0.365

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References

[1] C. Mira, chaotic dynamic from the one dimensional endomorphism to the two dimensional diffeomorphism, World Scientific, 1987.

[2] Helena E. Nussea and James A. Yorkeb, Bifurcations of basins of attraction from the view point of prime ends, Topology and its Applications, Volume 154, No. 13, July 1, 2007, pp. 2567-2579.

[3] Wanda Szemplinska-Stupnicka and Elzbieta Tyrkiel1, Effects of Multi Global Bifurcations on Basin Organization, Catastrophes and Final Outcomes in a Driven Nonlinear Oscillator at the 2T-Subharmonic Resonance, Nonlinear Dynamics, Vol. 17, No. 1, September 1998, pp. 41--59.

[4] Michele V. Bartuccelli, Jonathan H.B. Deane and Guido Gentiley, Bifurcation phenomena and attractive periodic solutions in the saturating inductor circuit, Proceedings of the Royal Society A, vol. 463 No. 2085 , September 8, 2007, pp. 2351-2369.

[5] Munehisa Sekikawa, Naohiko Inaba, Tetsuya Yoshinaga and Hiroshi Kawakami, Bifurcation structure of fractional harmonic entrainments in the forced Rayleigh oscillator, Electronics and Communications in Japan, Part 3, Vol. 87, No. 3, 2004, pp. 30-40.

[6] Paul Bryant and Carson Jeffries, Bifurcations of a Forced Magnetic Oscillator near Points of Resonance, Physical Review Letters, Vol. 53, No. 3, July 16, 1984.

[7] Kenjiro Yamaguchi and Genji Yorimitsu, Bifurcation Phenomena of a Forced Self-Oscillatory System, Electronics and Communications in Japan, Part 3, Vol. 82, No. 9, 1999.

[8] J.D Jeng, Y. Kang and Y.P. Chang, An Alternative Poincare Section for Steady-State Responses and Bifurcations of a Duffing-Van der Pol Oscillator, WSEAS Transactions on systems}, Vol. 7, No. 6, June 2008.

[9] H. Khammari, C. Mira and J.P. Carcasses, Behavior of harmonics generated by a Duffing type equation with a nonlinear damping: partI, 12th IEEE International Conference on Electronics, Circuits and Systems ICECS 2005, 11-14 Dec, Gammarth, Tunisia, 2005.

[10]H. Khammari and M.Benrejeb, Tangent bifurcation in doubling period process of a resonant circuit's responses, IEEE International conference on industrial technology ICIT 2004, Hammamet, Tunisia, December 8-10, 2004.

[11]C. Mira, H. Kawakami, M. Touzani-Qriouet, Bifurcations structures generated by the non-autonomous duffing equation, International Journal of Bifurcation and Chaos, Vol. 9, No.7, 1999, pp. 1363-1379.

[12]H. Kawakami, Bifurcation of periodic responses in forced dynamic nonlinear circuits: computation of bifurcation values of the system parameters, IEEE transactions on circuits and systems, 1984, vol. 31, No. 3, pp. 248-260.

[13]Igor Gumowski, C. Mira, Recurrence and Discrete Dynamic Systems, Springer-Verlag, August 1980.

[14]C. Mira, D. Fournier-Prunaret, L. Gardini, H. Kawakami and J.C. Cathala, Basin bifurcations of two-dimensional noninvertible maps: fractalization of basins, International Journal of Bifurcation and Chaos in Applied Sciences and Engineering, vol. 4, No. 2, 1994, pp. 343-382.

[15]H. Khammari, J.P. Carcasses and M.Benrejeb, Bifurcations of periodic solutions and higher harmonic oscillations in the RLC-circuit, CESA 98, Tunisia, April 4-5th, 1998.

[16]C. Mira, J. P. Carcasses, C. Simo, J. C. Tatjer, Crossroad area-spring area transition. (II) Foliated parametric representation, International Journal of Bifurcation and Chaos, vol. 1, No. 2, 1991 pp. 339-348.

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