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A Tandem Communication Network with Dynamic Bandwidth Allocation and Modified Phase Type Transmission having Bulk Arrivals

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Abstract

This paper deals with the performance evaluation of a two node communication network with dynamic bandwidth allocation and modified phase type transmission having bulk arrivals. The performance of the statistical multiplexing is measured by approximating with the compound Poisson process and the transmission completions with Poisson processes. It is further assumed that the transmission rate at each node are adjusted depending upon the content of the buffer which is connected to it. The packets transmitted through the first node may be forwarded to the buffer connected to the second node or get terminated with certain probabilities. The performance measures of the network like, mean content of the buffers, mean delays, throughput, transmitter utilization etc. are derived explicitly under transient conditions. Sensitivity analysis with respect to the parameters is also carried through numerical illustration. It is observed that the dynamic bandwidth allocation and batch size distribution of arrivals has a tremendous influence on the performance measures.

Keywords: *Compound Poisson process, Modified phase type transmission, Performance evaluation, Bulk arrivals, Dynamic Bandwidth Allocation*

1. Introduction

The demand for data/voice communication is growing rapidly in many different fields. To satisfy this rapidly growing demand by many users, various kinds of effective Communication networks have been developed. With the development of sophisticated technological innovations in recent years, a wide variety of Communication networks are designed and analyzed with effective switching techniques. In general, a realistic and high transmission of data or voice over the transmission lines is a major issue of the Communication systems (Srinivasa Rao. K. et al (2006)).

For efficient utilization of resources, it is needed to analyze the statistical multiplexing of data/voice transmission through congestion control strategies. Usually bit dropping method is employed for congestion control. The idea of bit dropping is to discard certain portion of the traffic such as least significant bits in order to reduce the transmission time, while maintaining satisfactory quality of service as perceived by the end users, whenever there is congestion in buffers. Bit dropping methods can be classified as input bit dropping (IBD) and output bit dropping (OBD) respectively (Kin K. Leung, (2002)). In IBD bits may be dropped when the packets are placed in the queue waiting for transmission. In contrast bits are possibly discarding in OBD only from a packet being transmitted over the channel. This implies fluctuation in voice quality due to dynamically varying bit rate during a cell transmission (Karanam V.R et al (1988)).

To have an efficient transmission, some algorithms have been developed with various protocols and allocation strategy for optimum utilization of the bandwidth (Emre and Ezhan, 2008; Gundale and Yardi, 2008; Hongwang and Yufan, 2009; Fen Zhou et al. 2009; Stanislav, 2009). These strategies are developed based on flow control or bit dropping techniques. Very little work has been reported in literature regarding utilization of the idle bandwidth by adjusting the transmission rate instantaneously just before transmission of a packet.

The transmission strategy of adjusting the transmission rate depending upon the content of the buffer connected to it, just before the packet transmitted is referred as dynamic bandwidth allocation (DBA). Recently Suresh Varma et al. (2007) have considered a two node communication network with load dependent transmission. However, they assumed that the arrivals to the source node are single packets. But, in communication systems, the messages arrived to the source are converted into a number of packets depending upon the message size and hence the arrival formulate a batch or bulk packets arrival at a time

A little work has been seen regarding tandem communication network with bulk arrivals having DBA. In addition to this, in many of the Satellite and Tele communication systems, the packet getting transmitted after the first node get terminated or forwarded to the second buffer connected to the second node with certain probabilities. Generally, conducting laboratory experiments with varying load conditions of a communication system in particular with DBA and bulk arrivals is difficult and complicated. Hence, mathematical models of communication networks are developed to evaluate the performance of the newly proposed communication network models under transient conditions

In this paper, a two node communication network with DBA having modified phase type transmission with bulk arrivals is modeled through imbedded Markov chain techniques. Using the difference-differential equations, the performance measures of the communication network like, the joint probability generating function of the number of packets in each buffer, probabilities of emptiness of buffers, mean content of the buffers, mean delays in buffers, throughput of the nodes are derived explicitly under transient conditions. The steady state behavior of the model is also analyzed. The performance evaluation of communication network is studied through numerical illustration.

2. Communication Network Model and Transient Solution

A communication network model with DBA having bulk arrivals and modified phase type transmission is studied. Consider a communication network in which two nodes are in tandem and the messages arrive to the first node are converted into number of packets and stored in first buffer connected to the first node. After transmitting from the first node, the packet may be forwarded to the second buffer which is connected to the second node for forward transmission with the probability **θ** or the packet may terminated after the first node with probability (**1-θ**). It is further assumed that the arrival of packets to the first buffer is in bulk with random batch size having the probability mass function $\{C_k\}$. In both the nodes the transmission is carried with DBA. i.e. the transmission rate at each node is adjusted instantaneously depending upon the content of the buffer connected to it. This can be modeled as the transmission rates are linearly dependent on the content of the buffers.

Here, it is assumed that the arrival of packets follows compound Poisson process with parameter **λ** and the number of transmissions at nodes 1 and 2 follow Poisson processes with parameters μ_1 and μ_2 respectively. The queue discipline is First-In-First-Out (FIFO). The schematic diagram representing the communication network is shown in figure 1. Using the difference-differential equations, the joint probability generating function of the number of packets in the first buffer and number of packets in the second buffer is derived as

$$P(Z_1,Z_2,t)=\exp\left[\lambda\sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}C_k\,({}^kC_r)({}^rC_J)\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^J(Z_2-1)^J\left((Z_1-1)+\frac{\theta\mu_1(Z_2-1)}{\mu_2-\mu_1}\right)^{r-J}\left(\frac{1-e^{-[J\mu_2+(r-J)\mu_1]t}}{J\mu_2+(r-J)\mu_1}\right)\right] \quad (1)$$

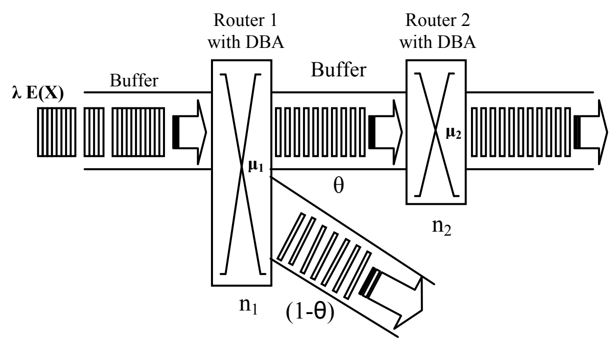

Fig.1 Communication network with dynamic bandwidth allocation and bulk arrivals having modified phase type transmission

3. Performance Measures of the Proposed Communication Network

The probability generating function of the first buffer size distribution is

$$P(Z_1,t)=\exp\left[\lambda\sum_{k=1}^{\infty}\sum_{r=1}^{k}C_k\,{}^kC_r\left(Z_1-1\right)^r\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right] \quad (2)$$

The probability that the first buffer is empty is

$$P_{0.}(t)=\exp\left[\lambda\sum_{k=1}^{\infty}\sum_{r=1}^{k}C_k\,{}^kC_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right] \quad (3)$$

The mean number of packets in the first buffer is

$$L_1=\frac{\lambda}{\mu_1}\left[\sum_{k=1}^{\infty}C_k.k\left(1-e^{-\mu_1 t}\right)\right] \quad (4)$$

The utilization of the first node is

$$U_1=1-P_{0.}(t)$$

$$=1-\exp\left[\lambda\sum_{k=1}^{\infty}\sum_{r=1}^{k}C_k\,{}^kC_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right] \quad (5)$$

Throughput of the first node is

$$Thp_1 = \mu_1 U_1 = \mu_1 \left[1 - \exp\left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^{k} C_k\, {}^kC_r \left(-1\right)^{3r} \frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1} \right] \right] \quad (6)$$

The average delay in the first buffer is

$$W(N_1) = \frac{L_1}{Thp_1} = \frac{\frac{\lambda}{\mu_1}\left[\sum_{k=1}^{\infty} C_k . k\left(1-e^{-\mu_1 t}\right)\right]}{\mu_1\left[1-\exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k} C_k\, {}^kC_r \left(-1\right)^{3r} \frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right]\right]} \quad (7)$$

The variance of the number of packets in the first buffer is

$$Var\left(N_1\right) = E\left[N_1^2 - N_1\right] + E\left[N_1\right] - \left(E\left[N_1\right]\right)^2$$

$$= \frac{\lambda}{2\mu_1}\left[\sum_{k=1}^{8} C_k k(k-1)\left(1-e^{-2\mu_1 t}\right)\right] + \frac{\lambda}{\mu_1}\left[\sum_{k=1}^{\infty} C_k k\left(1-e^{-\mu_1 t}\right)\right] \quad (8)$$

The coefficient of variation of the number of packets in the first buffer is

$$cv\left(N_1\right) = \frac{\sqrt{Var\left(N_1\right)}}{L_1} \quad (9)$$

The probability generating function of the second buffer size distribution is

$$P(Z_2,t) = \exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r} (-1)^{2r-J} C_k ({}^kC_r)({}^rC_J)\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r .(Z_2-1)^r \frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \quad (10)$$

The probability that the second buffer is empty is

$$P_0(t) = \exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r} (-1)^{3r-J} C_k ({}^kC_r)({}^rC_J)\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r \frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \quad (11)$$

The mean number of packets in the second buffer is

$$L_2 = \frac{\lambda\theta}{\mu_2}\left[\sum_{k=1}^{\infty} C_k . k\left[\left(1-e^{-\mu_2 t}\right) + \frac{\mu_2}{\mu_2-\mu_1}\left(e^{-\mu_2 t} - e^{-\mu_1 t}\right)\right]\right] \quad (12)$$

The utilization of the second node is

$$U_2 = 1 - P_0(t) = 1 - \exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r} (-1)^{3r-J} C_k ({}^kC_r)({}^rC_J) .\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r \frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \quad (13)$$

Throughput of the second node is

$$Thp_2 = \mu_2 U_2 = \mu_2\left[1-\exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r} (-1)^{3r-J} C_k ({}^kC_r)({}^rC_J) \left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r \frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right]\right] \quad (14)$$

The average delay in the second buffer is

$$W(N_2) = \frac{L_2}{Thp_2} = \frac{\frac{\lambda}{\mu_2}\left[\sum_{k=1}^{\infty} C_k . k\left[\left(1-e^{-\mu_2 t}\right) + \frac{\mu_2}{\mu_2-\mu_1}\left(e^{-\mu_2 t} - e^{-\mu_1 t}\right)\right]\right]}{\mu_2\left[1-\exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r} (-1)^{3r-J} C_k ({}^kC_r)({}^rC_J)\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r \frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right]\right]} \quad (15)$$

The variance of number of packets in the second buffer is

$$Var\left(N_2\right) = E\left[N_2^2 - N_2\right] + E\left[N_2\right] - \left(E\left[N_2\right]\right)^2$$

$$= \left\{\lambda\left(\sum_{k=1}^{\infty} C_k . k(k-1)\right)\left(\frac{\theta\mu_1}{\mu_1-\mu_2}\right)^2\left[\left(\frac{1-e^{-2\mu_1 t}}{2\mu_1}\right) - 2\left(\frac{1-e^{-(\mu_1+\mu_2)t}}{\mu_1+\mu_2}\right) + \left(\frac{1-e^{-2\mu_2 t}}{2\mu_2}\right)\right]\right\} + \left\{\lambda\theta\sum_{k=1}^{\infty} kC_k\left[\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right) - \left(\frac{e^{-\mu_2 t} - e^{-\mu_1 t}}{\mu_1-\mu_2}\right)\right]\right\} \quad (16)$$

The coefficient of variation of the number of packets in the second buffer is

$$cv\left(N_2\right) = \frac{\sqrt{Var\left(N_2\right)}}{L_2} \quad (17)$$

The probability that the network is empty is

$$P_{00}(t) = \exp\left[\lambda \sum_{k=1}^{\infty}\sum_{r=1}^{k}\sum_{J=0}^{r} (-1)^{2r} C_k ({}^kC_r)({}^rC_J)\left(\theta\mu_1\right)^J \frac{\left[(1-\theta)\mu_1-\mu_2\right]^{r-J}}{\left(\mu_2-\mu_1\right)^r} \frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \quad (18)$$

The mean number of packets in the entire network is

$$L_N = L_1 + L_2 \quad (19)$$

4. Particular Case when the Batch Size is Uniformly Distributed

For obtaining the performance of a communication network, it is needed to know the functional form of the probability mass function of the number of packets that a message can be converted (C_k). Let the batch size of packets follows a uniform (rectangular) distribution. Then, The probability distribution of the batch size of packets in a message is

$$C_k = \frac{1}{(b-a)+1} \quad \text{for} \quad k=a, a+1,\ldots, b.$$

The mean number of packets in a message is $\left(\frac{a+b}{2}\right)$ and its variance is $\frac{1}{12}\left[(b-a+1)^2-1\right]$.

Substituting the value of C_k in Eq. (1), we get the joint probability generating function of the number of packets in both the buffers is

$$P(Z_1,Z_2,t)=\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\left(\frac{1}{b-a+1}\right)({}^kC_r)({}^rC_J)\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^J(Z_2-1)^J \left((Z_1-1)+\frac{\theta\mu_1(Z_2-1)}{\mu_2-\mu_1}\right)^{r-J}\frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \tag{20}$$

The probability generating function of the first buffer size distribution is

$$P(Z_1,t)=\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\left(\frac{1}{b-a+1}\right){}^kC_r\left(Z_1-1\right)^r\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right] \tag{21}$$

The probability that the first buffer is empty is

$$P_{0.}(t)=\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\left(\frac{1}{b-a+1}\right){}^kC_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right] \tag{22}$$

The mean number of packets in the first buffer is

$$L_1=\frac{\lambda(a+b)}{2\mu_1}\left[1-e^{-\mu_1 t}\right] \tag{23}$$

The utilization of the first node is

$$U_1=1-P_{0.}(t)=1-\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\left(\frac{1}{b-a+1}\right){}^kC_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right] \tag{24}$$

Throughput of the first node is

$$Thp_1=\mu_1.U_1=\mu_1\left[1-\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\left(\frac{1}{b-a+1}\right){}^kC_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right]\right] \tag{25}$$

The average delay in the first buffer is

$$W(N_1)=\frac{L_1}{Thp_1}=\frac{\frac{\lambda(a+b)}{2\mu_1}\left[1-e^{-\mu_1 t}\right]}{\mu_1\left[1-\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\left(\frac{1}{b-a+1}\right){}^kC_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\mu_1 t}\right)}{r\mu_1}\right]\right]} \tag{26}$$

The variance of the number of packets in the first buffer is

$$Var(N_1)=E\left[N_1^2-N_1\right]+E\left[N_1\right]-\left(E\left[N_1\right]\right)^2$$

$$=\lambda\left[\sum_{k=a}^{b}\left(\frac{1}{b-a+1}\right)k(k-1)\left(\frac{1-e^{-2\mu_1 t}}{2\mu_1}\right)+\sum_{k=a}^{b}\left(\frac{1}{b-a+1}\right)k\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right] \tag{27}$$

The coefficient of variation of the number of packets in the first buffer is

$$cv(N_1)=\frac{\sqrt{Var(N_1)}}{L_1} \tag{28}$$

The probability generating function of the second buffer size distribution is

$$P(Z_2,t)=\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{2r-J}\left(\frac{1}{b-a+1}\right)({}^kC_r)({}^rC_J) .\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r(Z_2-1)^r\frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \tag{29}$$

The probability that the second buffer is empty is

$$P_{.0}(t)=\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\left(\frac{1}{b-a+1}\right)({}^kC_r)({}^rC_J) \left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r\frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \tag{30}$$

The mean number of packets in the second buffer is

$$L_2=\frac{\lambda\theta(a+b)}{2\mu_2}\left[\left(1-e^{-\mu_2 t}\right)+\frac{\mu_2}{\mu_2-\mu_1}\left(e^{-\mu_2 t}-e^{-\mu_1 t}\right)\right] \tag{31}$$

The utilization of the second node is

$$U_2=1-P_{.0}(t)=1-\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\left(\frac{1}{b-a+1}\right)({}^kC_r)({}^rC_J) \left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r\frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right] \tag{32}$$

Throughput of the second node is

$$Thp_2=\mu_2.U_2=\mu_2\left[1-\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\left(\frac{1}{b-a+1}\right)({}^kC_r)({}^rC_J) \left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r\frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right]\right] \tag{33}$$

The average delay in the second buffer is

$$W(N_2)=\frac{L_2}{Thp_2}$$

$$=\frac{\frac{\lambda\theta(a+b)}{2\mu_2}\left[\left(1-e^{-\mu_2 t}\right)+\frac{\mu_2}{\mu_2-\mu_1}\left(e^{-\mu_2 t}-e^{-\mu_1 t}\right)\right]}{\mu_2\left[1-\exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{J=0}^{r}(-1)^{3r-J}\left(\frac{1}{b-a+1}\right)({}^kC_r)({}^rC_J)\left(\frac{\theta\mu_1}{\mu_2-\mu_1}\right)^r\frac{\left(1-e^{-[J\mu_2+(r-J)\mu_1]t}\right)}{J\mu_2+(r-J)\mu_1}\right]\right]} \tag{34}$$

The variance of number of packets in the second buffer is

$$\mathrm{Var}(N_2) = E\left[N_2^2 - N_2\right] + E\left[N_2\right] - \left(E\left[N_2\right]\right)^2$$

$$= \left\{\lambda\left(\sum_{k=1}^{\infty}\frac{1}{(b+a)-1}.k(k-1)\right)\left(\frac{\theta\mu_1}{\mu_1-\mu_2}\right)^2\left[\left(\frac{1-e^{-2\mu_1 t}}{2\mu_1}\right)-2\left(\frac{1-e^{-(\mu_1+\mu_2)t}}{\mu_1+\mu_2}\right)+\left(\frac{1-e^{-2\mu_2 t}}{2\mu_2}\right)\right]\right\}$$

$$+\left\{\frac{\lambda\theta(a+b)}{2}\left[\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)-\left(\frac{e^{-\mu_2 t}-e^{-\mu_1 t}}{\mu_1-\mu_2}\right)\right]\right\} \tag{35}$$

The coefficient of variation of the number of packets in the second buffer is

$$cv(N_2) = \frac{\sqrt{\mathrm{Var}(N_2)}}{L_2} \tag{36}$$

The probability that the network is empty is

$$P_{00}(t) = \exp\left[\lambda\sum_{k=a}^{b}\sum_{r=1}^{k}\sum_{j=0}^{r}(-1)^{2r}\left(\frac{1}{b-a+1}\right)({}^{k}C_r)({}^{r}C_J)(\theta\mu_1)^J \frac{((1-\theta)\mu_1-\mu_2)^{r-j}}{(\mu_2-\mu_1)^r}\left(\frac{\left(1-e^{[J\mu_2+(r-J)\mu_1]t}\right)}{j\mu_2+(r-j)\mu_1}\right)\right] \tag{37}$$

The mean number of packets in the network is

$$L_N = L_1 + L_2 \tag{38}$$

5. Performance Evaluation of the Communication Network

The performance of the proposed network is discussed through numerical illustration. Different values of the parameters are considered for bandwidth allocation and arrival of packets. After interacting with the technical staff at the Internet providing station, it is considered that the message arrival rate (λ) varies from $1x10^4$ messages/sec to $5x10^4$ messages/sec. Then each message is converted into packets of size 53 bytes. The number of packets that can be converted into a message varies from 1 to 25. Hence, the number of arrivals of packets to the buffer are in batches of random size. The batch size is assumed to follow uniform distribution with parameters (a, b). The transmission rate of node $1(\mu_1)$ varies from $4x10^4$ packets/sec to $9x10^4$ packets/sec. The probability that the packets are forwarded to the buffer connected to the second node is θ varies from 0.1 to 0.9 with 0.2 interval and the probability of the packets that may terminate at node 1 is $(1-\theta)$. The packets leave the second node with a transmission rate (μ_2) which varies from $7x10^4$ packets/sec to $12x10^4$ packets/sec. In both the nodes, dynamic bandwidth allocation is considered i.e. the transmission rate of each packet depends on the number of packets in the buffer connected to it at that instant.

From equations (22), (30) and (37), the probability of network emptiness and different buffers emptiness are computed for different values of t, a, b λ, θ, μ_1, μ_2. It is observed that the probability of emptiness of the communication network and the two buffers are highly sensitive with respect to changes in time. As time (t) varies from 0.1 second to 1 second, the probability of emptiness in the network reduces from 0.17064 to 0.00083 when other parameters are fixed at (5, 25, 2, 0.5, 4, 8) for (a, b λ, θ, μ_1, μ_2). Similarly, the probability of emptiness of the two buffers reduce from 0.81874 to 0.22616 and 0.82233 to 0.17702 for node 1 and node 2 respectively. The decrease in node 1 is more rapid when compared with node 2.

When the batch distribution parameter (a) varies from $1x10^4$ packets/sec to $5x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.00212 to 0.00083 when other parameters are fixed at (1, 25, 2, 0.5, 4, 8) for (t, b, λ, θ, μ_1, μ_2). The same phenomenon is observed with respect to the first and second nodes. The probability of emptiness of the first and second buffers decrease from 0.25325 to 0.22616 and 0.22295 to 0.17702 respectively.

When the batch size distribution parameter (b) varies from $10x10^4$ packets/sec to $30x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.02875 to 0.00025 when other parameters are fixed at (1, 5, 2, 0.5, 4, 8) for (t, a, λ, θ, μ_1, μ_2). The same phenomenon is observed with respect to the first and second node. The probability of emptiness of the first and second buffers decrease from 0.28643 to 0.21559 and 0.42073to 0.13264 respectively.

The influence of arrival of messages on system emptiness is also studied. As the arrival rate (λ) varies from $0.5x10^4$ messages/sec to $2.5x10^4$ messages/sec, the probability of emptiness of the network decreases from 0.16955 to0.00014 when other parameters are fixed at (1, 5, 25, 0.5, 4, 8) for (t, a, b, θ, μ_1, μ_2). The same phenomenon is observed with respect to the first and second nodes. This decline is more in first node and moderate in the second node. When the probability that the number of packets arrive at the buffers connected to the node 2, (θ) varies from 0.1 to 0.9, the probability of emptiness of the network and the second buffer decrease from 0.00225 to 0.00032 and 0.69887 to 0.04908 respectively and the probability of emptiness of the first buffer remains constant when other parameters remain fixed at (1, 5, 25, 2, 4, 8) for (t, a, b, λ, μ_1, μ_2).

When the transmission rate of node $1(\mu_1)$ varies from $3.5x10^4$ packets/sec to $5.5x10^4$ packets/sec, the probability of emptiness of the network and the first buffer increase from 0.00042 to 0.00347 and 0.19894 to 0.31598 respectively and the probability of emptiness of the second buffer decreases from 0.18161 to 0.17202 when other parameters remain fixed at (1, 5, 25, 2, 0.5, 8) for (t, a, b, λ, θ, μ_2). When the transmission rate of node 2 (μ_2) varies from $6\text{x}10^4$ packets/sec to $10\text{x}10^4$ packets/sec, the probability of emptiness of the network and the second buffer increase from 0.00054 to 0.00107 and 0.10484 to 0.24604 respectively when other parameters remain fixed at (1, 5, 25, 2, 0.5, 4) for (t, a, b, λ, θ, μ_1)..

From the equations (23), (24), (31), (32) and (38), the mean number of packets and the utilization of the network are computed for different values of t, a, b, λ, θ, μ_1, μ_2. The values for mean number of packets in the two buffers and mean delays are given in Table.1 and the relationship between mean number of packets in the two buffers and the input parameters t, a, b, λ, θ, μ_1, μ_2 is shown in Figures 2, 3, 4 and 5.

Table 1: Values of mean number packets and mean delays in the two buffers

t*	a	b	$\lambda^{\#}$	θ	$\mu_1^{\$}$	$\mu_2^{\$}$	L_1	L_2	W(N1)	W(N2)
0.1	1	5	3	0.5	4	7	0.74178	0.06302	0.74500	0.15320
0.3	1	5	3	0.5	4	7	1.57231	0.29603	0.74856	0.17318
0.5	1	5	3	0.5	4	7	1.94550	0.46574	0.75803	0.18615
0.8	1	5	3	0.5	4	7	2.15829	0.58488	0.76697	0.19518
1.0	1	5	3	0.5	4	7	2.20879	0.61617	0.76955	0.19756
1	**1**	25	2	0.5	4	8	6.38095	1.56602	2.13623	0.25192
1	**2**	25	2	0.5	4	8	6.62637	1.62625	2.19156	0.25748
1	**3**	25	2	0.5	4	8	6.97179	1.68648	2.25159	0.26308
1	**4**	25	2	0.5	4	8	7.11721	1.74671	2.31425	0.26874
1	**5**	25	2	0.5	4	8	7.36263	1.80695	2.37859	0.27445
1	5	**10**	2	0.5	4	8	3.68132	0.90347	1.28975	0.19496
1	5	**15**	2	0.5	4	8	4.90842	1.20463	1.65526	0.21991
1	5	**20**	2	0.5	4	8	6.13553	1.50579	2.01797	0.24644
1	5	**25**	2	0.5	4	8	7.36263	1.80695	2.37859	0.27445
1	5	**30**	2	0.5	4	8	8.58974	2.10810	2.73766	0.30381
1	5	25	**0.5**	0.5	4	8	1.84066	0.45174	1.48252	0.16071
1	5	25	**1.0**	0.5	4	8	3.68132	0.90347	1.75487	0.19496
1	5	25	**1.5**	0.5	4	8	5.52197	1.35521	2.05415	0.23298
1	5	25	**2.0**	0.5	4	8	7.36263	1.80695	2.37859	0.27445
1	5	25	**2.5**	0.5	4	8	9.20329	2.25868	2.72596	0.31896
1	5	25	2	**0.1**	4	8	7.36263	0.36139	2.37859	0.15001
1	5	25	2	**0.3**	4	8	7.36263	1.08417	2.37859	0.20773
1	5	25	2	**0.5**	4	8	7.36263	1.80695	2.37859	0.27445
1	5	25	2	**0.7**	4	8	7.36263	2.52972	2.37859	0.34833
1	5	25	2	**0.9**	4	8	7.36263	3.25250	2.37859	0.42755
1	5	25	2	0.5	**2.5**	8	11.01498	1.65142	5.22978	0.25878
1	5	25	2	0.5	**3.0**	8	9.50213	1.72602	3.84195	0.26613
1	5	25	2	0.5	**3.5**	8	8.31259	1.77483	2.96485	0.27108
1	5	25	2	0.5	**4.0**	8	7.36263	1.80695	2.37859	0.27445
1	5	25	2	0.5	**4.5**	8	6.59261	1.82820	1.96811	0.27677
1	5	25	2	0.5	4	**5**	7.36263	2.80612	2.37859	0.60400
1	5	25	2	0.5	4	**6**	7.36263	2.37503	2.37859	0.44220
1	5	25	2	0.5	4	**7**	7.36263	2.05388	2.37859	0.34151
1	5	25	2	0.5	4	**8**	7.36263	1.80695	2.37859	0.27445
1	5	25	2	0.5	4	**9**	7.36263	1.61188	2.37859	0.22737

* = seconds, # =Multiples of 10,000 Messages/sec,
$= Multiples of 10,000 Packets/sec

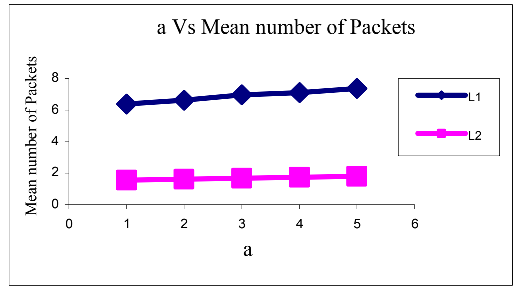

Fig. 2 Batch size distribution parameter **a** Vs Mean number of Packets in the buffers 1 and 2

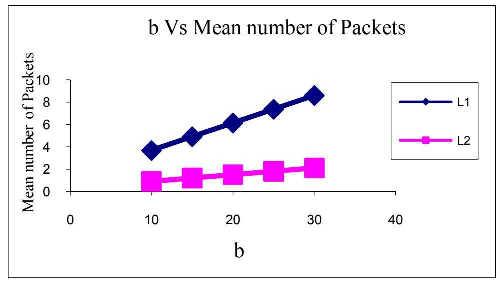

Fig. 3 Batch size distribution parameter **b** Vs Mean number of Packets in the buffers 1 and 2

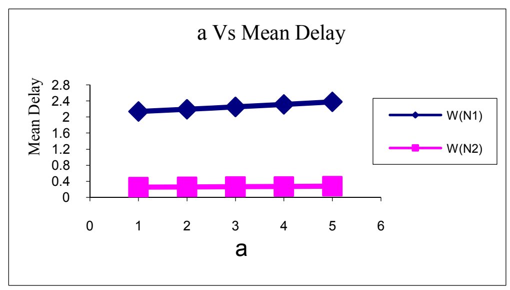

Fig. 4 Batch size distribution parameter **a** Vs Mean delay in the buffers 1 and 2

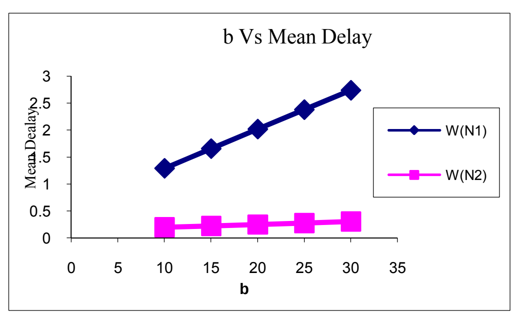

Fig. 5 Batch size distribution parameter **b** Vs Mean delay in the buffers 1 and 2

It is observed that after 0.1 seconds, the first buffer is having on an average of 7417.8 packets, after 0.3 seconds it rapidly raised to an average of 15723.1 packets. After 1 second, the first buffer is containing an average of

22087.9 packets and there after the system stabilizes and the average number of packets remains to be the same for fixed values of other parameters (5, 25, 2, 0.5, 4, 8) for (a, b, λ, θ, μ_1, μ_2). It is also observed that as time (t) varies from 0.1second to 1 second, average content of the second buffer and the network increase from 630.2 packets to 6161.7 packets and from 8048 packets to 28249.6 packets respectively.

As the batch size distribution parameter (a) varies from 1 to 5, the first buffer, second buffer and the network average content increase from 63809.5 packets to 73626.3 packets, 15660.2 packets to 18069.5 packets and 79469.7 packets to 91695.8 packets respectively when other parameters remain fixed. As the batch size distribution parameter (b) varies from 10 to 30, the first buffer, second buffer and the network average content increase from 36813.2 packets to 85897.4 packets, 9034.7 packets to 21081 packets and 45847.9 packets to 106978.4 packets respectively when other parameters remain fixed.

As the arrival rate of messages (λ) varies from $0.5\text{x}10^4$ messages/sec to $2.5\text{x}10^4$ messages/sec, the first buffer, second buffer and the network average content increase from 18406.6 packets to 92032.9 packets, 4517.4 packets to 22586.8 packets and 22923.9 packets to 114619.7 packets respectively when other parameters remain fixed at (1, 5, 25, 0.5, 4, 8) for (t, a, b, θ, μ_1, μ_2). When the joining probability of the number of packets arrive at the buffers connected to the second node (θ) varies from 0.1 to 0.9, the average content of the second buffer and the network increase from 3613.9 to 32525 and 77240.2 to 106151.3 respectively and the average content of the first buffer remain constant when other parameters remain fixed at (1, 5, 25, 2, 4, 8) for (t, a, b, λ, μ_1, μ_2).

As the transmission rate of node 1 (μ_1) varies from $2.5\text{x}10^4$ packets/sec to $4.5\text{x}10^4$ packets/sec, the first buffer and the network average content decrease from 110149.8 packets to 65926.1 packets and from 126664 packets to 84208.1 packets, the second buffer average content increases from 16514.2 packets to 18282 packets respectively when other parameters remain fixed at (1, 5, 25, 2, 0.5, 8) for (t, a, b, λ, θ, μ_2). As the transmission rate of node 2 (μ_2) varies from $5\text{x}10^4$ packets/sec to $9\text{x}10^4$ packets/sec, the second buffer and the network average content decrease from 28061.2 packets to 16118.8 packets and from 101687.5 packets to 89745.2 packets respectively when other parameters remain fixed at (1, 5, 25, 2, 0.5, 4) for (t, a, b, , λ, θ, μ_1).

It is revealed that the utilization characteristics are similar to mean number of packet characteristics. Here also, as the time (t) and the arrival rate of messages (λ) increase, the utilization of both the nodes increase for fixed values of the other parameters. As the batch size distribution parameters (a) and (b) increase, the utilization of both the nodes increase when the other parameters are fixed at (1, 2, 0.5, 4, 8) for (t, λ, θ, μ_1, μ_2). When the probability that the number of packets arrive at the buffers connected to the node 2 (θ) varies from 0.1 to 0.9, the utilization of the second node increases while the utilization of the first node remain constant when other parameters are fixed at (1, 5, 25, 2, 4, 8) for (t, a, b, λ, μ_1 μ_2).

It is also noticed that as the transmission rate of node 1 (μ_1) increases, the utilization of the second node increases while the utilization of the first node decreases when other parameters remain fixed. As the transmission rate of node 2 (μ_2) increases, the utilization of the second node decreases when other parameters remain fixed. Therefore in the communication network, dynamic bandwidth allocation strategy is necessary for control of congestion, efficient utilization of different nodes and to maintain satisfactory quality of service (QoS) with optimum speed.

From the equations (25), (26), (33)and (34) the throughput and the average delay of the network are computed for different values of t, a, b, λ, θ, μ_1, μ_2 and the values of mean delays are given in Table 1. It is observed that as the time (t) increases from 0.1second to 1 seconds, the throughput of the first and second nodes increase from 9956.8 packets to 28702.4 packets and from 4113.6 packets to 31189.1 packets respectively, when other parameters remain fixed at (1, 5, 3, 0.5, 4, 7) for (a, b, λ, θ, μ_1, μ_2). As the batch size distribution parameter (a) varies from 1 to 5, the throughput of the first and second nodes increase from 29870.1 packets to 30953.8 packets and 62164.4 packets to 65838.6 packets respectively when other parameters remain fixed at (1, 25, 2, 0.5, 4, 8) for (t, b, λ, θ, μ_1, μ_2). As the batch size distribution parameter (b) varies from 10 to 30, the throughput of the first and second nodes increase from 28542.8 packets to 31376.2 packets and 46341.3 packets to 69388.6 packets respectively when other parameters remain fixed at (1, 5, 2, 0.5, 4, 8) for (t, a, λ, θ, μ_1, μ_2).

As the arrival rate (λ) varies from $0.5\text{x}10^4$ messages/sec to $2.5\text{x}10^4$ messages/sec, it is observed that the throughput of the first and second nodes increase 12415.7 packets to 33761.7 packets and from 28108.8 packets to 70814.4 packets respectively, when the other parameters remain fixed at (1, 5, 25, 0.5, 4, 8) for (t, a, b, θ, μ_1, μ_2). When the joining probability of the number of packets arrive to the buffer connected to the second node (θ) varies from 0.1 to 0.9, the throughput of the second node increases from 24090.4 packets to 76073.8 packets while the throughput of the first node remain constant when other parameters are fixed at (1, 5, 25, 2, 4, 8) for (t, a, b, λ, μ_1, μ_2).

As the transmission rate of node 1(μ_1) varies from $2.5\text{x}10^4$ packets/sec to $4.5\text{x}10^4$ packets/sec, the throughput of first and second nodes increase from 21062.0 packets to

33497.1 packets and from 63815.6 packets to 66054.2 packets respectively, when other parameters remain fixed at (1, 5, 25, 2, 0.5, 8) for (t, a, b, λ, θ, μ_2). As the transmission rate of node 2 (μ_2) varies from $5x10^4$ packets/sec to $9x10^4$ packets/sec, the throughput of second node increases from 46459.2 packets to 70894.1 packets, when other parameters remain fixed at (1, 5, 25, 2, 0.5, 4) for (t, a, b, λ, θ, μ_1).

From Table 1, it is also observed that as time (t) varies from 0.1 second to 1 second, the mean delay of the first and second buffers increase from 74.500 µs to 76.955 µs and 15.32µs to 19.756µs respectively, when other parameters remain fixed (1, 5, 3, 0.5, 4, 7) for (a, b, λ, θ, μ_1, μ_2). As the batch size distribution parameter (a) varies from 1 to 5, the mean delay of the first and second buffers increase from 213.623µs to 237.859µs and 25.192µs to 27.445µs respectively when other parameters remain fixed at (1, 25, 2, 0.5, 4, 8) for (t, b, λ, θ, μ_1, μ_2). As the batch size distribution parameter (b) varies from 10 to 30, the mean delay of the first and second buffers increase from 128.975µs to 273.766µs and 19.496µs to 30.381µs respectively when other parameters remain fixed at (1, 5, 2, 0.5, 4, 8) for (t, a, λ, θ, μ_1, μ_2).

When the arrival rate (λ) varies from $0.5x10^4$ messages/sec to $2.5x10^4$ messages/sec, the mean delay of the first and second buffers increase from 148.252µs to 272.596µs and from 16.071µs to 31.896µs respectively, when other parameters remain fixed (1, 5, 25, 0.5, 4, 8) for (t, a, b, θ, μ_1, μ_2). As the joining probability of the number of packets arrive to the buffer connected to the second node (θ) varies from 0.1 to 0.9, the mean delay of the first node increases from 15.001µs to 42.755µs when the other parameters are fixed at (1, 5, 25, 2, 4, 8) for (t, a, b, λ, μ_1, μ_2). As the transmission rate of node 1 (μ_1) varies from $2.5x10^4$ packets/sec to $4.5x10^4$ packets/sec, the mean delay of the first buffer decreases from 522.978µs to 196.811µs and the mean delay of the second buffer increases from 25.878µs to 27.677µs, when other parameters remain fixed at (1, 5, 25, 2, 0.5, 8) for (t, a, b, λ, θ,μ_2). As the transmission rate of node 2 (μ_2) varies from $5x10^4$ packets/sec to $9x10^4$ packets/sec, the mean delay of the second buffer decreases from 60.4µs to 22.737µs, when other parameters remain fixed at (1, 5, 25, 2, 0.5, 4) for (t, a, b, λ, θ, μ_1).

If the variance increases then the burstness of the buffers will be high. Hence, the parameters are to be adjusted such that the variance of the buffer content in each buffer must be small. The coefficient of variation of the number of packets in each buffer will helps us to understand the consistency of the traffic flow through buffers. If coefficient variation is large then the flow is inconsistent and the requirement to search the assignable causes of high variation. It also helps us to compare the smooth flow of packets in two or more nodes. The variance of the number of packets in each buffer, the coefficient of variation of the number of packets in first and second buffers is computed. It is observed that, as the time (t) and the batch size distribution parameter (a) increase, the variance of first and second buffers increased and the coefficient of increased and the coefficient of variation of the number of packet in the first and second buffers decreased. As the batch size distribution parameter (b) increases, the variance of first and second buffers increased the coefficient of variation of the number of packets in the first buffer increased and for the second buffer it is decreased. As the joining probability of the number of packets arrive to the buffer connected to the second node (θ) varies from 0.1 to 0.9, the variance of the number of packets in the second buffer increased and the coefficient of variation of the number of packets in the second buffer decreased when the other parameters are fixed at (1, 5, 25, 2, 4, 8) for (t, a, b, λ, μ_1, μ_2).

From this analysis, it is observed that the dynamic bandwidth allocation strategy ha a significant influence on all performance measures of the network. It is further observed that the performance measures are highly sensitive towards smaller values of time. Hence, it is optimal to consider dynamic bandwidth allocation and evaluate the performance under transient conditions. It is also to be observed that the congestion in buffers and delays in transmission can be reduced to a minimum level by adopting dynamic bandwidth allocation. This phenomenon has a vital bearing on quality of transmission (service).

6. Sensitivity Analysis

Sensitivity analysis of the model is performed with respect to t, a, b, λ, μ_1, μ_2 on the mean number packets in the first and second buffers, the mean number of packets in the network, the mean delay in the first and second buffers, the utilization and throughput of the first and second nodes. The following data has been considered for the sensitivity analysis.
t = 0.1 sec, a=5 $x10^4$ packets/sec, b=25 $x10^4$ packets/sec $\lambda = 2x10^4$ packets/sec, $\mu_1 = 4x10^4$ packets/sec, $\mu_2 = 8x10^4$ packets/sec and θ=0.5

The performance measures of the model are computed with variation of -15%, -10%, 0%, +5%, +10% and +15% on the input parameters t, λ, θ, μ_1, μ_2 and -60%, -40%, -20%, 0%, +20%, +40% and +60% on the batch size distribution parameters **a** and **b** to retain them as integers. The performance measures are highly affected by time (t) and the batch size distribution of arrivals. As (t) increases to 15% the average number of packets in the two buffers and total network increase along with the average delays in buffers. Similarly, as arrival rate of messages (λ) increases by 15% the average number of packets in the two buffers and total network increases along with the average delays in buffers. The mean delays and mean content of the buffers are decreasing function of these parameters. Overall analysis of the parameters reflects that dynamic bandwidth allocation strategy for congestion control tremendously reduces the delays in communication and improves voice quality by reducing burstness in buffers.

7. Conclusions

In this paper, a two node tandem communication network with dynamic bandwidth allocation having bulk arrivals and modified phase type transmission is developed and analyzed. Here, the dynamic bandwidth allocation (DBA) strategy insists for the instantaneous change in rate of transmission of the nodes depending upon the content of the buffers connected to them. The emphasis of this communication network is on the bulk or batch arrivals of packets to the initial node with random size. The performance of the statistical multiplexing is measured by approximating the arrival process with a compound Poisson process and the transmission process with Poisson process. This is chosen such that the statistical characteristics of the communication network identically matches with Poisson process and uniform distribution. A communication network model with modified phase type transmission is more close to the practical transmission behavior in most of the communication systems. The sensitivity of the network with respect to input parameters is studied through numerical illustrations. It is observed that the dynamic bandwidth allocation strategy and the parameters of bulk size distribution have a significant impact on the performance measures of the network. It is further observed that transient analysis of the Communication network will approximate the performance measures more close to the practical situation. This network can also be extended to the multi node communication networks. It is interesting to note that this Communication network model includes some of the earlier Communication network model given by P.S.Varma and K.Srinivasa Rao (2007)

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