IJCSI International Journal of Computer Science Issues, Vol. 7, Issue 4, No 4, July 2010
ISSN (Online): 1694-0784
ISSN (Print): 1694-0814


# De Bruijn Pseudo Random Sequences Analysis For Modeling of Quaternary Modulation Formats

**Hadjira Badaoui[1], Yann Frignac[2], Petros Ramantanis[2], Badr Eddine Benkelfat[2] and Mohammed Feham[1]**

**[1] Laboratoire STIC, Département de Génie Electrique**
**Faculté de Technologie, Université Abou-Bekr Belkaïd -Tlemcen**
**BP 230, Pôle Chetouane, 13000 Tlemcen- Algeria**

**[2] Institut TELECOM & Management SudParis,**
**9, rue Charles Fourier - 91011 Evry Cedex - France**

**Abstract**

This paper refers to the generation and analysis of data sequences called De Bruijn quaternary sequences for modeling of quaternary modulation formats witch show very interesting properties. In particular, we will focus on the spectrum, autocorrelation function analysis and statistical properties. A De Bruijn quaternary sequence analysis for a length of 1024 bits has been made to confirm its properties. The simulations results are then presented and discussed.
*Key words*: ***De Bruijn sequence, PRBS sequence, PRQS sequence, irreducible primitive polynomial, autocorrelation function.***

## 1. Introduction

To estimate the performance of an optical transmission system, we have to test it experimentally or through numerical simulations for the principle of the latter lies in the numerical solution of nonlinear equation that describes the Schrödinger propagation of a light wave. But this equation is not solvable analytically, except for special cases (eg solitons). The basic pattern of an optical signal transmitted in fiber is called a symbol. At the beginning of transmission, the data sent by transmitter to receiver do no damage. During propagation, due to the interaction of different effects (chromatic dispersion, nonlinearities, noise, etc.) some transmitted symbols are more degraded than others. So as not to overestimate or underestimate the performance of the sequence data that is supposed to test the performance of the system should contain as many cases eventually degraded by the transmission case that not much affected [1-2]. The sequences of the most realistic (or rather the ideal case) are random sequences of infinite length. Simulating an optical transmission with such a sequence then requires a very long time to load and large memory space, which can easily exceed the power of the machine. So the best solution is the use of De Bruijn binary sequences [3].

In numerical simulations, we solve the equation of nonlinear Schrödinger method using the Split-Step Fourier Method (SSFM) [4], a method that requires many parts of the spectral time domain. To accelerate this approach, we use the algorithm of fast Fourier transform (FFT) which, however, requires a number of symbol sequences in power of 2[5]. So, for the propagation simulations over optical fiber, the PRBS sequences have a disadvantage: their length can not be a power of 2[6]. To overcome this disadvantage, it often makes PRBS sequences (pseudo random quaternary sequence) to De Bruijn sequences, the latter being well suited for this type of simulation.

## 2. De Bruijn Sequences

These sequences are defined so that they contain all the arrangements of m possible symbols. They have a length of $q^m$ symbols, where q is the number of different symbols in our alphabet and m is the number of cells constituting the LSFR register.

The whole De Bruijn sequence is constructed by two radically different ways, either:

- By use of a connected graph and balanced says “Euler graph”[7].
- Using a pseudo random binary sequence with the addition of a symbol.

So with the first method we can obtain all possible deBruijn sequences, we will focus only generate a deBruijn sequence by the second technique. A pseudo-random sequence involves a deBruijn sequence by adding a logical zero "0" in the longest train of "0". The reverse is not always true. That is to say, a deBruijn sequence does not always result in a pseudo-random binary sequence by removing a "0". Therefore, we can have pairs of sequences of length but also possess the pseudo-random properties.

Fig. 1 shows an example of a De Bruijn sequence of a 16 bits length. In this sequence one can be distinguished $2^4$ arrangements of 4 bits.

$$\left\{\begin{matrix}(0000)\\(0001)\\(0010)\\(0011)\\(0100)\\(0101)\\(0110)\\(0111)\\(1000)\\(1001)\\(1010)\\(1011)\\(1100)\\(1101)\\(1110)\\(1111)\end{matrix}\right. \in \text{De Bruijn sequence } (0000100110101111)$$

Fig. 1 The whole sub-sequence of 4 bits

## 3. De Bruijn Sequence Generation

A binary De Bruijn sequence is obtained from a PRBS sequence. The theory of pseudo-random binary generation based on the Galois field properties. They have an odd length, equal to 2m-1 bits. The pseudo random binary sequences can be constructed using a linear shift feedback register (LFSR) by one or more exclusive-or gates as shown in Fig. 2.

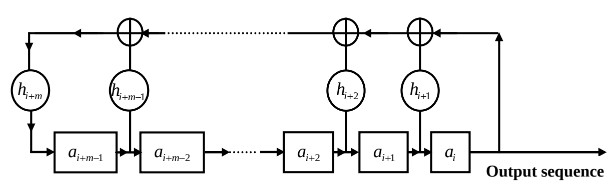


Fig. 2 LSFR Schematic register for a primitive polynomial

To obtain a De Bruijn sequence, we add a "0" in the longest train of "0" of a PRBS. Consequently there will be many "1" than "0" means that the length of the sequence is even. Taking the example of a PRBS sequence: 00010011010111. To construct a deBruijn sequence we add a logical zero in the middle of the sequence of 3 bits, we obtain the following sequence: 0000100110101111.

## 4. Pseudo Random Quaternary Sequence « PRQS»

The pseudo-random quaternary sequence or PRQS sequences have a length equal to a power of 4 minus one symbol and they are repeated periodically. They are created in two ways. Either:
By a method based on GF (4) because we can not define an XOR gate on objects with four levels.
Using a multiplexing between two pseudo-random binary.
Also, we can apply a second technique for the De Bruijn quaternary generation associated with PRQS sequence but we use multiplexing between two De Bruijn binary sequences. So, to explain this latter method we have chosen to illustrate it with example (Fig. 3).

## 5. Method of Multiplexing Between Two De Bruijn Sequences

First, we generate all De Bruijn sequences from a PRBS obtained by circular permutation, and then each sequence is shifted by a factor $\pm\frac{m}{2}$. Multiplexing between De Bruijn sequences one shifted from the other provides a sequence of De Bruijn associated with PRQS (See Fig. 3).

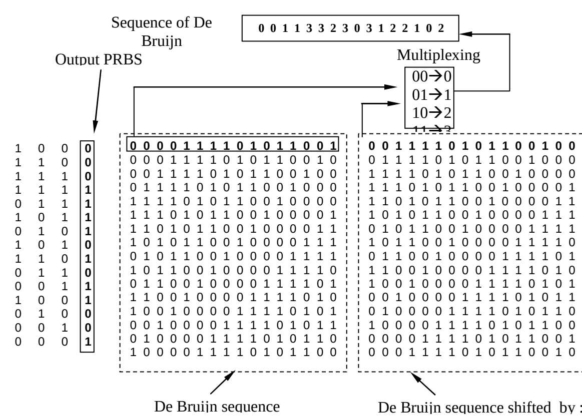


Fig. 3 Multiplexing principle between two De Bruijn sequences

Modelling transmission using QPSK formats concern the optimizing how to emulate the actual traffic data that is to say we are interested in the study and the generation of De Bruijn quaternary sequences associated with a PRQS sequence.
The calculation of the PRQS sequences is based on multiplexing between two De Bruijn sequences, which its length is equal to $4^m$.

In Fig. 4, we present an example of De Bruijn PRQS sequence of $4^2$ length, $q$ = 4, $m$ =2, obtained by multiplexing between two De Bruijn sequences.

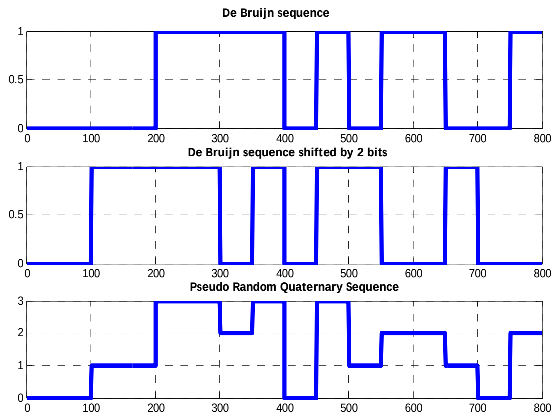


Fig .4 Multiplexing between two De Bruijn sequences to obtain a De Bruijn PRQS sequence of $4^2$ length, q = 4, m = 2.

## 6. Simulation Results and Discussion

The analysis carried out here are on the same principle as PRBS sequences generated through the primitive polynomial $h(x) = x^{10} + x^3 + 1$ . It is an addition to a logical zero "0" in the longest train of "0" of a PRQS.

Fig. 5 shows that adding a zero to a pseudorandom random sequence to obtain De Bruijn sequence disrupts completely its autocorrelation function. Also, we show an influence of the sequence length to its spectrum Fig. 5 (e).

Table 1 presents an example of statistical analysis of De Bruijn binary sequence for 1024 bits length. This analysis verifies the De Bruijn sequence properties: the probability of occurrence of each bit is identical. This is similar to the associated state changes sequence.

TABLE. 1 STATISTICAL ANALYSIS OF DE BRUIJN BINARY SEQUENCE FOR 1024 BITS LENGTH

| Logical state | Number | Probability |
|---|---|---|
| 0 | 512 | 0.5 |
| 1 | 512 | 0.5 |
| 0 | 256 | 0.5 |
| 1 | 256 | 0.25 |
| 2 | 256 | 0.25 |
| 3 | 256 | 0.25 |

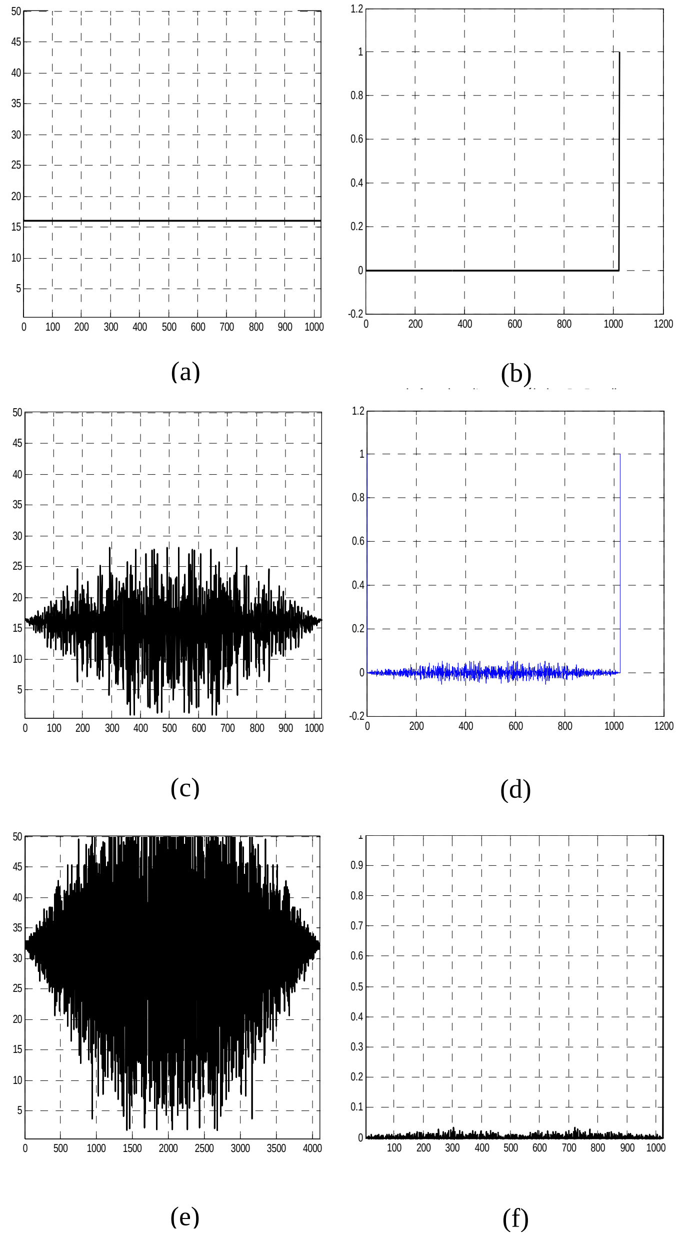


Fig. 5 (a) PRBS sequence spectrum of 1023 bits, (b) PRBS autocorrelation function of 1023 bits, (c) De Bruijn sequence spectrum of 1024 bits, (d) De Bruijn sequence autocorrelation function of 1024 bits, (e) De Bruijn sequence spectrum of 4096 bits, (f) De Bruijn quaternary sequence autocorrelation function of 1024 symbols.

## 7. Conclusion

To emulate a real traffic data transmission using a modulation format with four levels, we are moving towards pseudo-random quaternary say PRQS. Numerous tests have been made to verify its properties. The simulations require many sequences symbol power of 2.

For this, it often makes PRQS sequences to sequences of associate Quaternary DeBruijn, the latter being well suited for such simulations. It was noted initially that the autocorrelation ρ(n) and the spectrum function of a PRBS are constant for n different from 0. Second time, it was noted that the autocorrelation function ρ(n) and the spectrum of a binary De Bruijn sequence of length L is not constant for n equal to 0 due to the addition of logical zero at the middle (m-1) zeros of PRBS sequence. In addition, we find that the length of the sequence is proportional to the disturbance spectrum.
Then, We moved to the stage of multiplexing. Our tests have concluded that the Quaternary sequence De Bruijn takes the same properties as that of the binary sequence of De Bruijn.

## Acknowledgments

This work was made possible through collaboration between the institute & TELECOM Management Sud Paris (old INT, Institut National des Telecommunications) and
the Faculty of Technology, Department of Electrical Engineering of the University of Tlemcen. The objective of this research is included in the national project initiated by France of the teams Research Alcatel-Lucent.

## References

[1] RAMAMTANIS Petros, BADAOUI Hadjira, FRIGNAC Yann, Quaternary sequences comparison for the modeling of optical DQPSK dispersion managed transmission systems. OFC '09 : Optical Fiber Communication Conference, IEEE, 22-26 march 2009, San Diego, Ca, United States, 2009, ISBN 978-1-4244-2606-5.
[2] RAMANTANIS Petros, BADAOUI Hadjira, FRIGNAC Yann, Comparaison des séquences de données pour l'estimation de la perfomance des systèmes de transmission optique DQPSK. 28ièmes Journées Nationales d'Optique Guidée & Horizons de l'Optique , 06-09 juillet 2009, Lille, France, 2009.
[3] D. van den Borne, E. Gottwald, G.D. Khoe & H. de Waardt, « Bit Pattern Dependence in optical DQPSK », IEEE, Vol. 43, N°. 22, Oct. 2007.
[4] D. Van den born, E. Gottwald, G. D. Khoe and H. de. Waardt, ‘Pseudo Random Sequences for Modelling of Quatrenary Modulation formats’, 12 th Optoelectronics and communications conference, Technical digest, July 2007, Pacifico Yokohama. Pp. 722-723.
[5] GP. Agrawal, « Fiber-Optic Communication Systems », 3th edition
[6] Y. Frignac, « Contribution à l’Ingénierie des Systèmes de Transmission Terrestres sur Fibre optique utilisant le Multiplexage en Longueur d’onde de Canaux Modulés au Débit de 40 Gbits/s », Thèse de Doctorat, Ecole Nationale Supérieure des Télécommunications, Avr. 2003
[7] R. Aurélien, « Circuits dans le graphe de DeBruijn », Mémoire de Stage de Master, Université Montpellier II, Sept. 2006