IJCSI International Journal of Computer Science Issues, Vol. 7, Issue 4, No 4, July 2010
ISSN (Online): 1694-0784
ISSN (Print): 1694-0814

PRQS Sequences Characteristics Analysis by Auto-correlation Function and Statistical Properties

Hadjira Badaoui[1], Yann Frignac[2], Petros Ramantanis[2], Badr Eddine Benkelfat[2] and Mohammed Feham[1]

[1] Laboratoire STIC, Département de Génie Electrique
Faculté de Technologie, Université Abou-Bekr Belkaïd -Tlemcen
BP 230, Pôle Chetouane, 13000 Tlemcen- Algeria

[2] Institut TELECOM & Management SudParis,
9, rue Charles Fourier - 91011 Evry Cedex - France

Abstract

The goal of this paper is to describe the generation and present characteristics sequences analysis called pseudo random quaternary sequences witch show very remarkable properties. To emulate a real traffic data transmission using Differential Phase Shift keying (DQPSK) modulation format with four levels of phase, we are affecting towards pseudo random quaternary sequences. In particular, we will focus on the autocorrelation function study and statistical properties. Two PRQS Analysis respectively for a length of 63 symbols (m=3) and a sequence of 16383 symbols (m=7) have been used to confirm its properties. We will show an importance of the sequence length. The simulations results are then presented and discussed

Keywords: *PRQS sequence analysis, PRBS sequence, LSFR register, irreducible primitive polynomial, Galois field, autocorrelation function, state changes sequence.*

1. Introduction

Multi-level modulation formats are considered for next generation transmission systems. In particular, differential phase shift keying (DQPSK) is attractive as it encodes 2 bits/symbol and as such offers a combination of high spectral efficiency and large chromatic and polarisation-mode dispersion tolerance [1,3]. To asses the linear and nonlinear transmission properties of such modulation formats, inter-symbol interference as occurs along the transmission line should properly modeled [4]. Binary modulation formats are routinely modeled using pseudo random binary sequences (PRBS), but for quaternary modulation formats no such standard exists. Hence, to generate a quaternary sequence typically two PRBS are multiplexed with a cyclic shift for de-correlation. However, such a sequence does not necessary include all possible combinations of symbols up to a given length, which can result in inaccurate modeling of system penalties. This can be avoided by using pseudo-random quaternary sequences (PRQS).

In this paper we will focus on the autocorrelation function analysis and statistical properties.

2. Pseudo-random Sequences Quaternary "PRQS"

The pseudo-random quaternary sequence or PRQS sequences have a length equal to a power of 4 minus one symbol and they are repeated periodically. They are created in two ways. Either:

- By a method based on Galois field GF(4) because we can not define an XOR gate on objects with four levels.
- Using a multiplexing between two pseudo-random binary.

To explain these two techniques, we chose to illustrate them with examples. The first method follows the same principle as PRBS sequences [5]. The Figure illustrates a shift register according to an irreducible primitive polynomial $h(x)$ of m order (polynomial Galois), this time defined on the Galois field GF(4) [6].

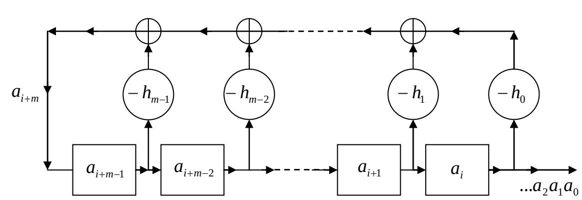


Fig.1 LSFR Schematic register for a primitive polynomial h(x)

$$h(x) = x^m + h_{m-1}x^{m-1} + ... + h_1 x + h_0 \quad (1)$$

With:

$$h_i \in GF(4),\ h_0 \neq 0$$

The LSFR is composed of m cells and each takes elements of GF (4), which are $a_{i+m-1}, a_{i+m-2}, \ldots, a_i$. The feedback element a_{i+m} and its contents will be given by the recurrence equation (2):

$$a_{i+m} = -h_{m-1}a_{i+m-1} - h_{m-2}a_{i+m-2} - \ldots - h_1 a_{i+1} - h_0 a_i \quad (2)$$

With:

$$a_i \in GF\{q = 4\}$$

As in the case of the pseudo random binary sequence generation, the coefficients of the polynomial correspond to the feedback connections from the register.
A quaternary sequence of symbols includes four states (0), (1), (2), (3) and belongs to the Galois field GF (4).
Equation (2) requires arithmetic operations of addition and multiplication. They are given in Table 1 [7]:

TABLE 1 ARITHMETIC OPERATIONS OF ADDITION AND MULTIPLICATION

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

Note that in Galois field GF (4), the arithmetic of addition and subtraction are similar.

2.1 Application Example

We present how we generate a sequence of length equal to PRQS 15 symbols using Galois field method GF (q = 4). We are interested in generating a sequence of symbols 4^m-1 for m = 2. We choose a primitive polynomial of order m = 2, for example $h(x) = x^2 + x + 2$.
From the equation 1, we can identify the coefficients with those of our polynomial generator selected. Yields:
$h_0 = 2$, $h_1 = 1$
In this case, the LSFR is illustrated in Fig. 2:
Firstly, we fix the initial state of the register, or for example: $(a_0, a_1) = (0,1)$

The recurrence equation (2) can then write:

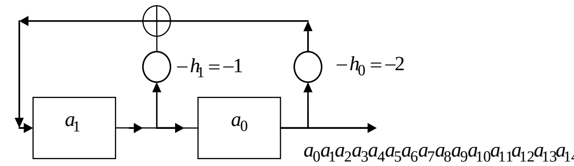


Fig. 2 LSFR register to generate a PRQS sequence

$$\begin{cases} i=0, & a_{0+2} = -h_{2-1}a_{0+2-1} - h_{2-2}a_{0+2-2} \\ i=1, & a_{1+2} = -h_{2-1}a_{1+2-1} - h_{2-2}a_{1+2-2} = -h_1a_2 - h_0a_1 \\ i=2, & a_{2+2} = -h_{2-1}a_{2+2-1} - h_{2-2}a_{2+2-2} = -h_1a_3 - h_0a_2 \\ i=3, & a_{3+2} = -h_{2-1}a_{3+2-1} - h_{2-2}a_{3+2-2} = -h_1a_4 - h_0a_3 \\ i=4, & a_{4+2} = -h_{2-1}a_{4+2-1} - h_{2-2}a_{4+2-2} = -h_1a_5 - h_0a_4 \\ i=5, & a_{5+2} = -h_{2-1}a_{5+2-1} - h_{2-2}a_{5+2-2} = -h_1a_6 - h_0a_5 \\ i=6, & a_{6+2} = -h_{2-1}a_{6+2-1} - h_{2-2}a_{6+2-2} = -h_1a_7 - h_0a_6 \\ i=7, & a_{7+2} = -h_{2-1}a_{7+2-1} - h_{2-2}a_{7+2-2} = -h_1a_8 - h_0a_7 \\ i=8, & a_{8+2} = -h_{2-1}a_{8+2-1} - h_{2-2}a_{8+2-2} = -h_1a_9 - h_0a_8 \\ i=9, & a_{9+2} = -h_{2-1}a_{9+2-1} - h_{2-2}a_{9+2-2} = -h_1a_{10} - h_0a_9 \\ i=10, & a_{10+2} = -h_{2-1}a_{10+2-1} - h_{2-2}a_{10+2-2} = -h_1a_{11} - h_0a_{10} \\ i=11, & a_{11+2} = -h_{2-1}a_{11+2-1} - h_{2-2}a_{11+2-2} = -h_1a_{12} - h_0a_{11} \\ i=12, & a_{12+2} = -h_{2-1}a_{12+2-1} - h_{2-2}a_{12+2-2} = -h_1a_{13} - h_0a_{12} \end{cases}$$

The output PRQS sequence is:

$$(a_0a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}a_{13}a_{14}) = (0\,1\,1\,3\,1\,0\,2\,2\,1\,2\,0\,3\,3\,2\,3)$$

A pseudo-random quaternary sequence has all the sequences of m symbols except 1, is a symbol 00.

3. PRQS Sequence Properties

We distinguish some interesting properties of the pseudo-random binary sequence (PRBS sequences $\{a_i\} \in \{0,1\}$) witch we will describe in the following subsections. It's the same properties then the PRQS sequences

3. 1. Autocorrelation Function

The autocorrelation function of a pseudo-random binary sequence $a_0, a_1, a_2, .., a_{L-1}$ of length L is given by [8]:

$$\begin{cases} \rho(0) = 1 \\ \rho(i) = -\dfrac{1}{L} \quad 1 \le i \le 2^m - 2 \end{cases} \quad (4)$$

Fig. 3 illustrates the principle of the autocorrelation function of a pseudo-random binary sequence.

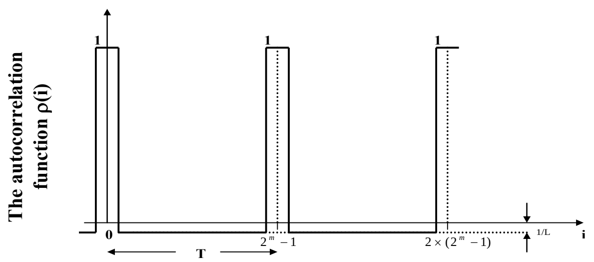


Fig.3 The autocorrelation function of a pseudo-random binary sequence

The autocorrelation function is periodic. That is to say:

$$\rho(i) = \rho(i + L) \quad (5)$$

The PRBS sequence is unusual, when correlated to guarantee only one maximum correlation at the origin. Indeed, it is a random sequence of bits of finite length so that two successive bits will be virtually uncorrelated, The autocorrelation function of a real sequence (or complex) $a_0, a_1, a_2, .., a_{L-1}$ is defined as:

$$\rho(i) = \frac{1}{L}\sum_{j=0}^{L-1} a_j \bar{a}_{i+j} \quad (6)$$

With: $i = 0, \pm 1, \pm 2, ...$
A correlation function involving a physical level -1 or 1 to a binary sequence (as $(-1)^{a_0}, (-1)^{a_1}, ..., (-1)^{a_{L-1}}$) is given by:

$$\rho(i) = \frac{1}{L}\sum_{j=0}^{L-1} (-1)^{a_j + a_{j+i}} \quad (7)$$

With: $i = 0 : 2^m - 2$,
L is the length of the sequence and $a_{i+L} = a_i$
Equation (7) is obtained by replacing all the bits "1" to a value of "-1" and bit "0" with value "1".

3. 2. Periodicity

PRBS sequences generated, are periodic sequences of period $2^m - 1$ bits. The PRBS sequence length L is defined as follows:

$$L = 2^m - 1 \quad (8)$$

Indeed, as their length is odd, there is necessarily an imbalance between the number of "1" and the number of "0". The equal probability of "1" and "0" in a PRBS is not perfectly satisfied. The number of "1" is equal to: 2^{m-1} and the number of "0" is equal to $2^{m-1} - 1$.

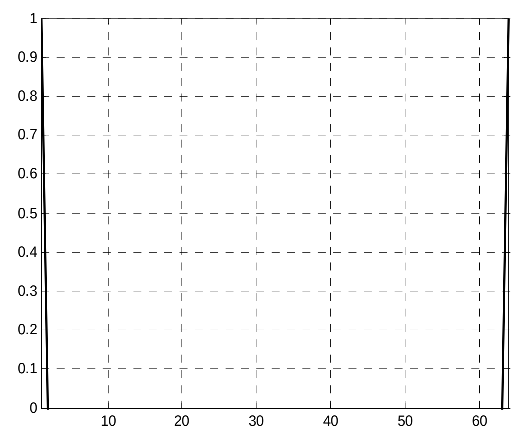
4. Simulation Results and Discussion

To characterize any sequence relative to its ability to possess the pseudo-random properties, we will be interested in testing the autocorrelation function of quaternary sequences commonly used for numerical simulations and the state changes sequence for a number of m cells constituting the LSFR register equal to m=3 and m=7. In our case we use two respectively primitive polynomials:

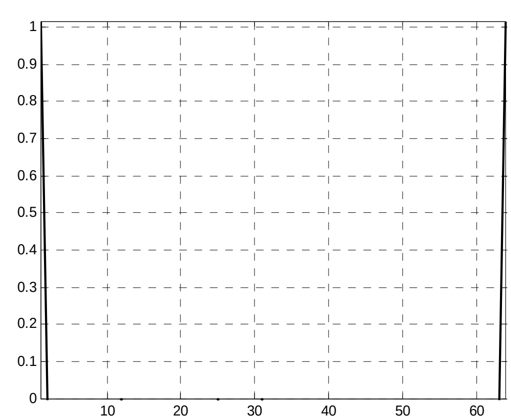
for m=3 : $h(x) = x^3 + x^2 + 2x + 3$

for m=7: $h(x) = x^7 + x^6 + 2x^5 + 3x^4 + 3x^2 + x + 2$

In the figures 4 (a) and (b), figures 5 (a) and (b) we represent respectively for m=3 and m=7 the autocorrelation function of a PRQS sequence and its of the state changes sequence. In addition, we will conduct a statistical analysis of a PRQS sequence and the state changes sequence.



(a)



(b)

Fig. 4 (a) PRQS autocorrelation function of 63 symbols; (b) PRQS State changes sequence autocorrelation function of 63 symbols

Notice that the state changes sequence is obtained by multiplexing between a symbol and the precedent symbol to get a new symbol. What Follows, we say that the state changes sequence is a sequence of sixteen state of symbols can take values ‘0’, ‘1’, ‘2’, ‘3’, ‘4’, ‘5’, ‘6’, ‘7’, ‘8’, ‘9’, ‘10’, ‘11’, ‘12’, ‘13’, ‘14’, or ‘15’. The autocorrelation function of a PRQS sequence is treated as a Dirac peak at the origin of amplitude equal to 1, this reflects the fact that, when correlated to guarantee only one maximum of correlation. On the other hand, if we shift the sequence of one symbol the autocorrelation function is always zero, that is to say, the PRQS sequence is uncorrelated beyond the origin confirming that the symbols are completely independent.

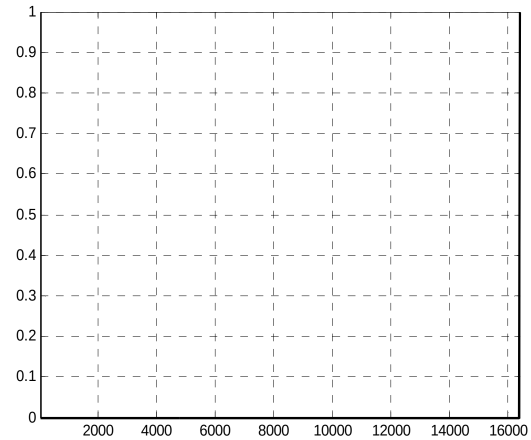
Similarly, and from Fig. 4 (a), we note that the autocorrelation function of the state changes sequence is similar to the PRQS. In the case of a PRQS sequence was 16 cases of correlation and the possibility of having two similar symbols is less likely than to have two different symbols. The correlation function quantifies the level of similarity between the symbols. If both symbols are equal we add a "1" and if we remove a different "1/3 ".

According to the statistics of a PRQS, we note that the numbers of "1", "2", and "3" are greater than the number of "0". This is similar to the state changes sequence which contains a number of zeros less than the other symbols. This is illustrated in the Table 2.

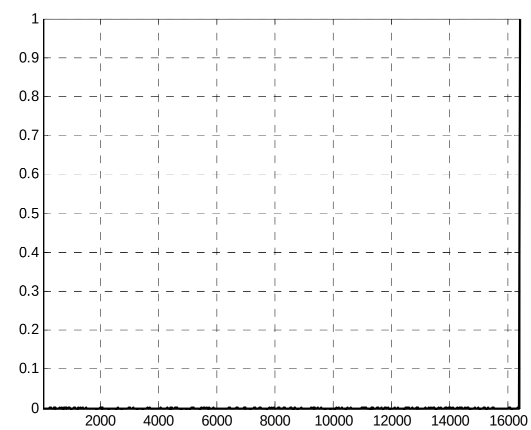
TABLE 2 A PRQS SEQUENCE OF 63 SYMBOLS STATISTICAL ANALYSIS

Logical state	Number	Probability
0	15	0.2381
1	16	0.2540
2	16	0.2540
3	16	0.2540
0	3	0.0476
1	4	0.0635
2	4	0.0635
3	4	0.0635
4	4	0.0635
5	4	0.0635
6	4	0.0635
7	4	0.0635
8	4	0.0635
9	4	0.0635
10	4	0.0635
11	4	0.0635
12	4	0.0635
13	4	0.0635
14	4	0.0635
15	4	0.0635

In reality, since the length of the sequence is large enough, the probability of having "0" is almost equal to that of "1", "2", and "3". So that, we will interest particularly to a PRQS sequence of a length equal to 16383 symbols.



(a)



(b)

Fig. 4 (a) PRQS autocorrelation function of 16383 symbols; (b) PRQS State changes sequence autocorrelation function of 16383 symbols

From figure 5 (a) and (b), we note that the Dirac peak at the origin of amplitude equal to 1 exactly when we increase the length of the sequence. In addition, we have a probability of ‘0’ is almost equal than the others symbols. This is illustrated in the Table 3.

TABLE 3 A PRQS SEQUENCE OF 16383 SYMBOLS STATISTICAL ANALYSIS

Etats logiques	Nombre	Probabilité
0	4095	0.2499
1	4096	0.2500
2	4096	0.2500
3	4096	0.2500
0	1023	0.0624
1	1024	0.0625
2	1024	0.0625
3	1024	0.0625
4	1024	0.0625
5	1024	0.0625
6	1024	0.0625
7	1024	0.0625
8	1024	0.0625
9	1024	0.0625
10	1024	0.0625
11	1024	0.0625
12	1024	0.0625
13	1024	0.0625
14	1024	0.0625
15	1024	0.0625

5. Conclusion

To emulate a real traffic data transmission using DQPSK modulation format with four levels of phase, we are moving towards pseudo-random quaternary sequence, PRQS Analysis for a length of 16383 symbols have been made to confirm its properties such as the statistics properties counting and its autocorrelation function. The length of the quaternary sequence to simulate is a very important parameter for the simulation results of a 40 Gbit/s are Significant.

Acknowledgments

This work was made possible through collaboration between the institute & TELECOM Management Sud Paris (old INT, Institut National des Telecommunications) and the Faculty of Technology, Department of Electrical Engineering of the University of Tlemcen. The objective of this research is included in the National project initiated by France of the teams Research Alcatel-Lucent.

References

[1] G. Charlet, et all., in proc. ECOC’05, PD paper Th 4.1.3.

[2] D. van den Borne, E. Gottwald, G.D. Khoe & H. de Waardt, « Bit Pattern Dependence in optical DQPSK », IEEE, Vol. 43, N°. 22, Oct. 2007.

[3] D. Van den born, E. Gottwald, G. D. Khoe and H. de. Waardt, ‘Pseudo Random Sequences for Modelling of Quatrenary Modulation formats’, 12 th Optoelectronics and communications conference, Technical digest, July 2007, Pacifico Yokohama. Pp. 722-723.

[4] I. Wickham, et. Al., IEEE PTL, no. 6, pp. 1591-1593, 2004.

[5] RAMAMTANIS Petros, BADAOUI Hadjira, FRIGNAC Yann, Quaternary sequences comparison for the modeling of optical DQPSK dispersion managed transmission systems. OFC '09 : Optical Fiber Communication Conference, IEEE, 22-26 march 2009, San Diego, Ca, United States, 2009, ISBN 978-1-4244-2606-5.

[6] H. Badaoui, «Analyse Numerique de L’implementation des Formats de Modulation en Phase dans les Futurs Systemes de Transmission sur Fibre Optique » Mémoire de Magister en systèmes et réseaux de télécommunications, Université de Tlemcen, 2008.

[7] C. Balza, « Four-Level Pseudorandom Sequences », IEEE, Vol. 3, N°. 7, July. 1967.

[8] F. Jessie, M. Williams & Neil J. A. Sloane, « Pseudo-Random Sequences and Arrays », IEEE, Vol. 64, N°. 12, Dec. 1976