Cramer-Rao Lower Bound for NDA SNR Estimation from Linear Modulation Schemes over Flat Rayleigh Fading Channel

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Abstract

In this contribution, Cramer-Rao lower bound (CRLB) for signal-to-noise ratio (SNR) estimation from linear modulation signals over flat Rayleigh fading channel is addressed. Therefore, we derive the analytical expressions of Fisher information matrix entries that assess the optimal variance of any unbiased SNR estimator. Based on statistical Monte Carlo computing method, simulation results are drawn from several constellation densities and observation window sizes. For the linear modulation schemes used here, it is shown that the lower bound is as higher as the modulation order increases. The derived bound provides an efficient standard for evaluating the performance of any unbiased non-data aided (NDA) SNR estimator from linear modulation signals over flat Rayleigh fading channel (FRFC).

Keywords: *Cramer-Rao lower bound, signal-to-noise ratio, non-data aided estimation, FRFC, complex AWGN.*

1. Introduction

Modern communication systems often require the knowledge of the SNR level at the receiver side in order to qualify the performance of the received signal quality. Then accurate SNR estimate is required for measuring the channel quality for adaptive modulation schemes as well as for soft decoding procedures as shown in [1], [2] and [3]. In addition to low-complexity requirement, it is essential to assess the truthfulness of SNR estimators in term of their statistical variances. For this purpose, the well-known CRLB is a prominent benchmark to evaluate the statistical variance performance of unbiased estimators.

Actually both data aided (DA) and non-data aided (NDA) trends are considered for either performance bounds derivation or estimation algorithms. Data aided approach, which relies on the transmission of known data streams such as training sequences and also pilot symbols, should expedite and ease the estimation process. Unfortunately, this approach limits the system through-put in the sense that adding known pilot symbols to the data stream should drop down the spectral efficiency of the communication system. Hence NDA SNR estimation approach receives substantial attention in recent literature. CRLB for NDA SNR estimation is derived in [4] from both BPSK and QPSK modulated signals with AWGN channel. Derived bounds are compared to those obtained for DA estimation. In [5], a straightforward approximation of the CRLB for NDA SNR estimation from BPSK modulated signals over AWGN channel is presented in efficient form that avoids tedious numerical integration. Authors, in [6], derive a lower bound for SNR estimation from general M-ary one/two dimensional modulation signals with axis/half plane symmetry over AWGN channel. Exact analytical CLRB of unbiased NDA SNR estimation from square QAM signals using I/Q received signal model is addressed in [7], where a generalization of the elegant CLRB expressions presented in [4] is also introduced.

In addition to AWGN channel, derivation of SNR estimates CRLB for fading channels deserves great attention regarding its significance for modern wireless communication systems. Hence, derivation of the CRLB for SNR estimation is addressed in [8] over a time-varying channel based on polynomial-in-time model according to Taylor's theorem. Recent works presented in [9] and [10] deal with the CRLB derivation for carrier phase and frequency estimation assuming transmission over fading channel. On this basis, the present work is devoted for analytically deriving the CRLB for NDA SNR estimation from linearly modulated signals over flat Rayleigh fading channel (FRFC) where the transmitted signal is scaled by a non-constant fading gain during the estimator observation window. Noise power and also signal amplitude are assumed as completely unknown at the receiver side. The lower bounds derived hereafter offer an efficient standard to assess NDA SNR estimator performance over FRFC.

2. System Model

Consider the transmission of linearly modulated signal over FRFC corrupted by a complex AWGN (CAWGN). In absence of carrier phase and frequency offsets and also under the assumption of ideal timing recovery, the complex sample at the output of the receiver matched filter x_k can be written as:

$$x_k = \rho_k a_k + \omega_k \quad ; \quad k = 0,\ldots,N-1 \tag{1}$$

where, a_k is the transmitted symbol and N is the observation window size. Note that the transmitted symbols $a_0,\ldots,a_{N-1}$ are assumed as independent and identically distributed. ω_k is the CAWGN sample. The vector $\boldsymbol{\omega} = \{\omega_0,\ldots,\omega_{N-1}\}$ is a set of randomly drawn samples from independent zero-mean complex Gaussian process with uncorrelated real and imaginary parts having equal variances σ^2. ρ_k is a Rayleigh distributed positive random variable, where the well-known Rayleigh probability density function (PDF) is given by [11]:

$$f(\rho,\sigma_0) = \frac{\rho}{\sigma_0^2} e^{-\frac{\rho^2}{2\sigma_0^2}} \tag{2}$$

The signal-to-noise ratio (SNR) is then given by:

$$SNR_{FRFC} = \gamma = \frac{\sigma_0^2}{\sigma^2} \tag{3}$$

We expect to estimate γ based on the observation samples vector $\boldsymbol{x} = \{x_0,\ldots,x_{N-1}\}$. Then two parameters are involved in this estimate. For convenience, we note:

$$\alpha = \sigma_0^2 \quad ; \quad \beta = \sigma^2 \tag{4}$$

Let us define a parameter vector $\boldsymbol{\theta}$ such that:

$$\boldsymbol{\theta} = [\alpha \quad \beta] \tag{5}$$

While the estimated SNR unit is usually the decibel, thus we consider the following function:

$$\boldsymbol{g}(\boldsymbol{\theta}) = 10\log(\frac{\alpha}{\beta}) \tag{6}$$

The CRLB of the SNR estimation is given by [11, pp.45-46]:

$$CRLB(\gamma) = \frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \boldsymbol{I}^{-1}(\boldsymbol{\theta}) \frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}^T \tag{7}$$

where $\boldsymbol{I}(\boldsymbol{\theta})$ is the 2×2 Fisher information matrix (FIM) defined as:

$$\boldsymbol{I}(\boldsymbol{\theta}) = \begin{bmatrix} -E_{\boldsymbol{x}}\left(\frac{\partial^2 \mathrm{Ln}(p(\boldsymbol{x}\,|\,\boldsymbol{\theta}))}{\partial \alpha^2}\right) & -E_{\boldsymbol{x}}\left(\frac{\partial^2 \mathrm{Ln}(p(\boldsymbol{x}\,|\,\boldsymbol{\theta}))}{\partial \alpha \partial \beta}\right) \\ -E_{\boldsymbol{x}}\left(\frac{\partial^2 \mathrm{Ln}(p(\boldsymbol{x}\,|\,\boldsymbol{\theta}))}{\partial \beta \partial \alpha}\right) & -E_{\boldsymbol{x}}\left(\frac{\partial^2 \mathrm{Ln}(p(\boldsymbol{x}\,|\,\boldsymbol{\theta}))}{\partial \beta^2}\right) \end{bmatrix} \tag{8}$$

and $\frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ is the 1×2 Jacobian matrix given by:

$$\frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{10}{Ln(10)\alpha} & -\frac{10}{Ln(10)\beta} \end{bmatrix} \tag{9}$$

3. CRLB Derivation for SNR Estimation

To derive the CRLB expressions, we have to evaluate the probability $p(\boldsymbol{x}\,|\,\boldsymbol{\theta})$ given in (8). The PDF $p(x_k\,|\,\boldsymbol{\theta},a_i,\rho_k)$ for a single received sample x_k parameterized by $\boldsymbol{\theta}$, a_i and ρ_k is given by:

$$p(x_k\,|\,\boldsymbol{\theta},a_i,\rho_k) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}|x_k-\rho_k a_i|^2} \tag{10}$$

Then the PDF $p(x_k\,|\,\boldsymbol{\theta},a_i)$ parameterized by $\boldsymbol{\theta}$ and a_i is computed by integration over the Rayleigh fading gain ρ_k as follows:

$$p(x_k\,|\,\boldsymbol{\theta},a_i) = \int_0^{+\infty} f(\rho,\sigma_0) p(x_k\,|\,\boldsymbol{\theta},a_i,\rho) d\rho \tag{11}$$

After several algebraic handling, we obtain the following expression from (11):

$$p(x_k\,|\,\boldsymbol{\theta},a_i) = \frac{C_k(\boldsymbol{\theta})}{4(A_i(\boldsymbol{\theta}))^{3/2}} F(A_i(\boldsymbol{\theta}), B_{k,i}(\boldsymbol{\theta})) \tag{12}$$

where:

$$F(u,v) = 2\sqrt{u} + \sqrt{\pi} v e^{\frac{v^2}{4u}} \left(1 + erf\left(\frac{v}{2\sqrt{u}}\right)\right) \tag{13}$$

$$A_i(\boldsymbol{\theta}) = \frac{1}{2\sigma_0^2} + \frac{|a_i|^2}{2\sigma^2} \tag{14}$$

$$B_{k,i}(\boldsymbol{\theta}) = \frac{\Re(x_k a_i^*)}{\sigma^2} \tag{15}$$

$$C_k(\boldsymbol{\theta}) = \frac{e^{-\frac{|x_k|^2}{2\sigma^2}}}{2\pi\sigma^2\sigma_0^2} \tag{16}$$

and erf(.) is the error function defined by:

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{17}$$

We consider that transmitted symbols $\{a_i\}$ fit in an M-ary constellation C , then the PDF $p(x_k \mid \boldsymbol{\theta})$ can be expressed as follows:

$$p(x_k \mid \boldsymbol{\theta})=\sum_{a_i\in C} p(a_i)p(x_k \mid \boldsymbol{\theta},a_i) \tag{18}$$

Assuming that the received samples $\{x_k\}$ are independent random variables and also that the transmitted symbols $\{a_i\}$ are equally likely random variables $(p(a_i)=p(a))$, then the probability $p(\boldsymbol{x} \mid \boldsymbol{\theta})$ is given by:

$$p(\boldsymbol{x} \mid \boldsymbol{\theta})=\prod_{k=0}^{N-1} p(x_k \mid \boldsymbol{\theta}) =\prod_k \sum_{a_i\in C} p(a_i)p(x_k \mid \boldsymbol{\theta},a_i) \tag{19}$$

$$=\prod_k p(a)\sum_{a_i\in C} p(x_k \mid \boldsymbol{\theta},a_i) \tag{20}$$

We inject (12) in (20), then we obtain:

$$p(\boldsymbol{x} \mid \boldsymbol{\theta})=\prod_k p(a)\frac{C_k(\boldsymbol{\theta})}{4}\sum_{a_i\in C}\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}F\left(A_i(\boldsymbol{\theta}),B_{k,i}(\boldsymbol{\theta})\right) \tag{21}$$

Taking the logarithm of (21) and retaining the $\boldsymbol{\theta}$ dependent terms only, we obtain the following expression:

$$\mathrm{Ln}\left\{\prod_k\left(\frac{C_k(\boldsymbol{\theta})}{4}\sum_{a_i\in C}\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}F\left(A_i(\boldsymbol{\theta}),B_{k,i}(\boldsymbol{\theta})\right)\right)\right\} =\sum_k \mathrm{Ln}\left(\mathrm{P_k}(\boldsymbol{\theta})\right) \tag{22}$$

where:

$$\boldsymbol{P_k}(\boldsymbol{\theta})=\frac{C_k(\boldsymbol{\theta})}{4}\sum_{a_i\in C}\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}F\left(A_i(\boldsymbol{\theta}),B_{k,i}(\boldsymbol{\theta})\right) \tag{23}$$

Then, the first diagonal element of the Fisher information matrix $\boldsymbol{I}(\boldsymbol{\theta})$ can be expressed as follows:

$$[\boldsymbol{I}(\boldsymbol{\theta})]_{11}=-E_{\boldsymbol{x}}\left[\frac{\partial^2 Ln\left(p(\boldsymbol{x} \mid \boldsymbol{\theta})\right)}{\partial\alpha^2}\right] =\sum_{k=0}^{N-1}\left\{\frac{\frac{\partial^2 \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha^2}}{\boldsymbol{P_k}(\boldsymbol{\theta})}-\left(\frac{\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha}}{\boldsymbol{P_k}(\boldsymbol{\theta})}\right)^2\right\} \tag{24}$$

In order to derive $\frac{\partial^2 Ln\left(p(\boldsymbol{x} \mid \boldsymbol{\theta})\right)}{\partial\alpha^2}$, we compute $\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha}$ and also $\frac{\partial^2 \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha^2}$.

The first partial derivative $\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha}$ may be written as:

$$\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha}=\frac{C_k(\boldsymbol{\theta})}{4}\times \sum_{a_i\in C}\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}\left\{F\left(A_i(\boldsymbol{\theta}),B_{k,i}(\boldsymbol{\theta})\right)\left[\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}G_i(\boldsymbol{\theta})-\frac{1}{\sigma_0^2}\right] + H_{k,i}(\boldsymbol{\theta})\right\} \tag{25}$$

$G_i(\boldsymbol{\theta})$ and $H_{k,i}(\boldsymbol{\theta})$ are given in appendix A.

The expression of the second derivative $\frac{\partial^2 \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha^2}$ is given by:

$$\frac{\partial^2 \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha^2}=\frac{C_k(\boldsymbol{\theta})}{4}\times \sum_{a_i\in C}\left\{\left[\frac{2}{\sigma_0^4}\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}+\dot{G}_i(\boldsymbol{\theta})\right]F\left(A_i(\boldsymbol{\theta}),B_{k,i}(\boldsymbol{\theta})\right) + 2G_i(\boldsymbol{\theta})H_{k,i}(\boldsymbol{\theta})+\left(A_i(\boldsymbol{\theta})\right)^{-\frac{3}{2}}\dot{H}_{k,i}(\boldsymbol{\theta})\right\} \tag{26}$$

$\dot{G}_i(\boldsymbol{\theta})$ and $\dot{H}_{k,i}(\boldsymbol{\theta})$ denote the first derivatives of $G_i(\boldsymbol{\theta})$ and $H_{k,i}(\boldsymbol{\theta})$ with respect to α , respectively. Their expressions are detailed in appendix B.
Applying the same procedure, then the derivation of the remaining elements of $\boldsymbol{I}(\boldsymbol{\theta})$ is given by:

$$[\boldsymbol{I}(\boldsymbol{\theta})]_{12}=[\boldsymbol{I}(\boldsymbol{\theta})]_{21}=-E_{\boldsymbol{x}}\left[\frac{\partial^2 Ln\left(p(\boldsymbol{x} \mid \boldsymbol{\theta})\right)}{\partial\alpha\partial\beta}\right] =\sum_{k=0}^{N-1}\left\{\frac{\frac{\partial}{\partial\alpha}\left(\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\beta}\right)}{\boldsymbol{P_k}(\boldsymbol{\theta})}-\frac{\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\alpha}\times\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\beta}}{\left[\boldsymbol{P_k}(\boldsymbol{\theta})\right]^2}\right\} \tag{27}$$

$$[\boldsymbol{I}(\boldsymbol{\theta})]_{22} = -E_{\boldsymbol{x}}\left[\frac{\partial^2 Ln(p(\boldsymbol{x}\,|\,\boldsymbol{\theta}))}{\partial\beta^2}\right] = \sum_{k=0}^{N-1}\left\{\frac{\frac{\partial^2 \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\beta^2}}{\boldsymbol{P_k}(\boldsymbol{\theta})} - \left(\frac{\frac{\partial \boldsymbol{P_k}(\boldsymbol{\theta})}{\partial\beta}}{\boldsymbol{P_k}(\boldsymbol{\theta})}\right)^2\right\} \tag{28}$$

After derivation of several elements of both the matrix $\boldsymbol{I}(\boldsymbol{\theta})$ and $\frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$, the CRLB for SNR estimation is given by:

$$CRLB(\gamma) = \frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \boldsymbol{I}^{-1}(\boldsymbol{\theta}) \frac{\partial \boldsymbol{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}^T \tag{29}$$

4. Simulation Results

We use Monte Carlo simulation techniques to evaluate the statistical expectation in (24), (27) and (28) with respect to the N-dimensional vector $\boldsymbol{x}$. Note that once $\{x_k\}$ are statistically independent variables, we estimate the value of this expectation by generating a sequence of random samples at each SNR value, then computing the average of $\frac{\partial^2 \mathrm{Ln}(p(\boldsymbol{x}\,|\,\boldsymbol{\theta}))}{\partial\alpha\partial\beta}$ for each sample. Note that a minimum of 1000 trials is considered to ensure that the estimate stemming from Monte Carlo integration converges to the statistical expectation value. Figures 1 and 2 depict the CRLB curves from M-QAM signals for an observation window size N=100 and N=1000. It is shown that CRLB values decrease as far as the observation window size increases. Moreover, CRLBs for large modulation order take higher values at low SNR range.
The method described here stand useful to determine the CRLB for larger M-QAM constellation densities and also general linear modulation schemes.

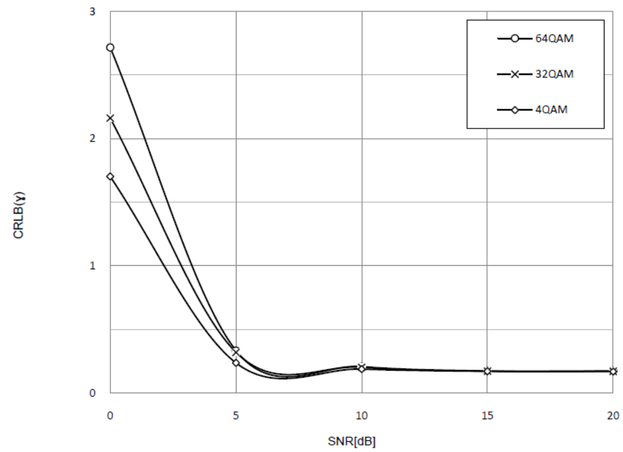

Fig. 1 CRLB versus SNR for 4, 32 and 64-QAM and N=100.

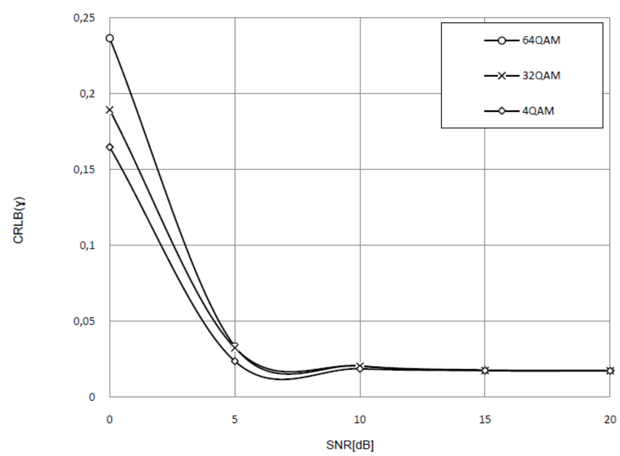

Fig. 2 CRLB versus SNR for 4, 32 and 64-QAM and N=1000.

5. Conclusions

True Cramer-Rao lower bound for NDA signal-to-noise ratio estimation from linearly modulated signals over FRFC with CAWGN is derived. At low SNR range, simulation results show that CRLB values decrease as far as the modulation order increases. For high SNR levels, CRLBs almost coincide either for various modulation orders or various observation window sizes. The method introduced here represents a standard for NDA SNR estimator over FRFC from linearly modulated signals.

Appendix A

The first derivative of $A_i(\boldsymbol{\theta})$ with respect to α is given by:

$$\frac{\partial}{\partial\alpha}\left(A_i(\boldsymbol{\theta})\right) = -\frac{1}{2\sigma_0^4} \tag{A.1}$$

The expressions of $G_i(\boldsymbol{\theta})$ and $H_{k,i}(\boldsymbol{\theta})$ are given by:

$$G_i(\boldsymbol{\theta}) = -\frac{3}{2}\frac{\partial}{\partial\alpha}\left(A_i(\boldsymbol{\theta})\right)\left(A_i(\boldsymbol{\theta})\right)^{-\frac{5}{2}} \tag{A.2}$$

$$H_{k,i}(\boldsymbol{\theta}) = \frac{\partial}{\partial \alpha}\left(F\left(A_i(\boldsymbol{\theta}), B_{k,i}(\boldsymbol{\theta})\right)\right) \quad \text{(A.3)}$$

$$= A_i(\boldsymbol{\theta})^{-\frac{1}{2}} \frac{\partial}{\partial \alpha}\left(A_i(\boldsymbol{\theta})\right)$$

$$-\frac{\sqrt{\pi}}{4} A_i(\boldsymbol{\theta})^{-2} \frac{\partial}{\partial \alpha}\left(A_i(\boldsymbol{\theta})\right) B_{k,i}(\boldsymbol{\theta})^3 e^{\frac{B_{k,i}(\boldsymbol{\theta})^2}{4A_i(\boldsymbol{\theta})}} \left(1 + erf\left(\frac{B_{k,i}(\boldsymbol{\theta})}{2\sqrt{A_i(\boldsymbol{\theta})}}\right)\right)$$

$$-\frac{1}{2} A_i(\boldsymbol{\theta})^{-\frac{3}{2}} \frac{\partial}{\partial \alpha}\left(A_i(\boldsymbol{\theta})\right) B_{k,i}(\boldsymbol{\theta})^2$$

(A.4)

Appendix B

$$\dot{G}_i(\boldsymbol{\theta}) = -\frac{3}{2} A_i(\boldsymbol{\theta})^{-\frac{5}{2}} \times$$

$$\left(\frac{\partial^2}{\partial \alpha^2}\left(A_i(\boldsymbol{\theta})\right) - \frac{5}{2}\left(\frac{\partial}{\partial \alpha}\left(A_i(\boldsymbol{\theta})\right)\right)^2 A_i(\boldsymbol{\theta})^{-1}\right) \quad \text{(B.1)}$$

$$\dot{H}_{k,i}(\boldsymbol{\theta}) = \frac{\partial}{\partial \alpha} T_{11} + \frac{\partial}{\partial \alpha} T_{12} + \frac{\partial}{\partial \alpha} T_{13} \quad \text{(B.2)}$$

$$\frac{\partial}{\partial \alpha} T_{11} = A_i(\boldsymbol{\theta})^{-\frac{1}{2}} \frac{\partial^2\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha^2} - \frac{1}{2} A_i(\boldsymbol{\theta})^{-\frac{3}{2}} \left(\frac{\partial\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha}\right)^2 \quad \text{(B.3)}$$

$$\frac{\partial}{\partial \alpha} T_{12} =$$

$$\frac{\sqrt{\pi}}{2} A_i(\boldsymbol{\theta})^{-3} \left(\frac{\partial\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha}\right)^2 B_{k,i}(\boldsymbol{\theta})^3 e^{\frac{B_{k,i}(\boldsymbol{\theta})^2}{4A_i(\boldsymbol{\theta})}} \left(1 + erf\left(\frac{B_{k,i}(\boldsymbol{\theta})}{2\sqrt{A_i(\boldsymbol{\theta})}}\right)\right)$$

$$+\frac{\sqrt{\pi}}{16} A_i(\boldsymbol{\theta})^{-4} \left(\frac{\partial\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha}\right)^2 B_{k,i}(\boldsymbol{\theta})^5 e^{\frac{B_{k,i}(\boldsymbol{\theta})^2}{4A_i(\boldsymbol{\theta})}} \left(1 + erf\left(\frac{B_{k,i}(\boldsymbol{\theta})}{2\sqrt{A_i(\boldsymbol{\theta})}}\right)\right)$$

$$-\frac{\sqrt{\pi}}{4} A_i(\boldsymbol{\theta})^{-2} \frac{\partial^2\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha^2} B_{k,i}(\boldsymbol{\theta})^3 e^{\frac{B_{k,i}(\boldsymbol{\theta})^2}{4A_i(\boldsymbol{\theta})}} \left(1 + erf\left(\frac{B_{k,i}(\boldsymbol{\theta})}{2\sqrt{A_i(\boldsymbol{\theta})}}\right)\right)$$

$$+\frac{1}{16} A_i(\boldsymbol{\theta})^{-\frac{5}{2}} \left(\frac{\partial\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha}\right)^2 B_{k,i}(\boldsymbol{\theta})^4$$

(B.4)

$$\frac{\partial}{\partial \alpha} T_{13} = \frac{3}{4} A_i(\boldsymbol{\theta})^{-\frac{5}{2}} \left(\frac{\partial\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha}\right)^2 B_{k,i}(\boldsymbol{\theta})^2$$

$$-\frac{1}{2} A_i(\boldsymbol{\theta})^{-\frac{3}{2}} \frac{\partial^2\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha^2} B_{k,i}(\boldsymbol{\theta})^2 \quad \text{(B.5)}$$

$$\frac{\partial^2\left(A_i(\boldsymbol{\theta})\right)}{\partial \alpha^2} = \frac{1}{\sigma_0^6} \quad \text{(B.6)}$$

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