IJCSI International Journal of Computer Science Issues, Vol. 10, Issue 1, No 3, January 2013
ISSN (Print): 1694-0784 | ISSN (Online): 1694-0814
www.IJCSI.org

Some Models for Multiple Attribute Decision Making with Intuitionistic Fuzzy Information and Uncertain Weights

Yujun Luo[1], Xianfu Li[2], Ying Yang[1] and Zhenglong Liu[1]

[1] **Department of Computer Science and Mathematics, North Sichuan Medical College, Nanchong, Sichuan 637007, China**

[2] **Department of Imaging Medicine, North Sichuan Medical College, Nanchong, Sichuan 637000, China**

Abstract

Multiple attribute decision making problems with uncertain weights in intuitionistic fuzzy setting are investigated. Some concepts related to the theory of intuitionistic fuzzy set (IFS), including intuitionistic fuzzy weighted averaging (IFWA) operator, score function, and accuracy function, are reviewed. Based on the technology for order preference by similarity to idea solution (TOPSIS) method and the score matrix converted from decision matrix given in the form of intuitionistic fuzzy number (IFN), some quadratic programming models, by which the attribute weights can be derived, are established. Then, the alternatives are ranked, and the most desirable one is selected according to the score and accuracy degree of the collective IFN aggregated by IFWA operator. Finally, an example about evaluation of teaching quality is discussed to verify the effectiveness of the proposed approach.

Keywords: *Multiple Attribute Decision Making, Intuitionistic Fuzzy Set, Uncertain Attribute Weights, TOPSIS, Quadratic Programming Models, Evaluation of Teaching Quality.*

1. Introduction

As a generalization of fuzzy set proposed by Zadeh, Atanassov [1, 2] introduced intuitionistic fuzzy set (IFS), which models the various uncertainty or vagueness by membership function and nonmembership function. As a more suitable way to deal with vagueness than classical fuzzy set, IFS plays an important role in solving the complicated multiple attribute decision making (MADM) problems, especially in the circumstances where the assessment information given by decision makers is imprecise or uncertain due to time pressure, lack of data, or the decision maker's limited attention and information processing capabilities [3]. Recently, some researchers addressed the intuitionistic fuzzy MADM problems in the situations where attribute weights are completely known, developed intuitionistic fuzzy aggregation operators [4-9] and proposed score function and accuracy function to solve the intuitionistic fuzzy MADM [10]. The other researchers showed great interest in intuitionistic fuzzy MADM problems with uncertain attribute weights and established some optimization models to derive the vector of attribute weights [11-15].

In this paper, based on the score matrix converted from decision matrix given in the form of IFN by the decision makers, based on the technology for order preference by similarity to idea solution (TOPSIS) method, we establish some quadratic programming models, by which the optimal attribute weights can be derived from the given incomplete information about attribute weights. Then, the procedures for ranking the alternatives in intuitionistic setting are developed.

The remainder of this paper is organized as follows. In section 2, we briefly review some concepts about IFS. In section 3, the MADM problems with attribute assessment given in the form of IFN and attribute weights incompletely known are discussed. Based on TOPSIS and score matrix, some programming models for determining the optimal attribute weights are established. Then, we rank the alternatives and select the most desirable one according to score function and accuracy function of the overall IFN. In section 4, an illustrative example is presented to verify the proposed method. In section 5, we conclude the paper.

2. Preliminaries

Atanassov [1, 2] introduced the concept of IFS, which is defined as follows:

Definition 1 Let X be a finite set. An IFS in X is an object having the form:

$$A=\left\{\left\langle x, m_A(x), n_A(x)\right\rangle \middle| x\in X\right\}, \tag{1}$$

where $\mu_A(x): X \to [0,1]$ and $\nu_A(x) \to [0,1]$ are the degree of membership and the degree of nonmembership of the element x to the set X, respectively, such that $0 \le \mu_A(x)+\nu_A(x) \le 1, \forall x \in X$.

IJCSI
www.IJCSI.org

For convenience, we call $a = (m_a, n_a)$ an intuitionistic fuzzy number (IFN), where $\mu_a \in [0,1]$, $v_a \in [0,1]$, and $0 \le \mu_a(x) + v_a(x) \le 1$.

Definition 2 [4] Let $a_i = (m_i, n_i)\ (i = 1,2,...,n)$ be a collection of the IFNs. The intuitionistic fuzzy weighted averaging (IFWA) operator is a mapping, which is defined as follows:

$$IFWA_w(a_1, a_2, \mathbf{L}, a_n) = \sum_{i=1}^{n} w_i a_i$$
$$= (1 - \prod_{i=1}^{n}(1-u_i)^{w_i}, \prod_{i=1}^{n} v_i^{w_i}), \quad (2)$$

where $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$ is the weight of IFN $a_i\ (i = 1,2, \mathbf{L}\ n)$.

Definition 3 [10] Let $a = (m_a, n_a)$ be an IFN, then we call $S(a) = m_a - n_a$ and $H(a) = m_a + n_a$ a score function of a and an accuracy degree function of a, respectively.

Let $S(a)$ and $S(b)$ be the scores of IFN a and b. Let $H(a)$ and $H(b)$ be the accuracy degree of a and b. We can compare two IFNs according to the following principles:
1) If $S(a) < S(b)$, then a is smaller than b, denoted by $a < b$;
2) If $S(a) = S(b)$ and $H(a) < H(b)$, then $a < b$;
3) If $S(a) = S(b)$ and $H(a) = H(b)$, then $a = b$.

3. MADM Problems with Uncertain Attribute Weights in Intuitionistic Fuzzy Setting

Let $A = \{a_1, a_2, \mathbf{L}, a_m\}$ be a finite set of alternatives, $C = \{c_1, c_2, \mathbf{L}, c_n\}$ be a finite set of attributes, and $W = \{w_1, w_2, \mathbf{L}, w_n,\}$ be a finite set of attribute weights. Let $R = (r_{tj})_{m \times n}$ be a decision matrix with attribute assessment values given in the form of IFN, where IFN $r_{tj} = (m_{ij}, n_{ij})$ denotes the degree to which the alternative a_i satisfies and does not satisfy the attribute c_j.

Obviously, if the information about attribute weights given by the decision makers is crisp real value, we can weight each attribute value and aggregate all the weighted attribute values corresponding to each alternative into the collective attribute values by formula (2). According to the score and accuracy degree of the collective values, we can rank the alternatives and choose the most desirable one. In the real world, however, it is very often that the information about attribute weights is incompletely known because of the inherent complexities of the MADM problems, time pressure or lack of knowledge. Let W be the set of the known information about attribute weights, which can be constructed by the following forms [16]:
1) A weak ranking: $w_i \ge w_j$;
2) A strict ranking: $w_i - w_j \ge d_i$, $d_i > 0$;
3) A ranking of differences: $w_i - w_j \ge w_k - w_l$, for $j \ne k \ne l$;
4) A ranking with multiples: $w_i \ge k_i w_j$, $0 \le k_i \le 1$;
5) An interval form: $0 \le d_i \le w_i \le d_i + e_i \le 1$.

In the MADM problems with incomplete attribute weights, before choosing the most desirable one among the candidate alternatives, we must determine the weight of each attribute. In the following, let us construct some optimization models based on TOPSIS method to derive the optimal attribute weights from uncertain information about attribute weights.

Step 1 Transform the intuitionisitc fuzzy decision matrix into the score matrix.

Base on the score function of IFN given in definition 3, we can transform the IFN decision matrix into the score matrix $S = (s_{tj})_{m \times n}$, where $s_{tj} = m_{ij} - n_{ij}$.

Step 2 Decide the positive idea solution and the negative idea solution.

Based on the score matrix, the positive idea solution S^+ and the negative idea solution S^- can be written as follows:

$$S^+ = (s_1^+, s_2^+, \mathbf{L}, s_n^+), \quad (3)$$
$$S^- = (s_1^-, s_2^-, \mathbf{L}, s_n^-), \quad (4)$$

where $s_j^+ = \max_i(s_{ij})$ and $s_j^- = \min_i(s_{ij})$.

Step 3 Calculate the distance between each alternative and the positive/negative idea solution.

The distance between alternative a_i and the positive idea solution S^+ can be calculated as follows:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left[w_j \left(s_{ij} - s_j^+ \right) \right]^2} . \qquad (5)$$

The distance between alternative a_i and the negative idea solution S^- can be calculated as follows:

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left[w_j \left(s_{ij} - s_j^- \right) \right]^2} . \qquad (6)$$

Step 4 Decide attribute weights from the given information about attribute weights.

According to the traditional TOPSIS method, the shorter the distance between each alternative and the positive ideal solution is, the better the alternative is; the longer the distance between each alternative and the negative ideal solution is, the better the alternative is. Therefore, for each alternative, we can construct the multi-objective programming models (7) and (8) to maximize d_i^+ and minimize d_i^- ($i = 1,2,\ldots m$).

$$\begin{cases} \min d_i^+(w) = \sqrt{\sum_{j=1}^{n} \left[w_j \left(s_{ij} - s_j^+ \right) \right]^2}, i = 1,2,\ldots m \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} , \quad (7)$$

$$\begin{cases} \max d_i^-(w) = \sqrt{\sum_{j=1}^{n} \left[w_j \left(s_{ij} - s_j^- \right) \right]^2}, i = 1,2,\ldots m \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} . \quad (8)$$

The models (7) and (8) can be reduced to the models (9) and (10).

$$\begin{cases} \min D_i^+(w) = \sum_{j=1}^{n} \left[w_j \left(s_{ij} - s_j^+ \right) \right]^2, i = 1,2,\ldots m \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} , \quad (9)$$

$$\begin{cases} \max D_i^-(w) = \sum_{j=1}^{n} \left[w_j \left(s_{ij} - s_j^- \right) \right]^2, i = 1,2,\ldots m \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} . \quad (10)$$

Because we have none of the preference for any alternative, the models (9) and (10) can be transformed into the two single objective programming models as follows:

$$\begin{cases} \min D^+(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_j^2 \left(s_{ij} - s_j^+ \right)^2 \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} , \qquad (11)$$

$$\begin{cases} \max D^-(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_j^2 \left(s_{ij} - s_j^- \right)^2 \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} . \qquad (12)$$

Combining model (11) and (12), we can get the model (13), which is a quadratic programming model.

$$\begin{cases} \min D(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_j^2 [(s_{ij} - s_j^+)^2 \\ \quad - (s_{ij} - s_j^-)^2] \\ \text{s.t. } w = (w_1, w_2, \ldots, w_n) \in W \end{cases} . \qquad (13)$$

Solving the quadratic programming model (13), we can get the vector of the attribute weights.

Step 5 Aggregate the attribute assessment values of each alternative into collective attribute value. Then calculate the score of collective attribute value corresponding to each alternative and order the alternatives.

4. Evaluation of Teaching Quality by the Proposed Approach

Suppose that we want to solve an evaluation of teaching quality problems in which the alternatives are four young teachers to be evaluated ($a_1 \sim a_4$) according to their teaching performances by the expert committee. The evaluation system includes the four indexes: teaching altitude (c_1), teaching content (c_2), teaching method (c_3) and teaching result (c_4). The assessment information provided in the form of IFN in the following denotes the membership degree and nonmenbership degree to which the alternatives corresponding to each attribute belong to the fuzzy concept “excellence”.

$$\begin{pmatrix} (0.80,0.10) & (0.75,0.20) & (0.80,0.05) & (0.85,0.10) \\ (0.65,0.25) & (0.85,0.05) & (0.80,0.05) & (0.75,0.20) \\ (0.75,0.20) & (0.75,0.15) & (0.85,0.10) & (0.70,0.25) \\ (0.80,0.15) & (0.70,0.20) & (0.75,0.10) & (0.75,0.15) \end{pmatrix}.$$

Step 1 Get the score decision matrix according to score function.

According to definition 3, we can transform the decision matrix with intuitionistic fuzzy information into the score matrix shown as follows:

$$\begin{pmatrix} 0.70 & 0.55 & 0.75 & 0.75 \\ 0.40 & 0.80 & 0.75 & 0.55 \\ 0.55 & 0.60 & 0.75 & 0.45 \\ 0.65 & 0.50 & 0.65 & 0.60 \end{pmatrix}.$$

Step 2 Get the positive idea solution and the negative idea solution.

The positive idea solution S^+ and the negative idea solution S^- can be calculated as follows:

$$S^+ = (0.70, 0.80, 0.75, 0.75),$$
$$S^- = (0.40, 0.50, 0.65, 0.45).$$

Step 3 Construct the optimization model to decide the vector of attribute weights.

Suppose that the known information about attribute weights is given in the form of the following:

$$W = \{\ 0.2 \le w_1 \le 0.3,\ 0.15 \le w_2 \le 0.3,\ w_2 - w_1 \ge 0.05, w_4 - w_3 \ge 0.1, w_3 \ge 0.5 w_2.\ \sum_{j=1}^{4} w_j = 1, w_j \ge 0,\ (\mathrm{j}=1,2,3,4)\ \}$$

According to model (13), we can construct the quadratic programming model (14)

$$\begin{cases} \min\ D(w) = -0.06 w_1^2 + 0.09 w_2^2 \\ \qquad\qquad - 0.02 w_3^2 + 0.03 w_4^2 \\ \text{s.t. } 0.2 \le w_1 \le 0.3, 0.15 \le w_2 \le 0.3, \\ \quad w_3 \ge 0.5 w_2, w_2 - w_1 \ge 0.05, \\ \quad w_4 - w_3 \ge 0.1, \\ \quad \sum_{j=1}^{4} w_j = 1, w_j \ge 0, (j = 1,2,3,4) \end{cases} . \qquad (14)$$

Solving model (14), we can get the vector of attribute weights $w = (0.200, 0.250, 0.225, 0.325)$.

Step 4 Calculate the collective values of each alternative and rank the alternatives according to their scores.

The collective values of the alternatives in the form of IFN are a_1: (0.8074, 0.1017), a_2: (0.7762, 0.1083), a_3: (0.7635, 0.1712), a_4: (0.7498, 0.1471).

The scores of collective attribute value of each alternative are:

$$s_1 = 0.7057, s_2 = 0.6679, s_3 = 0.5923, s_4 = 0.6027.$$

Since $s_1 > s_2 > s_4 > s_3$, hence, $a_1 \succ a_2 \succ a_4 \succ a_3$ and the most desirable alternatives is a_1.

5. Conclusions

In this paper, we investigate the MADM problems with attribute values given in the form of IFN and uncertain attribute weights. In order to derive attribute weights, some quadric programming models based on TOPSIS method are constructed. We aggregate the IFNs by IFWA operator into the collective values, based on which, the alternatives are ranked, and the most desirable one is chosen. The proposed models can be extended to solve the MADM problems with interval-valued intuitionistic fuzzy information and uncertain attribute weights.

Acknowledgments

This work was supported by Sichuan Center for Education Development Research (CJF10018) and Sichuan Provincial Education Commission (11ZA214).

References

[1] K. T. Atanassov, "Intuitionistic fuzzy sets", Fuzzy sets and systems, Vol. 20, No. 1, 1986, pp. 87-96.
[2] K. T. Atanassov, "More on intuitionistic fuzzy sets", Fuzzy sets and systems, Vol. 33, 1989, pp. 37-45.
[3] Z. S. Xu, and R. R. Yager, "Dynamic intuitionistic fuzzy multi-attribute decision making", International Journal of Approximate Reasoning, Vol. 48, 2008, pp. 246-262.
[4] Z. S. Xu, "Intuitionistic fuzzy aggregation operators", IEEE Transactions on Fuzzy Systems, Vol. 15, 2007, pp. 1179-1187.
[5] Z. S. Xu, and M. M. Xia, "Induced generalized intuitionistic fuzzy operators", Knowledge-Based Systems, Vol. 24, 2011, pp. 197-209.

[6] W. Yang, and Z.P. Chen, "The quasi-arithmetic intuitionistic fuzzy OWA operators", Knowledge-Based Systems, Vol. 27, 2012, pp. 219-233.

[7] M. M. Xia, Z. S. Xu, and B. Zhu, "Generalized intuitionistic fuzzy Bonferroni means", International Journal of Intelligent Systems, Vol. 27, 2012, pp.23-47.

[8] G. W. Wei. "Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making", Applied Soft Computing, Vol. 10, 2010, pp. 423-431.

[9] G. W. Wei, X. F. Zhao, R. Lin. "Some Induced Aggregating Operators with Fuzzy Number Intuitionistic Fuzzy Information and their Applications to Group Decision Making", International Journal of Computational Intelligence Systems, Vol. 3, 2010, pp. 84-95.

[10] D. H. Hong, and C. H. Choi, "Multicriteria fuzzy decision-making problems based on vague set theory", Fuzzy Set and Systems, Vol. 114, 2000, pp. 103-113.

[11] D. F. Li, Y. C. Wang, S. Liu, and F. Shan, "Fractional programming methodology for multi-attribute group decision-making using IFS", Applied Soft Computing, Vol. 9, 2009, pp. 219-225.

[12] G. W. Wei, "Maximizing deviation method for multiple attribute decision making in intuitionistic fuzzy setting", Knowledge-Based Systems, Vol. 21, 2008, pp. 833-836.

[13] G. W. Wei, "GRA method for multiple attribute decision making with incomplete weight information in intuitionistic fuzzy setting", Knowledge-Based Systems, Vol. 23, 2010, pp. 243-247.

[14] G. W. Wei, "Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making", Expert Systems with Appllications, Vol. 38, 2011, pp. 11671-11677.

[15] Z. J. Wang, K. W. Li, and W. Z. Wang, "An approach to multiattribute decision making with interval-valued intuitionistic fuzzy assessments and incomplete weights", Information Sciences, Vol. 179, 2009, pp. 3026-3040.

[16] Z. S. Xu, "Models for multiple attribute decision making with intuitionistic fuzzy information", International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, Vol. 15, 2007, pp. 285-297.

Yujun Luo received B.S. degree from Sichuan Normal University in 1996 and M. S. degree from Chengdu University of Technology in 2002. He currently works as an associate professor at Department of Computer Science and Mathematics, North Sichuan Medical College, Nachong, China. His current research interests include uncertain multiple attribute decision making with fuzzy information and computer applications.

Xianfu Li received B.S. degree from Sichuan Normal University in 1996 and M.S. degree from China West Normal University in 2008. He currently works as an associate professor at Department of Imaging Medicine, North Sichuan Medical College, Nachong, China. His current research interest is digital image processing in three-dimension conformal radiation therapy for tumor.

Ying Yang received M.S. degree from China West Normal University in 2006. She currently works as a lecturer at Department of Computer Science and Mathematics, North Sichuan Medical College, Nachong, China. Her current research interest is computer applications.

Zhenglong Liu received B.S. degree from China West Normal University in 2000 and M.E. degree from Sichuan University in 2008. He currently works as an associate professor at Department of Computer Science and Mathematics, North Sichuan Medical College, Nachong, China. His current research interests include intelligent optimization algorithms, computer applications and mathematical methods in biomedicine.