

Research on Spatial Estimation of Soil Property Based on Improved RBF Neural Network

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Abstract

To seek optimal network parameters of Radial Basis Function (RBF) Neural Network and improve the accuracy of this method on estimation of soil property space, this study utilizes genetic algorithm to optimize three network parameters of RBF Neural Network including the number of hidden layer nodes, expansion speed and root-mean-square error. Then, based on optimized RBF Neural Network, spatial interpolation is conducted for arable soil property under different sampling scales in the study area. The estimation result is superior to RBF Neural Network method without optimization and geostatistical method in terms of the fitting capacity and interpolation accuracy. Compared with the result of space estimation by RBF Neural Network method without optimization, among the 5 schemes, the forecast errors of RBF Neural Network optimized by genetic algorithm reduce greatly. Mean absolute error (MAE) reduces 0.4868 on the average and root-mean-square error (RMSE) reduces 1.492 on the average. Therefore, RBF Neural Network method optimized by genetic algorithm can gain the information about regional soil property spatial variation more accurately and provides technical support for arable land quality evaluation, accurate farmland management and rational application of fertilizer.

Keywords: Genetic algorithm, RBF Neural Network, Spatial forecast, Error analysis

1. Introduction

Soil is the loose surface with certain fertility covering the earth surface on which the plants can grow. Soil is formed through combined actions of parent material, climate, living beings, terrain, time and human factor. It has highly spatial heterogeneity [1]. Nutrient contents of arable soil are different in spatial distribution in different locations due to interactions of physical, chemical and biological processes. This is the specific representation of soil spatial heterogeneity [2]. Full understanding of changes in arable soil nutrients plays a vital role in soil nutrient management, rational application of fertilizer and improvement of farmland management efficiency [3, 4].

Foreign scholars put forward soil spatial variability as early as 1960s. The research methods underwent initial

Fisher statistical method, geostatistical method in late 1980s, geographic information technique, neural network and high-accuracy curve modeling. Since geostatistical method was introduced in soil property spatial variation study, it has become the major method. But in some circumstances, some preconditions cannot be met as follows: during use of Kriging interpolation, the study area must be homogeneous; different parts of the landscape different use semi variograms. So, Kriging cannot well describe spatial distribution of soil property with nonlinear characteristics [5,6,7]. Meanwhile, the complexity and peculiarity of geosciences phenomena make it hard to apply theoretical models established under various ideal conditions in practices. The parameters and even the structure of deterministic models need continuously modifying with the changes in the place and time. Thus, to a large extent, the universality of models is lost [8]. Artificial neural network is an approach to simulate biomechanism by computer. It has strong ability to deal with nonlinear system. In recent years, it has been gradually applied to study of soil property spatial variation [6,9]. José A. C. Ulson et al. [7] utilized back propagation network (BP Network) algorithm to train the soil property data collected from the field on the basis of designing a neural network with hidden-layer multi-layer perception, and then conducted spatial interpolation. The forecast accuracy of the result is higher than that of Kriging interpolation. Shen Zhangquan [5,10] compared soil nutrient spatial forecast by generalized regression network, integrated BP Network and Kriging interpolation under three sampling scales through designing three different soil sampling point collection schemes. The result showed in most cases, spatial forecast accuracy of generalized regression network, integrated BP Network was higher than that of Kriging interpolation. Besides, with the decrease in the number of samples, interpolation accuracy of generalized regression network, integrated BP Network showed more superiority. But these studies just established a mapping relation between space coordinates and soil properties, and overlooked other ecological processes at sampling point locations. Moreover, soil property spatial variability is very complex, which makes this method unable to fully reflect spatial

variation characteristics of soil property. Later, Li Qiquan et al. [6] utilized Radial Basis Function (RBF) Neural Network to study on spatial interpolation of soil property with different degrees of variation by RBF Neural Network under the condition of adding adjacent sampling point information as network input and compared RBF Neural Network method only taking space coordinates as network input and Kriging interpolation. The result showed the ability of RBF Neural Network adding adjacent sampling point information input to describe spatial distribution of soil property information improved greatly and could well reflect local variation information of soil property. However, RBF Neural Network has some problems in network topology, width and center confirmation as well as weight calculation from the hidden layer to the output layer, thus imposing great influence on interpolation accuracy. The researches of Chai Jie et al. [11] and Li Yu et al. [12] show genetic algorithm can optimize the weight of RBF Neural Network and network hidden-layer structure. Based on this, Dong Min et al. [13] utilized genetic RBF Neural Network model to optimize the weight of from the hidden layer to the output layer of RBF Network, then adopted optimized RBF Network to carry out spatial interpolation for available zinc in the soil in the study area and then compared it with the interpolations of RBF Network without optimization and Kriging. The result showed the error of the interpolation of genetic RBF Neural Network was small and that the interpolation chart could better reflect practical spatial distribution of available zinc element in the soil. But, there are many parameters needing confirming in RBF Neural Network. Only through optimizing the weight from the hidden layer to the output layer, the improvement effect of forecast accuracy is not obvious.

This study utilizes three network parameters of RBF Neural Network including the number of hidden layer nodes, expansion speed and root-mean-square error to design 5 different soil sampling point layout schemes on the basis of 637 soil samples in line with the thought of gradual improvement of sampling scales. Besides, under different sampling scales, the fitting capacity and estimation accuracy of soil property space of RBF Neural Network optimized by genetic algorithm, Ordinary Kriging as well as RBF Neural Network without optimization are respectively compared, which provides technical support for accurately estimating soil property spatial variability, reducing the number of soil samples and decreasing the cost of soil sampling.

2. Research Methods

2.1 Preprocessing of soil sampling point data

To check the accuracy of interpolation method on spatial interpolation of available phosphorus in soil, the soil sampling points should be first divided. Create Subsets function in Geostatistical Analyst module of ArcGIS software was used to sample. Besides, spatial distribution of soil samples should be even. Through referring to the studies of Lei Nengzhong et al., the modeling scheme of 5 different sampling scales was set up at the interval of 100 sampling points [14]. 500 training samples were drawn from 637 available phosphorus sampling points. The sampling point layout composed of this dataset is Scheme e. Then, based on sampling point layout of Scheme e, 400 sampling points were drawn at random from 500 training samples to form Scheme d. Then, based on Scheme d, 300 sampling points were drawn at random as Scheme c. Scheme d and a can be formed by parity of reasoning. Finally, after Scheme e was formed, 100 sampling points were drawn from 137 soil sampling points as the check sample of interpolation results of all schemes.

2.2 Interpolation of RBF neural network

Radial Basis Function Neural Network (hereinafter referred to as RBF Neural Network) was formed through Broomhead and Lowe [15,16] applying radial basis function raised by Powell (1985) to artificial neural network. Initially, it was used to interpolate data points in a group of multi-dimensional space. The objective of interpolation was to seek a function which could map each vector to corresponding target values.

In the process of spatial interpolation of RBF Neural Network, Gaussian kernel function was adopted in this study as the basis function:

$$\mu_j = \exp\left(-\frac{(X - C_j)^T(X - C_j)}{2\delta_j^2}\right), j = 1, 2, \dots, N_h \quad (1)$$

Where, μ_j is the output of nodes at the j^{th} hidden layer; X is output sample; C_j is the central value of Gaussian kernel function; δ_j is a standard constant; N_h is the number of nodes at the hidden layer. The output range of the nodes is between 0 and 1. Besides, the input sample is closer to the center of nodes, and the output value is larger. This paper adopts Matlab neural network tool kit to realize RBF spatial interpolation. The main steps are as follows:

(1) Data preprocessing

In ArcEngine, geodetic coordinates were transformed to decimal plane rectangular coordinates. The property row

was formed for longitude and latitude coordinates of sampling points and added in ArcGIS. Derivation function of ArcGIS was utilized to derive the file in dbf format. Then, decimal system longitude and decimal system latitude were drawn from soil sampling point coordinated system and put into a new Excel. Then the data in Excel were processed by Matlab.

To prevent excessive variable value of Matlab in operational process and improve learning speed, normalization processing is required for coordinate data. Assume left bottom and top right corner within spatial area coverage of interpolations are (x_{\min}, y_{\min}) and (x_{\max}, y_{\max}) respectively, and

$$x' = (x - x_{\min}) / (x_{\max} - x_{\min}) \quad (2)$$

$$y' = (y - y_{\min}) / (y_{\max} - y_{\min}) \quad (3)$$

As well, normalization processing is conducted for main physicochemical index values of soil. Assume the minimum value and the maximum value of a physicochemical index value of soil are Z_{\min} and Z_{\max} respectively, and normalization express is:

$$Z' = (Z - Z_{\min}) / (Z_{\max} - Z_{\min}) \quad (4)$$

(2) Generation of interpolation grid point

The grid can be established according to the size by use of Matlab order meshgrid in line with specific conditions of the study area. Denser grid means higher interpolation accuracy, the operating rate reduces exponentially. For grid coordinate points set up, the coordinate value should be calculated under normalization according to coordinate normalization formula.

(3) Seeking 5 data points nearest to the training point

Ergodic program was adapted to train sampling point dataset. Euclidean distance between other sampling points in the dataset and the training point was calculated. The computational formula is:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (5)$$

d_{ij} means spatial distance between the sampling point i and j ; (x_i, y_i) refers to the coordinate of i^{th} sampling point; (x_j, y_j) refers to the coordinate of j^{th} sampling point.

Sort algorithm was used to sequence spatial distance stored in the array. 5 nearest sampling points were selected according to the principle of $d_1 \leq d_2 \leq d_3 \leq d_4 \leq d_5$ and all information was stored in the array.

(4) Network establishment and analog simulation

Matlab neural network tool kit function was used to establish RBF Neural Network. Firstly, training dataset array was substituted into RBF Neural Network for training so as to gain non-linear relationship between the property of soil sampling points and adjacent points

respectively. Then, grid generated in (2) was substituted into Neural Network for simulation, thus getting soil property value of any grid unit in the whole grid.

(5) Comparison of data recovery and interpolation effect Recovery operation was implemented for interpolation-generated soil property values and geographical coordinates. Testing data were used to evaluate the accuracy of interpolation model and obtained the value of accuracy evaluation factor under current operation mode. Finally, the data can be written in Excel through Matlab function. Latticed data can be shown in ArcGIS through ArcGIS grid analysis.

2.3 RBF Neural Network interpolation improved by genetic algorithm

Genetic algorithm is a theory and approach with initiative significance jointly studied by a psychology professor in University of Michigan – Holland as well as his colleagues and students in 1975. Such approach was a highly concurrent, random and self-adapting search algorithm developed by referring to natural selection and evolutionary mechanism in the biosphere. This paper adopts genetic algorithm to seek optimized the number of nodes at the hidden layer, expansion speed and root-mean-square error of RBF Neural Network. Binary coding is adopted as gene code system in accordance with roulette model [17] as realization model of genetic algorithm natural selection. In this paper, the main steps to combine genetic algorithm and RBF Neural Network are as follows:

(1) Rewrite interface function of RBF interpolation algorithm; possible value scopes of the number of nodes at the hidden layer, expansion speed and root-mean-square error in RBF Neural Network are revealed in variable form for use.

(2) Solve the maximal length of binary coding necessary in genetic algorithm. In this study, the maximum value of parameter scope subtracts the minimum value of the scope. Then, compare the value with the figure expressed by binary coding so as to solve the shortest binary system length required by parameter scope, i.e. chromosome length.

(3) Produce initial group. An initial group is produced by Random function of Matlab. The length is the large random matrix of the sum of all lengths expressed by binary of chromosome.

(4) Calculate the fitness of each individual in the group. Draw individual gene length information and read binary coding of the gene by sections. Then, utilize transformation relation between binary number and

decimal numeral to calculate segmental gene information to decimal integers or decimals. Establish fitness function of genetic algorithm with the measurement standard of root-mean-square error and mean value error of spatial interpolation of RBF Neural Network model. Interpolation is carried out for the given points through modifying RBF spatial interpolation program and utilizing well-trained network. Then, solve root-mean-square error and mean value error.

(5) Method to gain optimized RBF network structure. In given iterative algebra range, calculate root-mean-square error and mean value error for all groups in each iteration and store them in temporary array. After solving fitness function of each iteration, solve the fitness of the most excellent individual and the genotype of this individual through sort algorithm and record them into external temporary array. The program will record the optimized individuals and their genes (RBF network parameter) of corresponding algebras according to specified iterative algebra, and sequence again and compare the numerical values in this array when finally returning to optimal individuals. The program will ultimately gain the algebra and individual with the best fitness.

(6) Implementation method of natural selection. The process of practical programming and running shows most fitness function values solve previously are between 0 and 1. Therefore, the large array can be produced through the way of generating pseudo-random numbers between 0 and 1 to simulate selection requirements of natural environment so as to retain individuals meeting natural selection conditions and weed out those not adapting environmental requirements.

(7) Implementation method of gene crossover. For individuals retained, every two are selected as parent individuals. Random function Rand is adopted to produce a random number between 0 and 1. If such random number is less than crossover probability, gene crossover operation is carried out. Under the effect of random function, a position in corresponding gene segments of both parents is produced as the cross point. Individual exchange is carried out for the corresponding gene of both parents at the central position of the cross point.

(8) Implementation method of genovariation. For each offspring individual, it is required to judge whether the random values between 0 and 1 produced by random function are less than variation probability specified by the function. If they are less than the variation probability, variation operation is implemented, or else, variation operation is not implemented for individuals. In corresponding gene segment of individuals, specific variation points may be gained through the values between 0 and 1 produced by random function multiplying by the length of the gene segment. Then,

variation operation is conducted under binary condition. This study is realized through judging binary values on the gene points. If the value is 1, it changes to 0; if the value is 0, it changes to 1.

In the end, the parameters of the number of nodes at the hidden layer, expansion speed and root-mean-square error gained through genetic algorithm optimization are input into RBF Neural Network. Interpolation grid is generated by use of meshgrid order in Matlab. The dataset of ergodic program is adopted to train sampling points and calculate Euclidean distance between other sampling points in the dataset and the training sample point so as to seek 5 adjacent points (soil features participating in the training). In the interpolation process, nonlinear function between soil property values z and x hide in the network after convergence. The specific expression is unknown. Geographical coordinates are regarded as network input to realize forecast of soil property space at unknown points.

2.4 Evaluation of interpolation accuracy

Spatial interpolation accuracy is tested by authentication dataset. Interpolation accuracy evaluation is conducted through comparing mean absolute error (MAE) and root-mean-square error (RMSE) of the forecasted value of soil property and measured value at the verification point. MAE reflects actual measurement error range of estimated values. The error can be given quantitatively. RMSE reflects the effect of the estimated value and the extreme value of sampling point data. The computational formulas are as follows:

$$MAE = \frac{1}{n} \sum_{k=1}^n |\hat{Z}_k - Z_k| \quad (6)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^n (\hat{Z}_k - Z_k)^2} \quad (7)$$

In the expression, \hat{Z}_k refers to the forecasted data of he points to be tested; Z_k means measured data at the testing point; n means the number of sampling points tested. It can be seen from the expression that two smaller parameters indicates higher accuracy when evaluating interpolation method.

3 Result and analysis

3.1 Parameters of RBF Neural Network interpolation

When spatial interpolation is implemented by use of RBF Neural Network, since optimized parameters of the

network cannot be gained in advance, “test method” is generally adopted for repeated trials. Then, the parameters can be confirmed in line with the whole accuracy of the network. In this study RBF Neural Network parameters based on adjacent points and geographical coordinates use defaults of Matlab software. The number of nodes at the hidden layer is 8; the expansion parameter is 1.0; the error coefficient is 0.001.

3.2 RBF Network Parameter parameters improved by genetic algorithm

RBF Neural Network improved by genetic algorithm can give full play to the characteristic of global optimization

of genetic algorithm to seek optimized parameters of RBF Neural Network. The researches of Zhou Ming et al. [18] show the parameters of RBF Neural Network improved by genetic algorithm can be given through experience. Generally, the expansion parameter is 1; the group size is 20-100; the crossover probability is 0.4-0.99; variation probability is 0.0001-0.1; the end algebra is 100-1000. The number of nodes at the hidden layer is gained by trial method. Through trial and estimated empirical values, genetic algorithm network structure and parameters are shown in Table 1; structural parameters of RBF Neural Network improved by genetic algorithm are shown in Table 2.

Table 1 Structure and parameters of genetic algorithm neural network

| Scheme | Expansion parameter | Number of individuals | Crossover probability | Variation probability | Genetic algebra |
|--------|---------------------|-----------------------|-----------------------|-----------------------|-----------------|
| A | 1 | 30 | 0.7 | 0.1 | 100 |
| B | 1 | 30 | 0.7 | 0.1 | 100 |
| C | 1 | 30 | 0.7 | 0.2 | 100 |
| D | 1 | 30 | 0.7 | 0.2 | 100 |
| E | 1 | 30 | 0.7 | 0.2 | 100 |

Table 2 RBF Neural Network improved by genetic algorithm

| Scheme | Number of nodes at hidden layer | Expansion speed | Error coefficient |
|--------|---------------------------------|-----------------|-------------------|
| A | 2 | 0.128014 | 0.001713 |
| B | 2 | 3.374016 | 0.078953 |
| C | 30 | 0.411811 | 0.058685 |
| D | 2 | 3.374016 | 0.078953 |
| E | 1 | 1.191339 | 0.020488 |

To evaluate the fitness capacity of RBF Neural Network improved by genetic algorithm, this study will utilize such method for contrastive analysis of spatial forecast result of soil organic matter under 5 sampling scales with that of RBF Neural Network method without optimization and Ordinary Kriging method through scatter diagram so as to solve regression equation of the forecasted value and measured value of the training sample and the determination coefficient (R^2). The matching degree of the forecasted value and the measured value of the training sample is judged by the determination coefficient. If the

determination coefficient approaches 1, this indicates the matching degree is higher and the fitness capacity of interpolation method is stronger. Under five sampling scales, the scatter diagram of forecast results gained by 3 spatial estimation methods are shown in Fig.1-5 (Ordinary Kriging is Ordinary Kriging interpolation method; RBF is the method of RBF Neural Network spatial interpolation without optimization; GARBF is the method of RBF Neural Network spatial interpolation optimized by genetic algorithm).

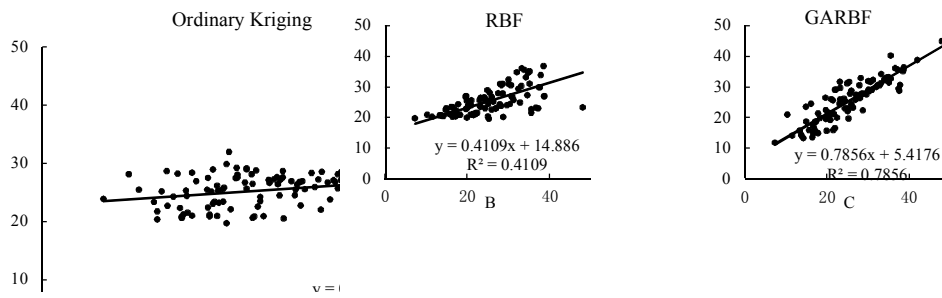


Fig.1 Scatter diagram of measured values and forecasted values of training sample gained by 3 methods in Scheme A

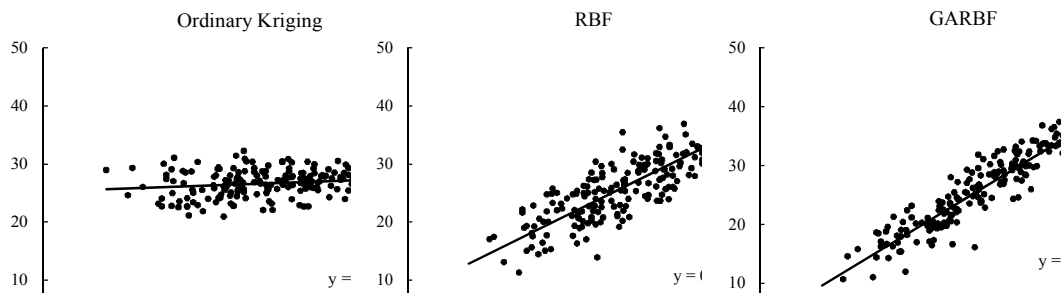


Fig.2 Scatter diagram of measured values and forecasted values of training sample gained by 3 methods in Scheme B

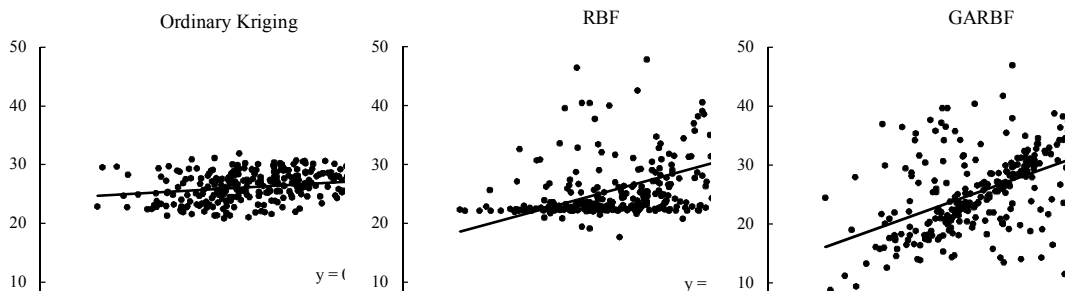


Fig.3 Scatter diagram of measured values and forecasted values of training sample gained by 3 methods in Scheme C

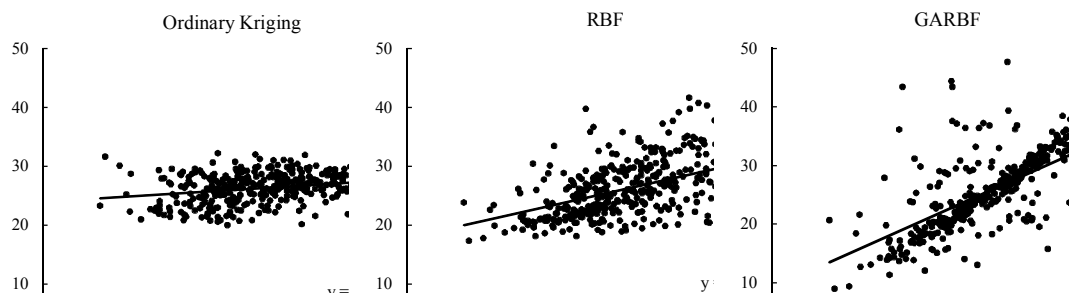


Fig.4 Scatter diagram of measured values and forecasted values of training sample gained by 3 methods in Scheme D

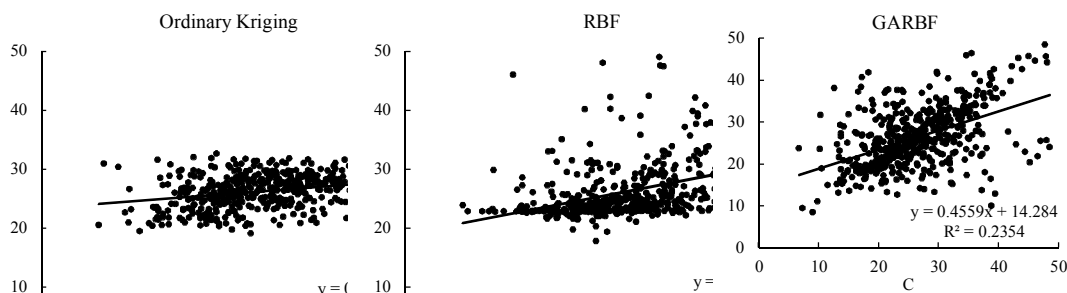


Fig.5 Scatter diagram of measured values and forecasted values of training sample gained by 3 methods in Scheme E

Note: Ordinary Kriging is Ordinary Kriging interpolation method; RBF is RBF Neural Network method based on adjacent points; GARBF is RBF Neural Network method optimized by genetic algorithm.

Table 3 Comparison of approximate errors of training samples in all cases

| | Scheme A | | Scheme B | | Scheme C | | Scheme D | | Scheme E | |
|------------------|----------|-------|----------|-------|----------|-------|----------|-------|----------|-------|
| | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE | MAE | RMSE |
| Ordinary Kriging | 6.547 | 8.495 | 6.056 | 7.520 | 5.275 | 7.099 | 5.147 | 6.247 | 5.187 | 6.313 |
| RBF | 6.221 | 7.724 | 5.647 | 7.086 | 5.478 | 6.864 | 5.383 | 7.346 | 6.035 | 7.507 |
| GARBF | 5.673 | 6.059 | 5.321 | 5.835 | 4.892 | 5.391 | 4.903 | 5.631 | 4.941 | 6.152 |

Note: MAE is mean absolute error; RMSE is root-mean-square error; Ordinary Kriging is Ordinary Kriging method; RBF is RBF Neural Network method based on adjacent points; GARBF is RBF Neural Network method optimized by genetic algorithm.

5. Conclusion and discussion

Since spatial variability of soil property is large, three important preconditions during application of geostatistics and smooth effect of Kriging interpolation cannot be met. These to some extent cause the inaccuracy of expressing abnormal area of soil property, thus reducing reliability of the forecast result [5,13]. Infinite approximation capability of RBF Neural Network can well solve this problem, but some problem still exists in optimization of network parameters. Therefore, in this study three parameters of RBF Neural Network optimized by genetic algorithm including the number of nodes at the hidden layer, expansion speed and RMSE are used to improve the accuracy and reliability of spatial estimation of soil property. The study result shows: spatial interpolation ability of RBF Neural Network optimized by genetic algorithm is superior to RBF Neural Network without optimization and geostatistics. Such superiority is not just reflected in the fitness capacity of RBF Neural Network method. In the aspect of testing MAE of RMSE between the forecasted value and the measured value of the samples, RBF Neural Network optimized by genetic algorithm is obviously less than other two methods. Besides, the superiority is more significant when the quantity of interpolation sampling points is less.

Very complex non-linear relationship exists between soil properties and various influencing factors. Besides, mutational boundary exists among different influencing factors [19]. So, when spatial estimation of soil property is conducted by use of neural network, if relevant factors such as parent material, soil type, planting system, elevation, climate element as well as other soil properties can be blended in the network, not only the stability and forecast accuracy of the network can be improved, but also synchronous estimation of multiple properties of soil can be realized. What's more, if soil property estimation process can well comply with geoscience laws, spatial estimation will tend to realer and more reasonable, and can better describe detailed information of soil property vibration.

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