IJCSI International Journal of Computer Science Issues, Vol. 10, Issue 1, No 3, January 2013
ISSN (Print): 1694-0784 | ISSN (Online): 1694-0814
www.IJCSI.org

The Analysis of Vibration Characteristics and Motion Stability of the Tracked Ambulance Nonlinear Damping System

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Abstract

Considering the impact of the nonlinear stiffness, a 2 DOF dynamic nonlinear vibration model with cubic terms was established according to the structural feature and nonlinear behavior of the tracked ambulance. In the case of primary resonance and 1: 1 internal resonance, multiple scale method was used to obtain a first-order approximate solution for this model. Taking the parameters of the tracked ambulance for instance, the approximate solution was verified and the influence of the parameters on damping effect was investigated. Finally, the motion stability of the damping system was analyzed with singularity theory and the theoretical bases for improving efficiency of the damping system were provided.

Keywords: *Damping System; Cubic Nonlinearity, Multiple Scale Method; Internal Resonance; Stability*

1. Introduction

The tracked ambulance can be delivered through a variety complex terrain and implement first aid to the sick and wounded. In order to achieve the safe transfer and implement first aid on the way, it is often necessary to demand the tracked ambulance has good mobility and meet the special needs of the sick and wounded of comfort. For the tracked ambulance is refitted by crawler chassis, he installation of the vehicle damping system becomes the main way to improve the ride comfort of the sick and wounded.

The tracked ambulance damping system is composed of the carriage, the stretcher base, the chassis and the nonlinear shock absorber. Hence, it can be easily converted into a multi-degree of freedom nonlinear vibration system. The use of the nonlinear vibration system presents various advantages, such as better performance in the inhibition of broadband vibration, especially low-frequency vibration. However, complex mechanical properties usually exist in a nonlinear vibration system such as chaos and bifurcation, which makes it difficult to be analytic calculation and analysis, therefore approximate analytic algorithm widely used. Christopher Lee [4] investigated suspended,elastic cables driven by planar excitation with near commensurable natural frequencies in a 2:1 ratio. The first order analysis shows that there are saturation and jump phenomena and the first order analysis reveals that the cubic nonlinearities disrupt saturation. Li Jian et al [5] applied multiple scales method and Runge-Kutta to study the nonlinear vibration characteristics of the axial movement, multi-layered cylindrical shells made from composites. The results show some nonlinear properties of the system such as the phenomenon of internal resonance and indicate that excitation amplitude, damping and speed can affect the response amplitude, range of interval resonance and soft feature of the system. Zhang Xin et al [6] used average method to analyze piecewise nonlinear characteristics of the viscoelastic shocker absorber and the relationship of amplitude-frequency characteristics and system parameters. Li Xinye [7] used average method to study the possibility of delay feedback control over the gyroscope system under force. Tsuyoshi Inoue [8] investigates the vibration phenomena of the one-degree-of-freedom magnetically levitated system considering the effect of the nonlinearity of the electromagnet, using a shooting method.

In this paper, the differential equations of the 2 DOF tracked ambulance nonlinear damping system, including the cubic nonlinear spring was presented. In the case of primary resonance and 1: 1 internal resonance, multiple scale method was used to obtain a first-order approximate solution of the differential equations. Taking the parameters of the tracked ambulance for instance, the accuracy of the approximate solution was established by compared to numerical results. The influence of the parameters on damping effect and motion stability was also investigated. Furthermore, the theoretical bases for improving efficiency of the damping system were put forward.

2. Damping System Physical Model

The tracked ambulance damping system is shown in Figure 1. Damping system is mainly composed of rubber damping shock absorber and zero stiffness damper. The linear model is used to describe the stiffness and damping of the rubber damping shock absorber. For zero stiffness damper, the damping is described by linear model and stiffness is described by positive and negative stiffness parallel model[9], shown in Figure 2.

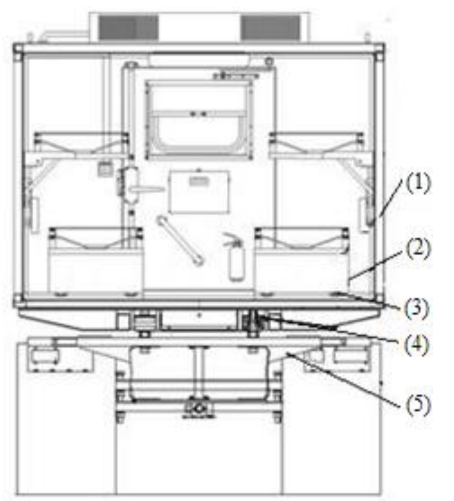


Fig.1 Ambulance damping system

(1) Carriage(2)Stretcher base(3) Zero stiffness damper (4)Rubber damping shock absorber(5) Coach chassis

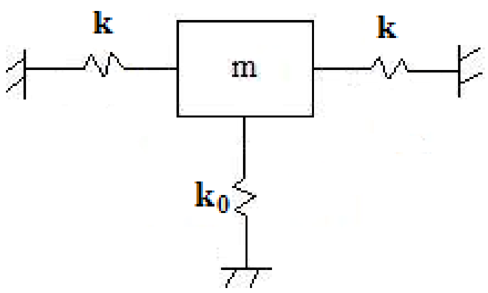


Fig.2 Positive and negative stiffness parallel model

The stiffness, original length and initial deformation of horizontal spring, in Figure1, are defined as k , L and λ. k_0 Stands for the stiffness of vertical spring. The vertical elastic restoring force of the model can be expressed in form

$$f(x)=k_0x-k[x-\frac{L-\lambda}{\sqrt{1-(x/L)^2}}\frac{x}{L}] \quad (1)$$

Using the Taylor series, I seek a second-order expansion in the form

$$\frac{1}{\sqrt{1-(x/L)^2}}=1+\frac{1}{2}(\frac{x}{L})^2+\frac{13}{24}(\frac{x}{L})^4+\cdots \quad (2)$$

Substitute the first two into the Eq.(1) result in

$$f(x)=(k_0-\frac{k\lambda}{L})x+\frac{L-\lambda}{L^3}\frac{k}{2}x^3 \quad (3)$$

Hence, the restoring force of quasi-zero stiffness damper can be expressed in form

$$f(z)=K_s x+\beta K_s x^3 \quad (4)$$

Where, $K_s=(k_0-\frac{k\lambda}{L})$, $\beta K_s=\frac{L-\lambda}{L^3}\frac{k}{2}$ and β is a small parameter.

According to the occupant of the vehicle ride(lying) comfort evaluation standards, occupant comfort is mainly affected by the vertical vibration acceleration. Ignoring the other two directions of vibration, the 2 DOF model of the tracked ambulance damping system is shown in Figure 3.

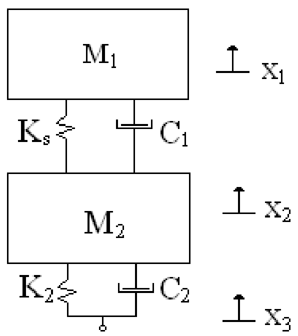


Fig.3 The 2 DOF damping system

Including:

M_1—The quality of stretcher and decubital body;

M_2—The quality of carriage;

K_s—The stiffness of quasi-zero stiffness damper;

C_1—The damping of zero stiffness damper;

K_2—The stiffness of rubber damping shock absorber;

C_2—The damping of rubber damping shock absorber;

x_1、x_2、x_3—Stretcher base vibration displacement, Carriage vibration displacement, Chassis vibration displacement.

The differential equations describing the motion of the damping system are

$$M_1\ddot{x}_1 + C_1(\dot{x}_1-\dot{x}_2)+K_s(x_1-x_2)+\beta K_s(x_1-x_2)^3=0 \quad (5)$$

$$M_2\ddot{x}_2 - C_1(\dot{x}_1-\dot{x}_2)-K_s(x_1-x_2)-\beta K_s(x_1-x_2)^3 + C_2(\dot{x}_2-\dot{x}_3)+K_2(x_2-x_3)=0 \quad (6)$$

We rewrite the Eq.(5) and (6) as

$$\ddot{x}_1+\omega_1^2 x_1 = l_1 x_2 - 2u_1\dot{x}_1 + 2u_1\dot{x}_2 - b_1(x_1-x_2)^3 \quad (7)$$

$$\ddot{x}_2+\omega_2^2 x_2 = f\cos\Omega t + 2u_2\dot{x}_1 - 2u_3\dot{x}_2 + l_2 x_1 + b_2(x_1-x_2)^3 \quad (8)$$

Where $\omega_1^2 = K_s/M_1$, $l_1 = K_s/M_1$, $2u_1 = C_1/M_1$, $b_1 = \beta K_s/M_1$, $2u_2 = C_1/M_2$, $\omega_2^2 = (K_2+K_s)/M_2$, $f\cos\Omega t = K_2x_3 + C_2\dot{x}_3$, $2u_3 = (C_1+C_2)/M_2$, $l_2 = K_s/M_2$, $b_2 = \beta K_s/M_2$.

3. Perturbation Analysis

Using multi-scale method, the dynamic response of damping system is solved. The new independent time scales

$$T_n = \varepsilon^n t \qquad n = 0,1,\cdots \quad (9)$$

are introduced where ε represent a small positive parameter and T_n, $n=0,1,\cdots$ are 'slow' time scales which capture the response due to the nonlinearities, damping and external excitation. And we note that

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \cdots \quad (10)$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2(D_1^2 + 2D_0D_2)+\cdots \quad (11)$$

Where $D_n = \partial/\partial T_n$, $n=0,1,\cdots$. We expand the time-dependent variable x_1 and x_2 in powers of ε as

$$x_1 = x_{11}(T_0,T_1)+\varepsilon x_{12}(T_0,T_1) \quad (12)$$

$$x_2 = x_{21}(T_0,T_1)+\varepsilon x_{22}(T_0,T_1) \quad (13)$$

Then we substitute Eq.(10)-(13) into the Eq.(7) and (8), equate coefficients of like powers of ε ,and obtain the following:

Order(ε^0):

$$\begin{cases} D_0^2 x_{11} + \omega_1^2 x_{11} = 0 \\ D_0^2 x_{21} + \omega_1^2 x_{21} = 0 \end{cases} \quad (14)$$

Order(ε^1):

$$\begin{cases} D_0^2 x_{12} + \omega_1^2 x_{12} = -2D_0(D_1x_{11} + u_1x_{11} - u_1x_{21}) \\ +l_1x_{21} - b_1x_{11}^3 + 3b_1x_{11}^2x_{21} - 3b_1x_{11}x_{21}^2 + b_1x_{21}^3 \\ D_0^2 x_{22} + \omega_1^2 x_{22} = -2D_0(D_1x_{21} - u_2x_{11} + u_3x_{21}) \\ +l_2x_{11} + b_2x_{11}^3 - 3b_2x_{11}^2x_{21} + 3b_2x_{11}x_{21}^2 - b_2x_{21}^3 \\ +f\cos(\Omega T_0) \end{cases} \quad (15)$$

The solution of Eq. (14) can be expressed as

$$\begin{cases} x_{11} = A_1(T_1)\exp(i\omega_1 T_0) + cc \\ x_{21} = A_2(T_1)\exp(i\omega_2 T_0) + cc \end{cases} \quad (16)$$

To express 1:1 internal resonance and the nearness of the excitation frequency to the first order natural frequency, we introduce two detuning parameter σ_1 and σ_2 defined by $\omega_2 = \omega_1 + \varepsilon\sigma_1$, $\Omega = \omega_1 + \varepsilon\sigma_2$.

Substitution of Eq.(16) and $\omega_2 = \omega_1 + \varepsilon\sigma_1$, $\Omega = \omega_1 + \varepsilon\sigma_2$ into Eq.(15) leads to secular terms. Elimination of these secular terms leads to the two state equations

$$\begin{cases} -2A_1' i\omega_1 - 2u_1A_1 i\omega_1 + 2u_1 i\omega_2 A_2\exp(i\sigma T_0) \\ +l_1A_2\exp(i\sigma T_0) - 3b_1A_1^2\bar{A}_1 + 3b_1A_2^2\bar{A}_2\exp(i\sigma T_0) \\ +3b_1A_1^2\bar{A}_2\exp(-i\sigma T_0) + 6b_1A_1\bar{A}_1A_2\exp(i\sigma T_0) - \\ 6b_1A_1\bar{A}_2A_2 - 3b_1\bar{A}_1A_2^2\exp(2i\sigma T_0) = 0 \\ -2A_2' i\omega_2 - 2u_3A_2 i\omega_2 + 2u_2A_1 i\omega_1\exp(-i\sigma T_0) \\ +l_2A_1\exp(-i\sigma T_0) + 3b_2A_1^2\bar{A}_1\exp(-i\sigma T_0) - \\ 3b_2A_2^2\bar{A}_2 - 3b_2A_1^2\bar{A}_2\exp(-2i\sigma T_0) - \\ 6b_2A_1\bar{A}_1A_2 + 6b_2A_1\bar{A}_2A_2\exp(-i\sigma T_0) + \\ 3b_2\bar{A}_1A_2^2\exp(i\sigma T_0) + \frac{1}{2}f\exp(i\sigma_2T_1 - i\sigma_1T_1) = 0 \end{cases} \quad (17)$$

Where $A_n' = D_1A_n$, $\sigma = \varepsilon\sigma_1$ .Introducing the polar form

$$A_n = \frac{1}{2}a_n\exp(i\theta_n),\ n=1,2$$

into Eq.(17) and separating the equation into real and imaginary parts results in the following four state equations,

$$-a_1'\omega_1 - u_1a_1\omega_1 + u_1\omega_2a_2\cos\gamma + \frac{1}{2}l_1a_2\sin\gamma + \frac{3}{8}b_1a_2^3\sin\gamma + \frac{3}{8}b_1a_1^2a_2\sin\gamma - \frac{3}{8}b_1a_1a_2^2\sin 2\gamma = 0 \quad (18)$$

$$a_1\theta_1'\omega_1 - u_1\omega_2a_2\sin\gamma + \frac{1}{2}l_1a_2\cos\gamma - \frac{3}{8}b_1a_1^3 + \frac{3}{8}b_1a_2^3\cos\gamma + \frac{9}{8}b_1a_1^2a_2\cos\gamma - \frac{3}{4}b_1a_1a_2^2 - \frac{3}{8}b_1a_1a_2^2\cos 2\gamma = 0 \quad (19)$$

$$-a_2'\omega_2 - u_3a_2\omega_2 + u_2a_1\omega_1\cos\gamma - \frac{1}{2}l_2a_1\sin\gamma - \frac{3}{8}b_2a_1^3\sin\gamma + \frac{3}{8}b_2a_1^2a_2\sin 2\gamma - \frac{3}{8}b_2a_1a_2^2\sin\gamma + \frac{1}{2}f\sin\varphi = 0 \quad (20)$$

$$a_2\omega_2\theta_2' + u_2a_1\omega_1\sin\gamma + \frac{1}{2}l_2a_1\cos\gamma + \frac{3}{8}b_2a_1^3\cos\gamma - \frac{3}{4}b_2a_1^2a_2 - \frac{3}{8}b_2a_1^2a_2\cos 2\gamma + \frac{9}{8}b_2a_1a_2^2\cos\gamma - \frac{3}{8}b_2a_2^3 + \frac{1}{2}f\cos\varphi = 0 \quad (21)$$

where $\gamma = \sigma T_0 + \theta_2 - \theta_1$, $\varphi = \sigma_2T_1 - \sigma_1T_1 - \theta_2$. At steady state, $a_n' = \theta_n' = 0$ and the average Eq.(18)-(21) reduced to the form

$$-u_1a_1\omega_1 + u_1\omega_2a_2\cos\gamma - \frac{3}{8}b_1a_1a_2^2\sin 2\gamma + \frac{3}{8}b_1a_1^2a_2\sin\gamma + \frac{3}{8}b_1a_2^3\sin\gamma + \frac{1}{2}l_1a_2\sin\gamma = 0 \quad (22)$$

$$a_1\sigma_2\omega_1 - u_1\omega_2 a_2\sin\gamma + \frac{1}{2} l_1 a_2\cos\gamma - \frac{3}{8} b_1 a_1^3 + \frac{3}{8} b_1 a_2^3\cos\gamma + \frac{9}{8} b_1 a_1^2 a_2\cos\gamma - \frac{3}{4} b_1 a_1 a_2^2 - \frac{3}{8} b_1 a_1 a_2^2\cos 2\gamma = 0 \quad (23)$$

$$-u_3 a_2\omega_2 + u_2 a_1\omega_1\cos\gamma - \frac{3}{8} b_2 a_1^3\sin\gamma + \frac{3}{8} b_2 a_1^2 a_2\sin 2\gamma - \frac{1}{2} l_2 a_1\sin\gamma - \frac{3}{8} b_2 a_1 a_2^2\sin\gamma + \frac{1}{2} f\sin\varphi = 0 \quad (24)$$

$$a_2\omega_2(\sigma_2-\sigma_1) + u_2 a_1\omega_1\sin\gamma + \frac{1}{2} l_2 a_1\cos\gamma + \frac{3}{8} b_2 a_1^3\cos\gamma - \frac{3}{8} b_2 a_2^3 - \frac{3}{8} b_2 a_1^2 a_2\cos 2\gamma - \frac{3}{4} b_2 a_1^2 a_2 + \frac{9}{8} b_2 a_1 a_2^2\cos\gamma + \frac{1}{2} f\cos\varphi = 0 \quad (25)$$

which provide the steady state amplitudes and phases.

4. Simulation Analysis

To establish the accuracy of the average equations and illustrate the relationship between the tracked ambulance parameters and damping effect, simulation analysis is performed for the parameters: $M_1 = 180kg$, $M_2 = 2000kg$, $K_s = 217582N/m$, $C_1 = 4200N\cdot s/m$, $\beta = 0.1$, $K_2 = 2200000N/m$, $C_2 = 19540N\cdot s/m$, $f = 1500N$. For $\omega_1 = \omega_2 = 34.8rad/s$, the 1:1 resonance may occur. In Figure 4, we show the comparison between numerical solutions, obtained by Runge-Kutta, and perturbation solutions.

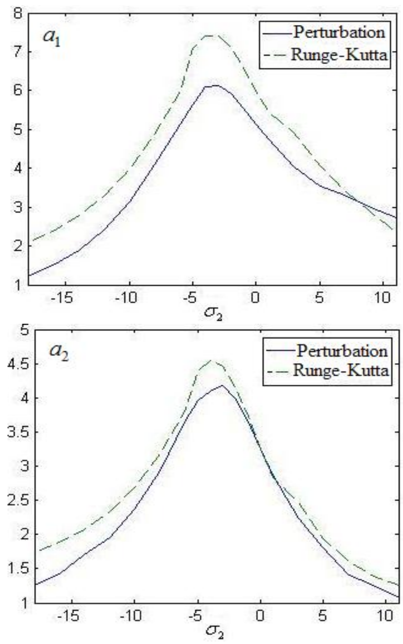


Fig.4 Amplitude-frequency curve

In Figure 4, the trend and resonance position of perturbation and numerical solutions are the same. But the amplitudes are different. Because we only use the first-order approximation, which don't affect our qualitative analysis of the behavior of the system dynamics. The damping system amplitude-frequency diagram is similar to the linear system, where jump phenomenon does not occur.

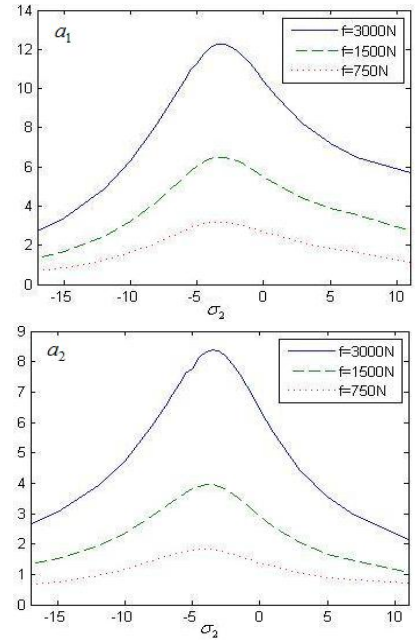


Fig.5 The influence of excitation amplitude

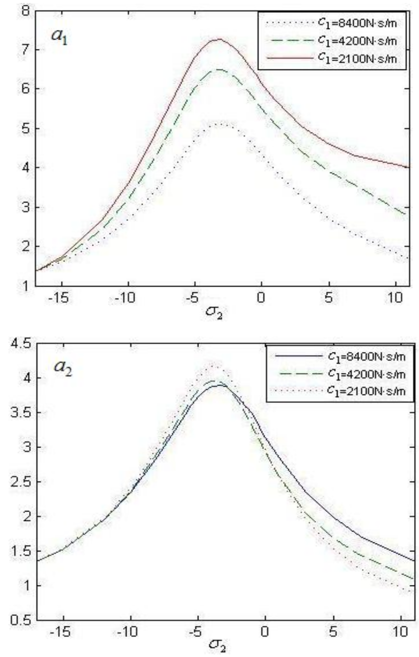
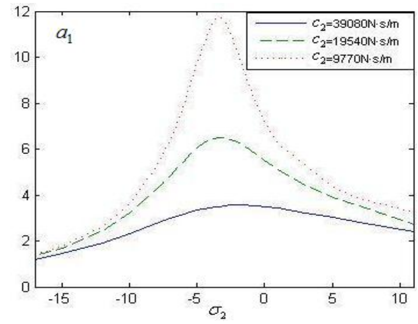


Fig. 6 The influence of C_1



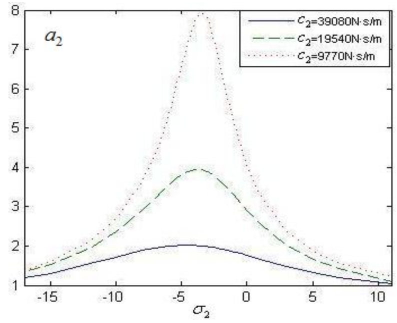


Fig. 7 The influence of C_2

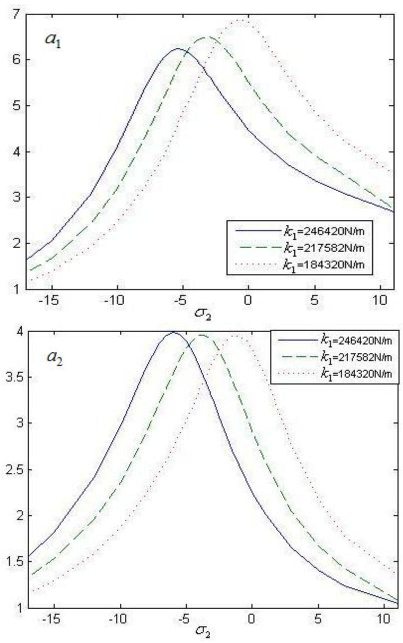


Fig. 8 The influence of K_s

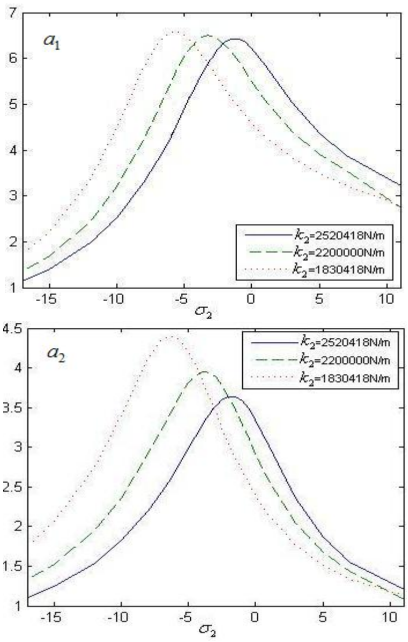


Fig. 9 The influence of K_2

Figure 5-9 show the impact of the tracked ambulance parameters on the system vibration, where a_1 and a_2 represent the vibrating amplitudes of the carriage and the stretcher base. As can be seen from Figure 5, the amplitude of the excitation force has a great impact on a_1 and a_2. Figure 6 and Figure 7 clearly demonstrate that he damping C_1 has a great impact on the amplitude of a_1, but little effect on the amplitude of a_2 and damping C_2 has a great impact on the amplitude of a_1 and a_2.The damping is greater and amplitude is smaller. Hence, increasing the damping, to some degree, is more effective to reduce vibration. Figure 8 and Figure 9 show that K_s only has a major impact on the amplitude of a_1 and K_2 only has a major impact on the amplitude of a_2 on the premise of meeting 1:1 internal resonance approximately. But both K_s and K_2 affect the resonance frequency greatly. Increasing K_2 or decreasing K_2 can increase the resonance frequency, which is beneficial to reduce vehicle vibration[10]. With comprehensive comparison of Figures 6 to 9, damping has a great impact on the amplitude of vibration and stiffness has a great impact on resonance frequency.

5. Stability Analysis

In order to analyze the stability of the system in the primary resonance, we need convert the average equations in polar form into a rectangular form by introducing $p_1 = a_1\cos(\gamma+\varphi)$, $q_1 = a_1\sin(\gamma+\varphi)$, $p_2 = a_2\cos\varphi$, $q_2 = a_2\sin\varphi$ [11-12]result in

$$\dot{p}_1 = -u_1 p_1 + u_1 p_2 - \frac{l_1}{2\omega_1} q_2 - \frac{3}{8\omega_1} b_1(p_2^2+q_2^2)q_2 - \sigma_2 q_1 - \frac{3}{8\omega_1} b_1(p_1^2+q_1^2)q_2 - \frac{3}{8\omega_1} b_1(q_1 p_2^2 - q_2^2 q_1 - 2p_1 p_2 q_2) - \frac{3}{4\omega_1} b_1(p_1 p_2 + q_1 q_2)q_1 + \frac{3}{4\omega_1} b_1(p_2^2+q_2^2)q_1 + \frac{3}{8\omega_1} b_1(p_1^2+q_1^2)q_1 \quad (26)$$

$$\dot{p}_2 = -u_3 p_2 + u_2 p_1 - \frac{l_2}{2\omega_2} q_1 - \frac{3}{8\omega_2} b_2(p_1^2+q_1^2)q_1 - (\sigma_2 - \sigma_1)q_2 + \frac{3}{8\omega_2} b_2(2p_1 q_1 p_2 - p_1^2 q_2 + q_1^2 q_2) - \frac{3}{8\omega_2} b_2(p_2^2+q_2^2)q_1 + \frac{3}{8\omega_2} b_2(p_2^2+q_2^2)q_2 + \frac{3}{4\omega_2} b_2(p_1^2+q_1^2)q_2 - \frac{3}{4\omega_2} b_2(p_1 p_2 + q_1 q_2)q_2 \quad (27)$$

$$\dot{q}_1 = -u_1 q_1 + u_1 q_2 + \frac{l_1}{2\omega_1} p_2 + \frac{3}{8\omega_1} b_1(p_2^2+q_2^2)p_2 + \sigma_2 p_1 + \frac{3}{8\omega_1} b_1(p_1^2+q_1^2)p_2 - \frac{3}{8\omega_1} b_1(p_1 p_2^2 - q_2^2 p_1 + 2q_1 p_2 q_2) + \frac{3}{4\omega_1} b_1(p_1 p_2 + q_1 q_2)p_1 - \frac{3}{4\omega_1} b_1(p_2^2+q_2^2)p_1 - \frac{3}{8\omega_1} b_1(p_1^2+q_1^2)p_1 \quad (28)$$

$$\dot{q}_2 = -u_3 q_2 + u_2 q_1 + \frac{l_2}{2\omega_2} p_1 + \frac{3}{8\omega_2} b_2 (p_1^2 + q_1^2) p_1 + \frac{f}{2\omega_2} - \frac{3}{8\omega_2} b_2 (p_1^2 q_1 - q_1^3 + 2 p_1 q_1 q_2) + \frac{3}{8\omega_2} b_2 (p_2^2 + q_2^2) p_1 + (\sigma_2 - \sigma_1) p_2 - \frac{3}{8\omega_2} b_2 (p_2^2 + q_2^2) p_2 - \frac{3}{4\omega_2} b_2 (p_1^2 + q_1^2) p_2 + \frac{3}{4\omega_2} b_2 (p_1 p_2 + q_1 q_2) p_2 \quad (29)$$

Where the average equations become more complex and the exact analytical solution cannot be obtained. At steady state, $\dot{p}_1 = 0$, $\dot{p}_2 = 0$, $\dot{q}_1 = 0$, $\dot{q}_2 = 0$ and we use Newton Method to calculate the value of the equilibrium point of the average Eq.(26)-(29) by repeatedly changing the initial value of the equilibrium point. There are three sets of equilibrium points, as follows

$$\phi_1 = \{-3.9041, -1.2622, 2.0696, 1.8797\}$$
$$\phi_2 = \{12.1124, 7.7186, -13.7048, -10.4106\}$$
$$\phi_3 = \{-10.9811, -6.7073, 11.9741, 8.9831\}$$

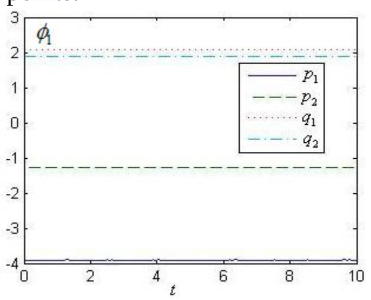
The stability of the system at the equilibrium point is governed by the eigenvalue of the Jacobian matrix of Eq.(26)-(29) based on the singularity theory. The eigenvalues are obtained:

$$\lambda_1 = \{-13.5988 + 22.7426i, -13.5988 - 22.7426i, -4.0362 + 5.5237i, -4.0362 - 5.5237i\}$$
$$\lambda_2 = \{-31.75 + 120.17i, 30.18, -31.75 - 120.17i, -1.95\}$$
$$\lambda_3 = \{-28.95 + 103.88i, -1.70, -28.95 - 103.88i, 24.33\}$$

The Eq.(30) is the Jacobian matrix of the Eq.(26)-(29) at equilibrium point, where the expressions of $n_{ij}\,(i = 1\ldots4,\ j = 1\ldots4)$ are given in appendix.

$$A = \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{31} & n_{32} & n_{33} & n_{34} \\ n_{41} & n_{42} & n_{43} & n_{44} \end{bmatrix} \quad (30)$$

After singularity analysis, the system is only stable in the first equilibrium point. Since there is only one stable equilibrium point, the jump phenomenon does not occur. Use Runge-Kutta method to validate the singularity analysis. The Figure10 presents the final stable position of the Eq.(26)-(29) whose the initial values are the three equilibrium points.



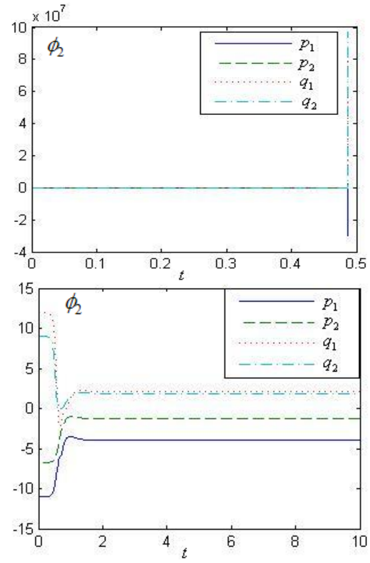


Fig. 10 System stability location

Figure 10 clearly illustrates that the system is only stable in the first equilibrium point, which is in line with the actual system and diverges to infinity(Figure b) or converge to the stable equilibrium point(Figure c) at unstable equilibrium point. Therefore, the system is impossible to maintain a stable state in the unstable equilibrium point.

6. Conclusion

This paper established the dynamics model of a tracked ambulance damping system containing three nonlinear terms. We used Multiple Scales Method to investigate the dynamics model and obtain the average equations. The average equations were verified with the actual parameters. The influence of damping system parameters for the damping effect as well as the stability of the damping system were analyzed. The result explained the reasons that there is no jump phenomenon. This analysis method is suitable for multi-degree-of-freedom bearing motion system, particularly suitable for vehicle. The research results are valuable for the vehicle damping system design as well as forecast the damping system dynamic behavior.

Appendix:

$$n_{11} = -u_1 - \frac{3b_1 q_2 p_1}{4\omega_1} + \frac{3b_1 q_2 p_2}{4\omega_1} - \frac{3b_1 q_1 p_2}{4\omega_1} + \frac{3b_1 q_1 p_1}{4\omega_1}$$

$$n_{12} = u_1 - \frac{3b_1 q_2 p_2}{4\omega_1} - \frac{3b_1 (2q_1 p_2 - 2p_1 q_2)}{8\omega_1} - \frac{3b_1 q_1 p_1}{4\omega_1} + \frac{3b_1 q_1 p_2}{2\omega_1}$$

$$n_{13} = -\frac{3b_1 q_2 q_1}{2\omega_1} - \frac{3b_1 (p_2^2 - q_2^2)}{8\omega_1} - \sigma_2 - \frac{3b_1 (p_1 p_2 + q_1 q_2)}{4\omega_1} + \frac{3b_1 (p_2^2 + q_2^2)}{4\omega_1} + \frac{3b_1 q_1^2}{4\omega_1} + \frac{3b_1 (p_1^2 + q_1^2)}{8\omega_1}$$

$$n_{14} = -\frac{l_1}{2\omega_1} - \frac{3b_1 q_2^2}{4\omega_1} - \frac{3b_1 (p_2^2 + q_2^2)}{8\omega_1} - \frac{3b_1 (p_1^2 + q_1^2)}{8\omega_1} - \frac{3b_1 q_1^2}{4\omega_1} + \frac{3b_1 (p_1 p_2 + q_1 q_2)}{4\omega_1} + \frac{3b_1 q_2 q_1}{2\omega_1}$$

$$n_{21}=u_2-\frac{3b_2p_1q_1}{4\omega_2}+\frac{3b_2(q_1p_2-p_1q_2)}{4\omega_2}+\frac{3b_2p_1q_2}{2\omega_2}-\frac{3b_2p_2q_2}{4\omega_2}$$

$$n_{22}=-u_3+\frac{3b_2p_1q_1}{4\omega_2}-\frac{3b_2p_2q_1}{4\omega_2}+\frac{3b_2p_2q_2}{4\omega_2}-\frac{3b_2p_1q_2}{4\omega_2}$$

$$n_{23}=-\frac{l_2}{2\omega_2}-\frac{3b_2q_1^2}{4\omega_2}-\frac{3b_2(p_1^2+q_1^2)}{8\omega_2}-\frac{3b_2(p_2^2+q_2^2)}{8\omega_2}+\frac{3b_2q_1q_2}{2\omega_2}+\frac{3b_2(p_1p_2+q_1q_2)}{4\omega_2}-\frac{3b_2q_2^2}{4\omega_2}$$

$$n_{24}=\frac{3b_2(-p_1^2+q_1^2)}{8\omega_2}-\frac{3b_2q_1q_2}{2\omega_2}-\sigma_2+\sigma_1+\frac{3b_2q_2^2}{4\omega_2}+\frac{3b_2(p_2^2+q_2^2)}{8\omega_2}+\frac{3b_2(p_1^2+q_1^2)}{4\omega_2}-\frac{3b_2(p_1p_2+q_1q_2)}{4\omega_2}$$

$$n_{31}=\frac{3b_1p_2p_1}{2\omega_1}-\frac{3b_1(p_2^2-q_2^2)}{8\omega_1}+\sigma_2+\frac{3b_1(p_1p_2+q_1q_2)}{4\omega_1}-\frac{3b_1(p_2^2+q_2^2)}{4\omega_1}-\frac{3b_1p_1^2}{4\omega_1}-\frac{3b_1(p_1^2+q_1^2)}{8\omega_1}$$

$$n_{32}=\frac{l_1}{2\omega_1}+\frac{3b_1p_2^2}{4\omega_1}+\frac{3b_1(p_2^2+q_2^2)}{8\omega_1}+\frac{3b_1(p_1^2+q_1^2)}{8\omega_1}+\frac{3b_1p_1^2}{4\omega_1}-\frac{3b_1(p_1p_2+q_1q_2)}{8\omega_1}-\frac{3b_1p_2p_1}{2\omega_1}$$

$$n_{33}=-u_1+\frac{3b_1q_1p_2}{4\omega_1}-\frac{3b_1q_2p_2}{4\omega_1}+\frac{3b_1q_2p_1}{4\omega_1}-\frac{3b_1q_1p_1}{4\omega_1}$$

$$n_{34}=u_1+\frac{3b_1q_2p_2}{4\omega_1}-\frac{3b_1(q_1p_2-p_1q_2)}{4\omega_1}+\frac{3b_1q_1p_1}{4\omega_1}-\frac{3b_1q_2p_1}{2\omega_1}$$

$$n_{41}=\frac{l_2}{2\omega_2}+\frac{3b_2p_1^2}{4\omega_2}+\frac{3b_2(p_1^2+q_1^2)}{8\omega_2}+\frac{3b_2(p_2^2+q_2^2)}{8\omega_2}-\frac{3b_2(p_1p_2+q_1q_2)}{4\omega_2}-\frac{3b_2p_1p_2}{2\omega_2}+\frac{3b_2p_2^2}{4\omega_2}$$

$$n_{42}=\frac{3b_2p_1p_2}{2\omega_2}+\sigma_2-\sigma_1-\frac{3b_2p_2^2}{4\omega_2}-\frac{3b_2(p_2^2+q_2^2)}{8\omega_2}-\frac{3b_2(p_1^2+q_1^2)}{4\omega_2}+\frac{3b_2(p_1p_2+q_1q_2)}{4\omega_2}$$

$$n_{43}=u_2+\frac{3b_2p_1q_1}{4\omega_2}-\frac{3b_2(p_1^2-3q_1^2+2p_1q_2)}{8\omega_2}-\frac{3b_2q_1p_2}{2\omega_2}+\frac{3b_2p_2q_2}{4\omega_2}$$

$$n_{44}=-u_3-\frac{3b_2p_1q_1}{4\omega_2}+\frac{3b_2p_1q_2}{4\omega_2}-\frac{3b_2p_2q_2}{4\omega_2}+\frac{3b_2p_2q_1}{4\omega_2}$$

Acknowledgment

An acknowledgment should be shown to Dr. Weihua Su who helped us during the writing of this paper.

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