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Synchronization Criteria of Chaos Systems with Time-delay Feedback Control

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Abstract

For the time-delay feedback control of chaos synchronization problem, an idea of Lyapunov functional with time-delay decomposition is presented. Some delay-dependent synchronization criteria are formulated in the form of matrix inequalities. The controller gain with maximum allowed time-delay can be achieved by solving a set of linear matrix inequalities (LMIs). A simulation example is given to illustrate the effectiveness of the design method.

Keywords: synchronization, chaos system, time-delay feedback, delay decomposition, linear matrix inequality (LMI).

1. Introduction

During the last two decades, chaotic synchronization has received considerable interests, e.g., [1–3] and references cited therein. It is found to be useful or has great potential in a variety of fields including physical, chemical and ecological systems, human heartbeat regulation, secure communications, and so on.

Recently, the effect of delay on synchronization between two chaotic systems has been reported in many literatures due to the propagation delay frequently encountered in remote master–slave synchronization scheme. In particular, some delay-independent[4] and delay-dependent synchronization criteria was derived in [5,6]. Liao and Chen in [7] improved some results in [6] and gave some simple algebraic conditions which are easy to be verified. In [8] Cao et al. further generalized and improved the results in [6,7]. However, when deriving delay-dependent sufficient conditions for master-slave synchronization, Yalcin et al. [6] and Cao et al. [8] employed model transformation, which led to some conservative synchronization criteria for inducing additional terms. In order to avoid using model transformation, some new approaches had been employed to derive much less conservative synchronization conditions. Xiang et al. [9] and He et al. [10] used integral inequality and free weighting matrix approach in the derivative of Lyapunov functional respectively. It is interesting and valuable issue to proposed new method to obtain a larger delay threshold below which synchronization can be ensured theoretically.

In this paper, we employ a delay decomposition approach recently proposed in [11,12] and fully use information from the nonlinear term of the error system to derive the synchronization criteria. Based on the synchronization criteria, we will give some sufficient conditions on the existence of a state error feedback controller. These sufficient conditions will be formulated in the form of matrix inequalities. Moreover, we will design the controller by solving a set of LMIs. We will use one simulation example to illustrate the effectiveness of synchronization criteria and the design method.

2. Problem statement

Consider a general master–slave synchronization scheme using time-delay feedback control.

$$M:\begin{cases}\dot{x}(t)=Ax(t)+B\varphi(Cx(t))\\ p(t)=x(t)\end{cases} \tag{1}$$

$$S:\begin{cases}\dot{y}(t)=Ay(t)+B\varphi(Cy(t))+u(t)\\ q(t)=y(t)\end{cases} \tag{2}$$

$$C:u(t)=K(p(t-\tau)-q(t-\tau)) \tag{3}$$

With master system M, slave system S and controller C, where the time-delay $\tau>0$. The master and slave system are chaos systems with state vectors $x,y\in R^n$, and the output vector $p,q\in R^l$, respectively. The matrices $A\in R^{n\times n}$, $B\in R^{n\times m}$, $C\in R^{m\times n}$, $H\in R^{l\times n}$ are known constant matrices. The nonlinearity $\varphi(\cdot)$ is time-invariant, decoupled, and satisfies a sector condition with $\varphi_i(\sigma)(i=1,2,....,m)$ belonging to a sector $[0,k]$, i.e.,

$$\varphi_i(\sigma)(\varphi_i(\sigma)-k\sigma)\le 0 \quad \forall t\ge 0 \quad \forall\sigma\in R \qquad (4)$$

Now, define the synchronization error as $e(t)=x(t)-y(t)$. Then, an error dynamical system is obtained in the form:

$$\dot{e}(t)=Ae(t)+B\eta(Ce(t),y(t))-Ke(t-\tau) \qquad (5)$$

where $\eta(Ce,y)=\varphi(Ce+Cy)-\varphi(Cy)$.

Let $C=[c_1,c_2,...c_m]^T$ with $c_i\in R^n, i=1,2,...m$ .The nonlinearity $\eta(Ce,y)$ is assumed to belong to the sector $[0,k]$, i.e., for $\forall t\ge 0$, $\forall e,y$

$$\eta_i(c_ie,y)(\eta_i(c_ie,y)-kc_ie)\le 0 \qquad (6)$$

The purpose of this paper is to study the master-slave synchronization of chaos systems and design the controller (3), i.e., to find the controller gain K, such that the system described by (5)-(6) is globally asymptotically stable, which means that the master system and the slave system synchronizes.

The following lemma is useful in deriving synchronization criteria.

Lemma1.(Ding [3]) For any constant matrix $R>0$, $R=R^T\in R^{n\times n}$,scalar $\tau>0$,and vector function e and $\dot{e}:[0,\tau]\to R^n$ such that the following inequality is well defined, then

$$-\tau\int_0^\tau \dot{e}^T(s)R\dot{e}(s)ds\le\begin{pmatrix}e(t)\\ e(t-\tau)\end{pmatrix}^T\begin{pmatrix}-R & R\\ R & -R\end{pmatrix}\begin{pmatrix}e(t)\\ e(t-\tau)\end{pmatrix}$$

3. Main results

Then we are in the position to give the main result.

Theorem 3.1. For a given scalar $\tau>0$,the error system (5) is globally asymptotically stable if there exist matrices $P=P^T>0$, $Q_i=Q_i^T>0$, $R_i=R_i^T>0(i=1,2,...N)$, and positive diagonal matrices $\Lambda=diag(\lambda_1,\lambda_2,...,\lambda_m)>0$, and any matrices G_1,G_2 such that

$$\Xi=\begin{pmatrix}\Xi_{11} & R_1 & 0 & \cdots & 0 & -G_1K & \Xi_{1N+2} & G_1B+kC^T\Lambda & 0\\ * & \Xi_{22} & R_2 & \cdots & 0 & 0 & 0 & 0 & 0\\ * & * & \Xi_{33} & \cdots & 0 & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & \cdots & \Xi_{NN} & R_N & 0 & 0 & 0\\ * & * & * & \cdots & * & \Xi_{N+1N+1} & -K^TG_2^T & 0 & 0\\ * & * & * & \cdots & * & * & -G_2-G_2^T & G_2B & h\left(\sum_{i=1}^N R_i\right)\\ * & * & * & \cdots & * & * & * & -2\Lambda & 0\\ * & * & * & \cdots & * & * & * & * & -\sum_{i=1}^N R_i\end{pmatrix}<0 \qquad (7)$$

where

$\Xi_{11}=A^TG_1+G_1^TA+Q_1-R_1$, $\Xi_{ii}=-R_{i-1}-R_i+Q_i-Q_{i-1}$, $\Xi_{N+1N+1}=-Q_N-R_N$, $\Xi_{1N+2}=-G_1^T+A^TG_2+P$

Proof. Consider the following Lyapunov-Krasovskii function candidate for system (5) as :

$$V(e(t))=V_1(e(t))+V_2(e(t))+V_3(e(t)) \qquad (8)$$

with

$$V_1(e(t))=e^TPe+2\sum_{i=1}^m\int_0^{c_i^Te}\lambda_i\varphi_i(s)ds$$

$$V_2(e(t))=\sum_{i=1}^N\int_{t-ih}^{t-(i-1)h}e^T(s)Q_ie(s)ds$$

$$V_3(e(t))=\sum_{i=1}^N\int_{-ih}^{-(i-1)h}\int_{t+\theta}^{t}\dot{e}^T(s)hR_i\dot{e}(s)dsd\theta$$

where $h=\tau/N$, N is the positive integer of division on the interval $[-\tau,0]$ and h is the length of each division.

According to (5), for any appropriately dimensioned matrices G_1,G_2, the following equations are true:

$$\begin{aligned}&2[e^T(t)\quad \dot{e}^T(t)\ ][G_1\quad G_2]^T\times[-\dot{e}(t)+Ae(t)\\ &+B\eta(Ce(t),y(t))-Ke(t-\tau)\ ]=0\end{aligned} \qquad (9)$$

From (4), (5) and Ti=diag$(t_{i1},t_{i2},...,t_{im})>0$(i=1,2), we have

$$-2\eta^T\Lambda\eta+2ke^TC^T\Lambda\eta\ge 0 \qquad (10)$$

Taking the derivative of $V(e(t))$ with respect to t along the trajectory of (5) yields

$$\dot{V}_1(e(t))=2e^TP\dot{e} \qquad (11)$$

$$\begin{aligned}\dot{V}_2(e(t))&=\sum_{i=1}^N e^T(t-(i-1)h)Q_ie(t-(i-1)h)\\ &-\sum_{i=1}^N e^T(t-ih)Q_ie(t-ih)\end{aligned} \qquad (12)$$

$$\begin{aligned}\dot{V}_3(e(t))&=\sum_{i=1}^N h^2\dot{e}^T(t)R_i\dot{e}(t)\\ &-\sum_{i=1}^N h\int_{t-ih}^{t-(i-1)h}\dot{e}^T(s)R_i\dot{e}(s)ds\end{aligned} \qquad (13)$$

Adding the left side of (9)-(10) to the right side of $\dot{V}(e(t))$, and using Lemma 1we have

$$\dot{V}(e(t))\le q^T(t)\hat{\Xi}q(t)$$

Where

$$q(t)=[e^T(t)\ \ e^T(t-h)\ \ e^T(t-2h)\ \ ......\ \ e^T(t-(N-1)h)\ \ e^T(t-Nh)\ \ \dot{e}^T(t)\ \ \eta^T(t)]$$

$$\hat{\Xi}=\begin{pmatrix}\Xi_{11} & R_1 & 0 & \cdots & 0 & -G_1K & \Xi_{1N+2} & G_1B+kC^T\Lambda\\ * & \Xi_{22} & R_2 & \cdots & 0 & 0 & 0 & 0\\ * & * & \Xi_{33} & \cdots & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & \cdots & \Xi_{NN} & R_N & 0 & 0\\ * & * & * & \cdots & * & \Xi_{N+1N+1} & -K^TG_2^T & 0\\ * & * & * & \cdots & * & * & -G_2-G_2^T+h^2\left(\sum_{i=1}^{N}R_i\right) & G_2B\\ * & * & * & \cdots & * & * & * & -2\Lambda\end{pmatrix}<0$$

It follows from Schur complement that $\hat{\Xi}<0$ is equivalent to (7), then $\dot{V}(e(t))\le q^T(t)\Xi q(t)<0$ for $q(t)\neq 0$, which means that the system described by (5)-(6) is globally asymptotically stable. This completes the proof.

Remark 3.2: In order to reduce the conservative, the delay-decomposition is proposed in the Lyapunov functional. Therefore, the delay-dependent stability criterion is expected to be less conservative than the existing ones, which will be illustrated through an example in the next section.

In order to get the controller gain we let $G_2=\mu G_1, Y=-G_1^TK$,then we can establish the following synchronization criterion.

Corollary 3.3. For a given scalar $h>0$,the error system (5) is globally asymptotically stable if there exist matrices $P=P^T>0$, $Q_i=Q_i^T>0$, $R_i=R_i^T>0(i=1,2,...N)$, and positive diagonal matrices $\Lambda=diag(\lambda_1,\lambda_2,...,\lambda_m)>0$, and any matrices Y,G_1 such that

$$\Xi=\begin{pmatrix}\Xi_{11} & R_1 & 0 & \cdots & 0 & Y & \hat{\Xi}_{1N+2} & G_1B+kC^T\Lambda & 0\\ * & \Xi_{22} & R_2 & \cdots & 0 & 0 & 0 & 0 & 0\\ * & * & \Xi_{33} & \cdots & 0 & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots\\ * & * & * & \cdots & \Xi_{NN} & R_N & 0 & 0 & 0\\ * & * & * & \cdots & * & \Xi_{N+1N+1} & Y^T & 0 & 0\\ * & * & * & \cdots & * & * & -uG_1-uG_1^T & G_1B & h\left(\sum_{i=1}^{N}R_i\right)\\ * & * & * & \cdots & * & * & * & -2\Lambda & 0\\ * & * & * & \cdots & * & * & * & * & -\sum_{i=1}^{N}R_i\end{pmatrix}<0 \quad (14)$$

Where $\hat{\Xi}_{1N+2}=-G_1+uA^TG_1^T+P$

Moreover, the delay feedback controller gain matrix is given by $K=-G_1^{-T}Y$. This completes the proof.

4. An example

Consider the following Chua's circuit

$$\begin{cases}\dot{x}=\alpha(y-h(x)),\\ \dot{y}=x-y+z,\\ \dot{z}=-\beta y,\end{cases}$$

with nonlinear characteristic:

$h(x)=m_1x+0.5(m_0-m_1)(|x+c|-|x-c|)$,

and parameters $m_0=-1/7$, $m_1=2/7$, $\alpha=9$, $\beta=14.28$ and $c=1$. The system can be represented in Lur'e form by Yalcin et al.[5] with

$$A=\begin{pmatrix}-\alpha m_1 & \alpha & 0\\ 1 & -1 & 1\\ 0 & -\beta & 0\end{pmatrix}, B=\begin{pmatrix}-\alpha(m_0-m_1)\\ 0\\ 0\end{pmatrix}, C=H=\begin{pmatrix}1 & 0 & 0\end{pmatrix}$$

and $\varphi(\varepsilon)=0.5(|\varepsilon+1|-|\varepsilon-1|)$ belonging to the sector $[0,k]$ with $k=1, \mu=0.36, N=3$. Applying Matlab LMI-toolbox to the inequality (11) with different τ, it is obtained that the gain matrix:

$$K=\begin{pmatrix}2.1502 & 2.1683 & -1.0209\\ 0.6802 & 0.6112 & 0.2531\\ -0.2862 & -1.2154 & 2.6651\end{pmatrix}$$

Which can stabilize the error system (5) for $\tau\in[0,0.229]$. No feasible point is found for $\tau>0.229$.

The initial conditions of the master and slave systems are $x(0)=[-0.2,-0.3,0.2]$ and $y(0)=[0.5,0.1,-0.6]$. The simulation result with $\tau=0.229$ is shown in Fig.1-6. The behaviors of the master system and slave system are shown in Fig.1 and Fig.2. The state variables of the master system and slave system are described in Fig.3-Fig5. In Fig.6, the error of the variables are shown. From the figures, we can see that the designed controller realize the synchronization of the two systems.

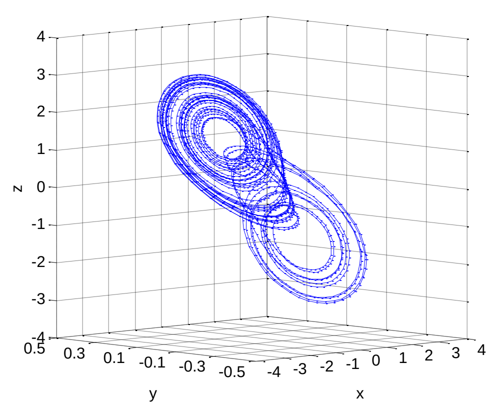

Fig.1 Master system

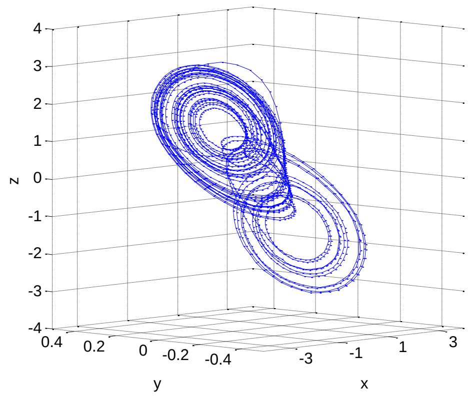

Fig.2 Slave system

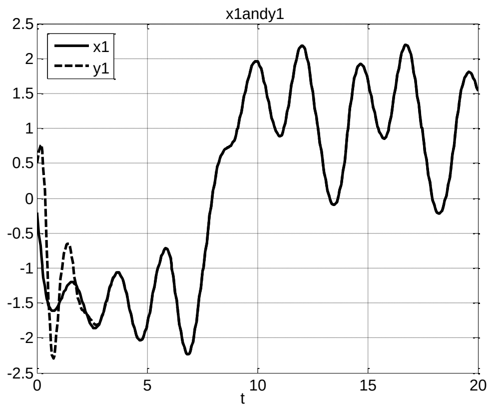

Fig.3 x1 and y1

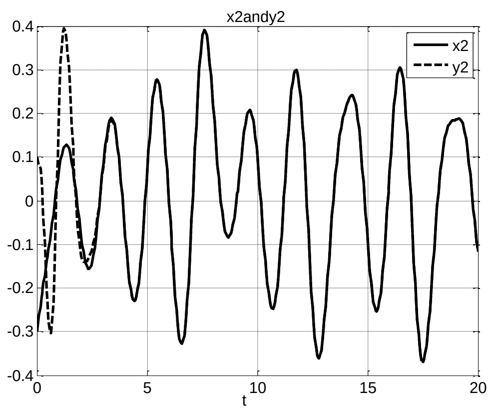

Fig.4 x2 and y2

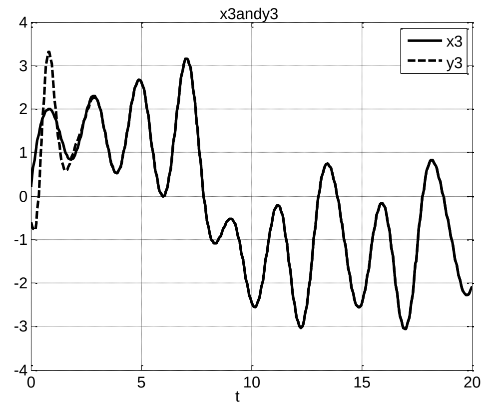

Fig. 5 x3 and y3

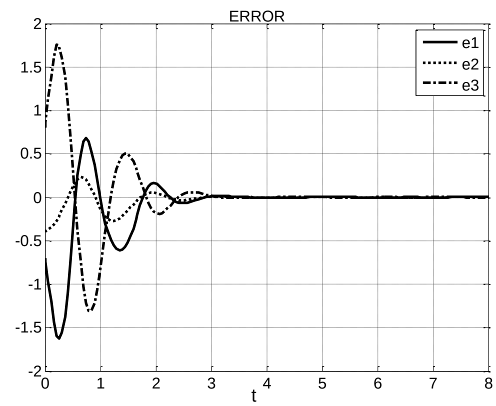

Fig.6 Error system

5. Conclusion

In this Letter, we have addressed the problem of master-slave synchronization criterion of Lur'e systems with time-delay feedback control. We have employed a delay decomposition approach to derive the synchronization criteria. Based on the synchronization criteria ,we have derived some sufficient conditions on the existence of a delayed error feedback controller. Moreover, we have designed the controller by solving a set of LMIs. An example has shown that the new sufficient conditions improve some of the previous results in the earlier references.

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