DOA Estimation of Multi-path Signals Using Transient Smoothing Method

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Abstract

The present work introduces a novel algorithm for Direction-of-arrival (DOA) estimation of signals in the multi-path environment. The technique relies on transient smoothing way to extract the coherence properties of sub-array, and restores the rank of the data covariance matrix. After processed, the multi-path channel signals were de-correlation and can be processed by traditional DOA estimation algorithms. Through extensive computer simulations, the relation between ratio of phase and position of signal sources has been demonstrated. The performance of technique is contrasted with other suggested methods in the different assumptions.

Keywords: *Transient smoothing, DOA estimation, Multi-path, Space Smoothing.*

1. Introduction

The problem of direction-of-arrival (DOA) estimation has been studied extensively both for the narrowband and wideband cases. A wide variety of DOA estimators have been proposed and analyzed in the past few decades, such as MUSIC algorithm [1], ESPRIT algorithm [2-4], the maximum likelihood (ML) algorithm [5,6], weighted subspace fitting (WSF)method [7,8] and so on. In the multi-path environment, signals are coherent or correlated when they are arriving at the receiver. In the coherent case, the rank of the covariance matrix of received data is reduced to 1. Obviously, this will cause the signal subspace dimension is less than the number of source. The solution idea is how to compensate the rank deficit caused by coherent. So far, domestic and foreign scholars have proposed many algorithms which can be broadly divided into two categories: dimensionality reduction and non dimensional reduction. For the former, representative algorithms were spatial smoothing method [9,10] and matrix decomposition method [11,12]. S.U.Pillai took conjugate transpose of forward spatial smoothing sub-array and proposed a new concept: forward/backward spatial smoothing. The method can realize the DOA estimation of 2M/3 (M is the number of array elements) coherent signals. Based on some improving, Wax M transformed the uniform circular array of array element space into virtual

uniform linear array of pattern space and opened up a new era of circular array de-coherence [13]. Then, many scholars had improved this spatial smoothing technique and proposed some new algorithms. Analyzing the essence of the technique, its performance of de-correlation depends on the number of sub-array and space interval of DOA. It must reduce the number of the degrees of freedom.

According to the current array pattern of base station, for the estimation of unknown signals arriving from different directions to a passive array, we propose a transient smoothing method, which is similar to spatial smoothing method. By extracting the coherence of sub-array and combining some subspace algorithm, the technique can be used to estimate DOA of coherent signals in fading channels.

2. Multi-path Signal Model

Let us consider the case in which narrow band wave is incident on an array antenna consisting of M antenna elements through Q multi-paths. In the multi-path environment, the signal source equates to Q signals when it arrives receiving array antenna. Suppose the direction of the waves was $\theta_k, k = 1, 2, \cdots, Q$, and the received signal at the i-th antenna ($i = 1, 2, \cdots, M$) be $x_i(t)$. Then, the received signal at the i-th antenna is given by

$$x_i^{\alpha}(t) = \sum_{k=1}^{Q} \alpha_k s(t - \tau_k - (i-1) d \sin\theta_k / c) \cdot \exp[j[\omega_c t + \phi_k(t) - (i-1) d \sin\theta_k / c]] + n_i(t) e^{j\omega_c t} \quad (1)$$

Where $\phi_k(t) = (\omega_d \cos\psi_k) t - \omega_c \tau_k(t)$. α_k, τ_k, θ_k, d and c denote respectively correlation coefficient, delay of travel time, DOA of signals, distance of antenna array and propagation velocity. ω_d, ψ_k and $n_i(t)$ denote respectively maximum Doppler frequency, angle between moving speed vector and *k*-th scattering and white noise. Transform Eq. (1) into base-band signal and assume the bandwidth of $s(t)$ is much smaller than the reciprocal of time delay in spanning array aperture, there be

$$x_i(t)=\sum_{k=1}^{Q}\alpha_k s(t-\tau_k)e^{j\phi_k(t)-(i-1)\frac{d}{c}\sin\theta_k]}+n_i(t) \qquad (2)$$

Transform Eq. (2) into the vector form and let $\gamma_k(t)$ be the product of α_l and time-varying phase term, then

$$x_i(t)=\sum_{k=1}^{Q}\gamma_k(t)a(\theta_k)s(t-\tau_k)+n_i(t) \qquad (3)$$

Where $a(\cdot)$ is the steering vector parameter. In this pattern, DOA of signals and delay of travel time is continuous. When the signal source is static, $\gamma_k(t)=\alpha_k e^{-j\omega_c\tau_k}$ is continuous value and all scattering of signals is static.

3. Transient Smoothing Algorithm

Based on Eq. (3), the covariance matrix of $x(t)$ is

$$R_{xx}=E[x(t)x^*(t)]=APA^{\mathrm{H}}+\sigma^2 I \qquad (4)$$

Where $(\bullet)^{\mathrm{H}}$ denotes the conjugate transpose,

$$A=[a(\theta_1),a(\theta_2),\cdots,a(\theta_Q)]$$

$$P=\begin{bmatrix} E[\|\gamma_1(t)\|^2\|s(t-\tau_1)\|^2] & \cdots & E[\gamma_1(t)\gamma_Q^*(t)s(t-\tau_1)s^*(t-\tau_Q)] \\ \cdots & \cdots & \cdots \\ E[\gamma_Q(t)\gamma_1^*(t)s(t-\tau_Q)s^*(t-\tau_1)] & \cdots & E[\|\gamma_Q\|^2\|s(t-\tau_Q)\|^2] \end{bmatrix}. \qquad (5)$$

Analyze Eq. (5) as follows:

Case 1.

If delay of travel time of multi-path signals is same, $\tau_1=\tau_2=\cdots=\tau_Q$, then Eq. (5) is following form:

$$P=E[\|s(t-\tau_1)\|^2]\begin{bmatrix} \|\gamma_1\|^2 & \cdots & \gamma_1\gamma_Q^* \\ \cdots & \cdots & \cdots \\ \gamma_Q\gamma_1^* & \cdots & \|\gamma_Q\|^2 \end{bmatrix} \qquad (6)$$

Case 2.

If the signal source is moving, the first term $(\omega_d\cos\psi_k)t$ of $\phi_l(t)$ should attribute to Doppler Effect and the second term $\omega_c\tau_k(t)$ be time delay of propagation. In a typical mobile communications environment, DOA of multi-path signal is slowly changing with time (be constant within a certain time, for example, within a few seconds) [14]. But $\phi_k(t)$ is different. If the signal source is moving fast enough, the $\phi_k(t)$ is changing fast with time t. Usually we consider $\phi_k(t)$ being the uniform random variable in $(-\pi,\pi]$ time interval. If $\phi_k(t)$ is random variable, *P* will be the same as Eq. (5).

If $\gamma_1(t)$, $\gamma_m(t)$ and $s(t-\tau_1)s(t-\tau_m)$ is uncorrelated and $\phi(t)$ is uniform random variable in $(-\pi,\pi]$, adding to statistical independence between $\phi(t)$ and α_l, $E[\gamma_1(t)]$ will be zero. Eq. (5) will transform into diagonal matrix:

$$P=E[\|s(t)\|^2]\begin{bmatrix} \|\alpha_1\|^2 & 0 & \cdots & 0 \\ 0 & \|\alpha_2\|^2 & \cdots & 0 \\ & \cdots & & \\ 0 & 0 & \cdots & \|\alpha_Q\|^2 \end{bmatrix} \qquad (7)$$

In the multi-path environment, the correlation coefficient $\alpha_k\neq 0$. So the matrix *P* is of full rank. The rank of matrix *P* is unassociated with signal correlation and the purpose of de-correlation is achieved.

Then we carry on eigen-value decomposition for R_{xx}. Its eigen-values are complied with this sequence:

$$\lambda_1\geq\lambda_2\cdots\geq\lambda_Q>\lambda_{Q+1}=\cdots=\lambda_M=\sigma^2 \qquad (8)$$

In Eq. (8), the first *Q* eigen-values are in connection with signal and their numeric value are all more than σ^2. The eigen-vectors corresponding with the *Q* larger eigen-values $\lambda_1,\lambda_2,\cdots,\lambda_Q$ are $v_1,v_2,\cdots v_Q$, and they construct signal subspace $V_S=[v_1,v_2,\cdots,v_Q]$. The smaller eigen-values are determined by the noise. Their numeric values are σ^2. The eigen-vectors corresponding with $\lambda_{Q+1},\lambda_{Q+2},\cdots\lambda_M$ are $v_{Q+1},v_{Q+2},\cdots v_M$, and they construct noise subspace. We can obtain a DOA estimation algorithm through conventional MUSIC algorithm.

References [15] indicated that unidirectional spatial smoothing MUSIC algorithm needs *N* sub-arrays to restore full-rank covariance matrix and bidirectional spatial smoothing MUSIC algorithm needs *N*/2 sub-arrays. For *N* signal sources, bidirectional spatial smoothing technique needs 1.5*N* antenna elements to estimate their DOA, and unidirectional spatial smoothing technique 2*N*. The algorithm in this paper needs *N*+1 antenna elements and it does not to cause array aperture loss. The performance of technique is contrasted with spatial smoothing methods through extensive computer simulations in the end.

4. The Key Parameter in Algorithm

Transient smoothing method applies the MUSIC algorithm to sample data. Be sure that the sampling interval is long enough, the attenuation channel phase term $\gamma_1(t)$ is independent stochastic process.

$\gamma_1(t)$ is the product of α_l and time-varying phase term. To define the condition of time-varying phase term $\phi_l(t)$ and estimate the DOA of coherent signals, we analyze the non-diagonal terms in Eq. (4)

$$p_{ij} = E[\alpha_i \alpha_j^* e^{j[\phi_i(t)-\phi_j(t)]}] \quad (9)$$

If the non-coherent term is a uniform random variable in $(-\pi,\pi]$, p_{ij} should be 0.

Assume that a two-path model consists of direct travel path and feedback path and the two paths form an equilateral triangle. The signal source moves along the path that is vertical to the receiving antennas.

The phase term (DOA) $\phi_l(t)$ will be

$$\phi_2(t)-\phi_1(t) = \omega_d(\cos\psi_2 - \cos\psi_1)t - \omega_c(\tau_2(t)-\tau_1(t)) \quad (10)$$

Where ω_d is maximum Doppler frequency, ψ_2 is the angle between moving speed vector and 2-th scattering signal.

Combining the above assumption, we can obtain:

$$\omega_c(\tau_2(t)-\tau_1(t)) = \omega_c\left(\frac{2r(t)}{c} - \frac{d(t)}{c}\right) \quad (11)$$

Because the signal source is vertical to the receiving antennas, ψ_1 is seen as π, and there is $\cos\psi_1 = -1$, to derivate the two sides of (10),

$$\phi_2(t)-\phi_1(t) = \omega_d\left(\frac{v}{2r(t)} - \frac{vd^2(t)}{8r^3(t)}\right)t - \omega_d\left(\frac{d(t)}{r(t)} - 2\right) \quad (12)$$

Where v denotes the moving speed.

When $t=0$,

$$\phi_2(t)-\phi_1(t) = \omega_d\left(2 - \frac{d(t)}{r(t)}\right) \quad (13)$$

Eq. (13) shows the ratio of phase to time. If the reflected path is close to direct path, i.e. the feedback path is close to direct path, $d(t)/r(t)$ will be close to 2. Then $\phi_2(t)-\phi_1(t) \approx 0$. That is the ratio of phase to time will be small.

In contrast, if the reflected path is far to direct path, the ratio will be big and achieve its maximum value $2\omega_d$. This will lead to p_{12} in p_{ij} close to 0 and the estimation of DOA is easier and more precise.

5. Simulation Result

In this section, we present the simulation results of the proposed theory. Assume that the direction of sending signals is vertical to moving direction of signal source and the receiver consists of uniform antennas. The number of antenna elements is 10. The distance of signal source and receiver is 1.5 km. The DOA of direct signals and two multi-path coherent signals is 0°，36°，45°。

The attenuation factor of three signals is $\beta = [1,\ 0.8+0.4\mathrm{j},\ 0.4+0.8\mathrm{j}]$. Signal sampling rate is 1000Hz. The number of snapshots is 75. The SNR in receiver is 10dB.

Simulation 1:

In this simulation, we compare the spatial smoothing MUSIC algorithm with the proposed algorithm of correlated sources in the multi-path environment. The received array consists of 10 antenna elements and the number of sub-array elements is 4. The signal source is moving at a speed of 10km/h.

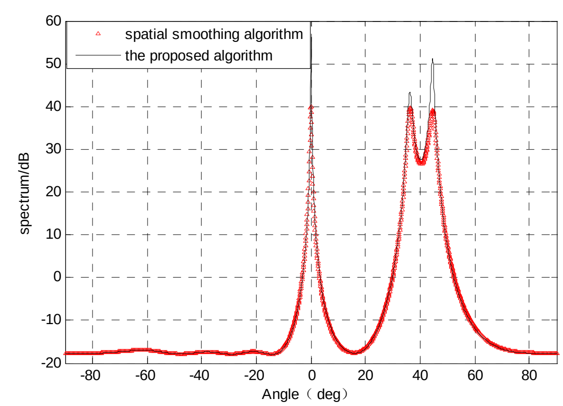

Fig.1 DOA spectra of the various algorithms in some assumptions

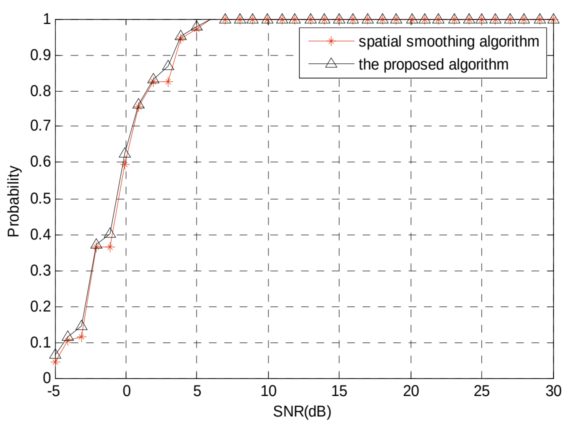

Fig.2 Probability of resolution versus SNR in some assumptions

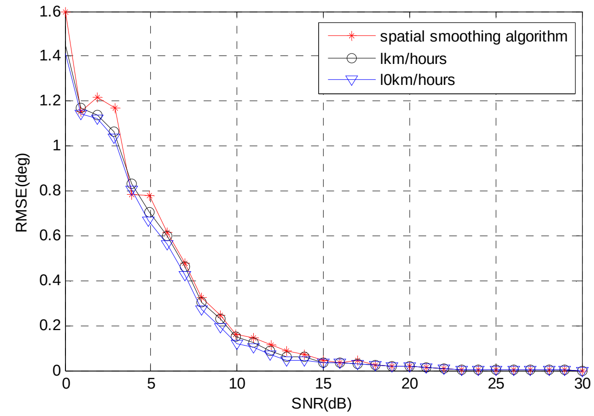

Fig.4 RMSE of resolution versus SNR

Fig.1 shows the spatial spectrums of signals in different algorithms. It is obvious that the DOA of signals can be estimated accurately. Two algorithms can all works in above assumptions. Fig.2 shows the probability of resolution versus SNR by performing 200 Monte Carlo runs. The performance of two methods is very similar when the number of arriving signals is less than $2M/3$. But they both perform poorly when the SNR is low. Also the de-correlation performance of spatial smoothing technique depends on the number of sub-array and space interval of DOA. It must reduce the number of the degrees of freedom. The proposed technique does not.

Simulation 2:

In this simulation, we compare the performance of algorithms when signal source is moving at a speed of 0km/h, 1km/h or 10km/h. Based on simulation 1，we compare the RMSE of algorithms by equation RMSE=$\sqrt{\sum_{k=1}^{K}(\theta_k-\hat{\theta}_k)^2}$.

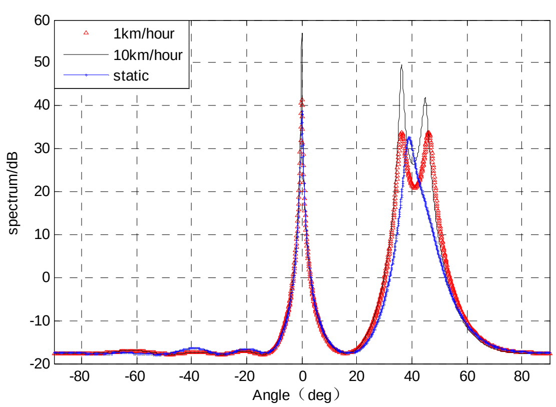

Fig.3 DOA spectra of proposed algorithms in different assumptions

From Fig.3 we can see the proposed algorithm has failed to resolve two closely spaced signals when the signal source is static. Even the signals are moving slowly the DOA can be estimated accurately. When it moves at high speed, the spectrum peaks are easily to tell and the estimation should be more precise. Fig.4 shows the RMSE versus SNR of the two algorithms in different cases. This verifies the DOA spectra in Fig.3.

4. Conclusions

In this paper, we propose the transient smoothing MUSIC algorithm and apply it on estimating algorithm of DOA in the presence of multi-path channel. By analyzing and deducing seriously, we transplant the spatial smoothing algorithm into the eigen-space and obtain the transient smoothing algorithm to estimate the DOA in the condition of coherent sources. Through extensive analyzing and computer simulations, we have confirmed that the proposed technique is valuable. Besides, the possibility and RMSE of estimation has been demonstrated. For a further subject, the proposed algorithm and spatial smoothing technique should be combined to estimate directions of more than M-1 coherent signals and achieve better performance.

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