EVALUATING EFFECTS OF CHATTEING ON THE SLIDING MODE SYSTEM

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ABSTRACT

This paper performs an analysis of switching time abut sliding mode controller. A mathematical method for determining and analysis the switching frequency about the sliding line are declared. A contribution to the SMC implementation based on Variable Structure Controllers (VSC) was adapted to analysis the chattering phenomenon and estimate the switching time of sliding mode controller. Also shows the behavior of switching frequency and find the relation between the switching intervals with the magnitude of the gain. The simulation results should be significance to design engineers who wish to express and study experience in control methods and their applications.

Keywords: Sliding mode, control, chattering, switching.

1. Introduction

A competent type of the variable-structure systems (VSSs) is a sliding mode control (SMC). It has been commonly used in practical applications by reason of its simplicity and robustness against parameter variations and disturbances[1]. The removal of the effects of unmodeled dynamics, disturbances, uncertainties, and improvement of robustness is the main task of SMC. But there are problems that grow on with the presence of a sliding mode controller called chattering phenomenon, this caused by high frequency switching with small time constant. This paper presents the analysis of chattering phenomenon, and indicates the actions of the switching signal resulting from sliding mode controller. As well as checking the effect of the switching time on the growth of chattering phenomenon. Identify and address the switching time delay and gain value of sliding mode controller and their relationship to decreeing the chattering phenomenon.

2. Design of SMC

Sliding mode Controller (SMC) is a powerful effective control deal with uncertainties and varying parameter for non-linear system[2]. However the SMC has a significant weakness, it is called chattering phenomenon. The chattering involves extremely high control activity and increase electric power consumption. In addition, the chattering produces unwanted highly nonlinearity of the system dynamics. Sliding Mode Control (SMC) design includes two parts. In the first part chooses a switching line structure where the sliding mode intersection will happen. In the second part who to enforce the system trajectory to achieve and stay in the region of desired switching line[3]. There are two steps to design the sliding mode controller. Step one choosing a parameter of sliding equation that describe a desired motion of states. Step two Designing an irregular control law that assurances the switching line will be done [4]. Sliding mode happens when the system trajectories are restricted to the sliding line and do not leave them even the system trajectory achieved to the equilibrium point[5].

The other main task in the SMC system design is the selection of a suitable control law this can be achieved either by assuming a certain type of the control law, and proving that this control satisfies reaching conditions or by applying the reaching law method [6].

3. Sliding Mode Equations

Sliding mode control system can be verified by taking the second-order system, to generalize a linear dynamic system single input single output (SISO) [7].

The canonical space-space can be described the system as

$$\dot{x_i} = x_{i+1},$$
$$\dot{x_n} = \sum_{i=1}^{n} a_i x_i + bu \qquad (1)$$

Where a_i, b are system parameters, u is the control signal (input). In the second-order systems control was designed as a bang-bang function of system output.

$$u = -\varphi x$$
$$\varphi = \begin{cases} +\beta & if\ x_1\sigma > 0 \\ -\beta & if\ x_1\sigma < 0 \end{cases} \qquad (2)$$

With switching plane

$$\sigma = \sum_{i=1}^{n} k_i x_i = 0, \quad k_i = const, k_n = 1$$

Design of SMC methodology relies on the foundations and rules:

- At any point a sliding mode should be exist when the switching equation is an act.
- Sliding mode must be stable.
- Eliminated or decrease the unwanted signal (chattering phenomenon).
- From any initial conditions the state trajectory should reach to the sliding line[8].

4. Chattering Phenomenon.

Chattering phenomenon is the oscillation about the sliding line with high frequencies, which is rising by the high-speed switching to the organization of a sliding mode[9]. It is, it is a weak point on slide mode because it may lead to unmodeled that caused the system goes to unstable regions, decreasing the accuracy of the system, In mechanical moving parts caused high wear, while in power electric produced high heat losses[10] .

5. Derive and Analysis of chattering phenomenon.

This section focuses on the chatter analysis. Consider an n[th] order linear of the SMC design as

$$\dot{x} = Ax + Bu \qquad (3)$$

$$y = Cx$$

$$\begin{bmatrix} \dot{x}_1 \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \cdot \\ \cdot \\ b_n \end{bmatrix} u$$

$$y = [c_1 \ \dots \ c_n] \times \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

Where $a_{11}, \dots, a_{nn}, b_1, \dots, b_n$ system parameters $x_1, \dots, x_n$ the states of the system, u control signal and y is the output of the system.

Let consider a second order system

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u$$

$$y = [c_1 \ \ c_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x_1} = a_{11}x_1 + a_{12}x_2 + b_1 u$$

$$\dot{x_2} = a_{21}x_1 + a_{22}x_2 + b_2 u \qquad (4)$$

$$y = c_1 x_1 + c_2 x_2$$

the Sliding mode equation

$$\sigma = k_1 x_1 + k_2 x_2 + \cdots + k_n x_n \qquad (5)$$

Where $k_1, k_2, \dots, k_n$ are the parameters of sliding line (slope of sliding line) generally? $k_n = 1$

The choice of linear operator (k_n or θ) to achieve sliding motion is governed by two conditions. The first (stability) is to ensure that the characteristic Eq. (5) is stable; the second condition will depend on the number of switched components [9, 11-13]

$$\varphi = \beta^{+} \ if\ \sigma x_1 \ > \ 0 \qquad (6)$$

$$\varphi = \beta^{-} \ if\ \sigma x_1 \ < \ 0$$

For second order system Eq. (5) becomes:

$$\sigma = k_1 x_1 + x_2 \qquad (7)$$

To ensure that equation pass through origin equilibrium point (0, 0) than Eq. (7) must be equal to zero (σ=0)

When $\sigma = 0$ then

$$k_1 = -\frac{x_2}{x_1} \quad \text{Or} \quad |k_1| = \left|\frac{x_2}{x_1}\right| \qquad (8)$$

Eq. (8) shows **-v_e** slope in a sliding mode that is defined in the fourth and second quarter the makes the system is stable, as this switching equation will be a slider which forces the system towards the equilibrium point (0,0). The purpose of the switching control law Eq. (6) is to force the system state trajectory onto a sliding line maintain the system state trajectory for the subsequent time as shown in figure (1).

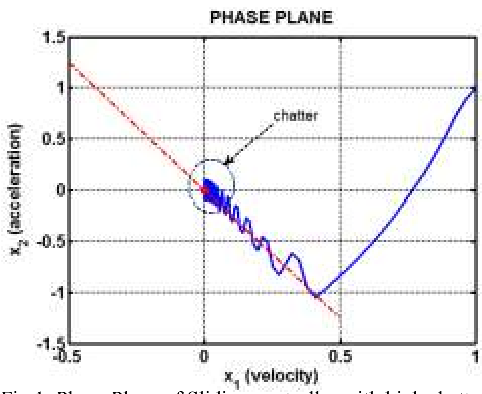
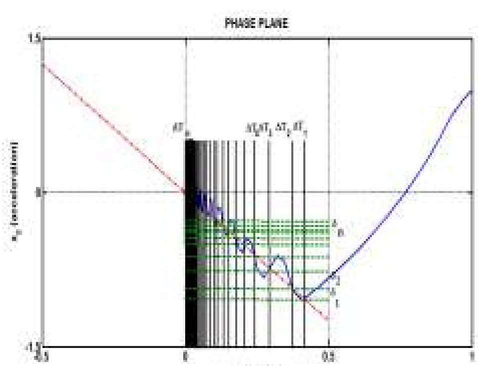


Fig 1. Phase Plane of Sliding controller with high chattering.

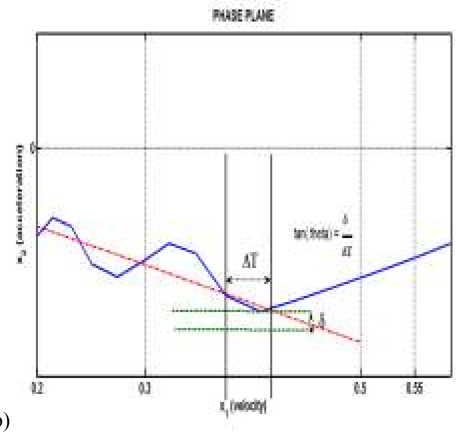
To simplify the relations between the slope and gain value, assume that θ, δ are fixed where (δ is the gain value of SMC,$\theta = k_1$ *slope of sliding line*), the slope of the sliding line can ($k_1 = \tan(\theta)$) Explained in the above equation:

$$k_1 = \tan(\theta) = \frac{\delta}{\Delta T} \qquad (9)$$

$\delta, \Delta T$ The value of sliding controller gain (α, β), sampling time of switching gain respectively. Figure 2 (a, b) shows in details the relation between the control signal and sliding line parameters.



(a)



(b)

Fig. 2 (a, b) Relationship between the gain of sliding mode and the slope of sliding line

Assume canonical form of a second order system that has $a_{11} = 0, a_{12} = 1$ *and* $a_{21} = -a_{21}, a_{22} = -a_{21}, b_1 = 0$, then the Eq. (4) becomes

$$\dot{x_1} = x_2 \qquad (10)$$

$$\dot{x_2} = -a_{21}x_1 - a_{22}x_2 + b_2 u \qquad (11)$$

$$u = -\psi x_1 \qquad (12)$$

The sliding mode exists (Lyapunov's stable exists) if the system state trajectories are bound and moving to the sliding line when,

$$\lim_{\sigma \to +0} \dot{\sigma} < 0, \lim_{\sigma \to -0} \dot{\sigma} > 0 \qquad (13)$$

and the sates of control equations are:

$$u_i = \varphi \, sign\,(\sigma)$$
$$u_e = S^{-1}B^{-1}S\,A\,[x_1 \quad x_2]$$
$$u = u_e - u_i$$
$$u = S^{-1}B^{-1}S\,A\,[x_1 \quad x_2] - \varphi \, sign\,(\sigma) \qquad (14)$$

Feedback representation of the system (A, B, C), if $CB = 0$ and $CAB > 0$, presents a 2nd order sliding-mode of measure zero. While the trajectories near to $\{x \in \sigma: |CA^2x| < CAB\}$ can rising a chattering phenomenon. The analysis of the chattering phenomenon is explained and a principle rule proves to show that in many cases the chattering can be estimated by a 2nd order sliding mode. In figures 2, this result is used to show the reality of chattering phenomenon. The parameterization corresponds to a linear system, when $CB = 0$ *and* $CAB = 1$.

While $CAB > 0$, the trajectories close to the set $\{x \in \sigma: |x_1| < 1\}$ will provide fast switching. This fast performance will happen in the state (x_2). As opposed to the non-chattering state x_1, as shown in figure 3.

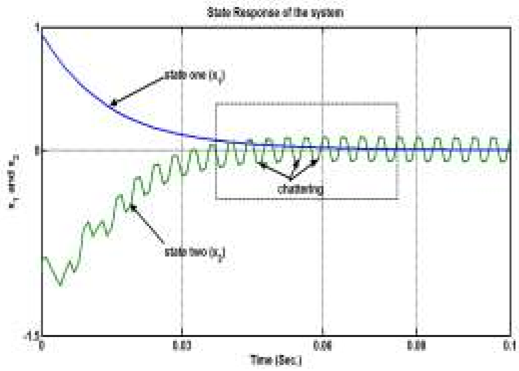


Fig. 3 Response of the second order system with chattering that appears on second state.

On the other hand the first-order sliding mode can be derived in the same way from 2nd order sliding mode. It is given by $x_1 = x_2 = 0$. When the trajectory starting at a point $x(0)$ with, $x_1(0) = 0, x_2(0) \neq 0$, properly small, will fluctuate around the sliding equation (σ). With parameterization of (A, B, C) and the order $n \geq 2$ over a duration of $[0, T], when\ T > 0$. See figure 4. On the other hand when initial state will be $x(0) = (0, x_2(0))^T$, where $x_2(0)$ is changeable and $x_1(0)$ is fixed, allow the switching times be indicated by $\Delta T \in [0, T]$, $f \geq 1$ $f =$ number of switching, i.e., let ΔT_f Be the time suggested such that $x_1(\Delta T) = 0$ .If $x_2(\Delta T) < 1$ for all $\Delta T \in [0, T]$, then the chattering variable x_2 Convinces

$$\frac{1}{|x_2(0)|} max|x_1(0)_{\to 0}, as\ x_2(0)_{\to 0}| \quad (15)$$

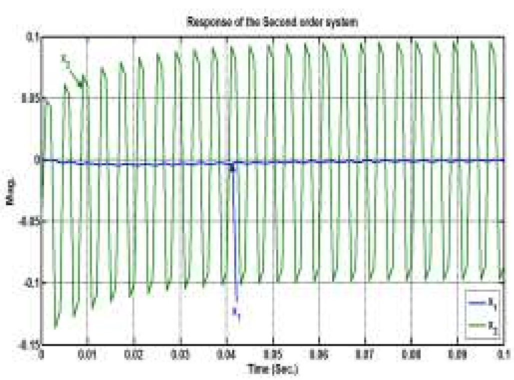


Fig. 4 Response of the system with initial condition ($x_{1o} = 0$ and $x_{2o} = 0.05$)

And the covering of the hit the highest points of the chattering variable $x_2(0)$ is given by

$$x(t) = e^{AT}x(0) + (e^{AT} - I)A^{-1}Bu \quad (16)$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}\, Bu(\tau)d\tau$$

$$x(t) = \varphi(t)x(0) + \int_0^t \varphi(t-\tau)\, Bu(\tau)d\tau$$

When initial time is given by t_o

$$t = t_o$$

$$x(t_o) = \varphi(t_o)x(0) + \int_0^{t_o} \varphi(t_o - \tau)\, Bu(\tau)d\tau$$

Multiply both the side by $\varphi(t_o)^{-1}$

$$\varphi(t_o)^{-1}x(t_o) = x(0) + \int_0^{t_o} \varphi(t_o)^{-1}\varphi(t_o - \tau)Bu(\tau)d\tau$$

$$\varphi(t_o)^{-1} = \varphi(-t_o)$$

$$\varphi(t_o)^{-1}x(t_o) = x(0) + \int_0^{t_o} \varphi(-t_o)\varphi(t_o - \tau)Bu(\tau)d\tau$$

$$\varphi(t_o)^{-1}x(t_o) = x(0) + \int_0^{t_o} \varphi(-t_o + t_o - \tau)Bu(\tau)d\tau$$

$$\varphi(t_o)^{-1}x(t_o) = x(0) + \int_0^{t_o} \varphi(-\tau)\, Bu(\tau)d\tau$$

$$x(0) = \varphi(-t_o)x(t_o) - \int_0^{t_o} \varphi(-\tau)\, Bu(\tau)d\tau$$

$$x(t) = \left[\varphi(-t_o)x(t_o) - \int_0^{t_o} \varphi(-\tau)\, Bu(\tau)d\tau\right]\varphi(t) + \int_0^{t_o} \varphi(t-\tau)\, Bu(\tau)d\tau$$

$$x(t) = .\varphi(t - t_o)x(t_o) + \int_{t_o}^{\Delta T} \varphi(\Delta T - \tau)Bu(\tau)d\tau \quad (17)$$

$$\varphi(t) = e^{At} = I + A\Delta T + \frac{A^2\Delta T^2}{2!}$$

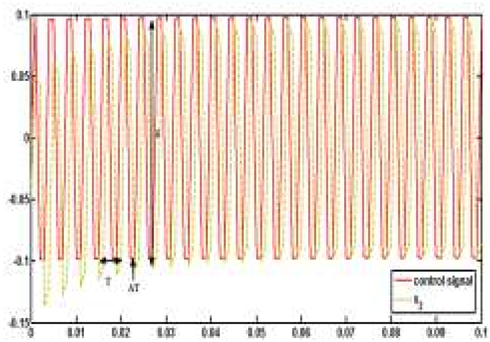
$$u = sign(\sigma)$$

$$x(\Delta T) = x(0) + \Delta T(Ax(0) + Bu) + \frac{\Delta T}{2}(A^2x(0) + ABu)$$

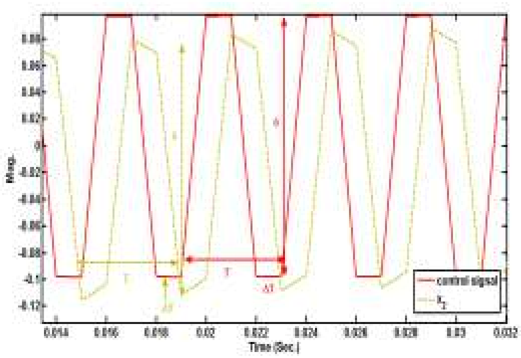
$$\Delta T \leq max\left|\frac{e^{A\gamma}A(Ax(0)+Bu)}{2}\right| \quad (18)$$

$, \gamma\ small\ switching\ time$

It's clear from Eq. (18) the periodic switching time the high chattering (ΔT) not depended only on Initial condition of states but also depend on the magnitude value of sliding gain.



(a)



(b)

Fig. 5(a, b) Relationship between x_2 and sliding mode gain

Now if $CAB = 1 > 0$ subsequently switching if $x_2(0)$ is suitably small that introduce

$$\alpha_1 = CAx(0) = x_2(0) + u \qquad (19)$$

$$\alpha_2 = CA^2x(0) + CABu \qquad (20)$$

Eq. (20) holds if the system order $n = 2$. If, this equation and the following at rest holds, but with $x_2 \equiv 0$ can be produced $\alpha_1 u = -|\alpha_1| < 0$ and $\alpha_1\alpha_2 < 0$. if presuppose that ΔT_f is the next switching instant, i.e $Cx(t) = x_1(\Delta T)$., then, it holds that

$$0 = x_1(\Delta T) = \alpha_1 \Delta T + \frac{\alpha_2 \Delta T^2}{2} \qquad (21)$$

$$CAx(\Delta T) = x_2(\Delta T) = \alpha_1 + \alpha_2 \Delta T \qquad (22)$$

in small periodic t establish ΔT as an estimate of t to the exactness of the Eq. (22).

$$\alpha_1 + \alpha_2 \Delta T = 0 \qquad (23)$$

$$\Delta T = -\frac{\alpha_1}{\alpha_2} \qquad (24)$$

By comparing between Eq. (9) & (24) that is clear $\delta = \alpha_1$ and $\tan(\theta) = -\alpha_2$

Then, since for small ρ

$$\frac{1}{1+\sqrt{1-\delta}} = \Delta T + \frac{\Delta T}{\varepsilon}\rho$$

$$\frac{1}{1+\sqrt{1-\delta}} = \Delta T\left(1 + \frac{\rho}{\varepsilon}\right)$$

$$\Delta T = \frac{\frac{1}{1+\sqrt{1-\delta}}}{\left(1+\frac{\rho}{\varepsilon}\right)} \qquad (25)$$

From above $\delta \leq 1$, $\varepsilon \ll \rho$ then ΔT decreasing as well as $\tan(\theta) = -\alpha_2 = \rho$ decreasing

Form an Eq. (9, 24) that is clear θ is directly proportional to (δ) & proportional inversely with (ΔT), to keep θ fixed then must be balance the two parameters (δ, ΔT). assume θ is fixed as long as $\theta \leq \frac{\delta}{\Delta T}$, and $x_1(0) = 0$, and $x_2(0) < 1$ for $\Delta T \in [0, T]$. Let the switch times be denoted by t_f , $f > 1$, then the chattering variable x_1 satisfies in equation (18), and the envelope of the peaks of the chattering variable x_2 is given by

$$x_2(t_f) = x_2(0)e^{\frac{-(a_{11}-b_1)t_f}{3}},$$
$$where\ t_f\ switching\ time = \Delta T \qquad (26)$$

$$x_2(\Delta T) = x_2(0)e^{\frac{-(a_{11}-b_1)\Delta T}{3}} \qquad (27)$$

In above Eq. (27) the chattering can be stable or unstable depend on the value of (a_{11}, b_1), and the simple necessary and sufficient condition for stability $a_{11} > b_1$ there in it is argued that for high frequencies

$$H(s) = \frac{m}{s(s+a_{11}-b_1)},\ where\ m > 0 \qquad (28)$$

So a root-locus argument gives that the catering is stable if and only if $(a_{11} - b_1) > 0$
Eq. (28) shows the behavior of chattering pretty well. Consider a chattering solution that starts with x_2 , so the Eq. (28) says that x_2 oscillates with exponentially decreasing amplitude. The length of the

switch intervals will decrease as x_2 decreases. Also the number of switching can be derived from Eq. (28) as follows:

$$f = \frac{1}{|x_2(0)|}\left[\frac{1}{2}\int_0^{\Delta T} e^{\frac{(a_{11}-b_1)\widetilde{\Delta T}}{3}}\right]dt \qquad (29)$$

As shown in figure (3) the switching time period (ΔT) was changes depend on criteria of sliding equations trajectory, and it is clear the change of structure (controller structure) that happened only on when σ (sliding equation) crosses over the sliding line (that pass through the original (0,0) point) in both directions, that's led to a fact that no way to control the ΔT only by the control to the crossing between the sliding equation trajectory and zero line sliding mode with two directions as . From above conclude the reduced the ΔT with fixed δ, the chattering was increased while the system has more speed, this is caused by high gain (δ is large). Therefore, we consider there is a proportional relationship and complex proportional maintain to generate the chattering. This leads us to conclude that chattering cannot be fully subsided because their sources of it may be from the internal parameters of the system itself.

6. Conclusion

This paper displayed a specified the effective of period time switching on Sliding mode controller SMC on the chattering phenomenon. Various aspects of the system's performance were discussed and illustrated by simulation results. The important conclusions from this paper can be summarized as follows

1 - First conclusion the chattering of the sliding mode controller is identified by fast switching time of the controller. This may take %60 of the reason to generate the chatter.

2 - The second conclusion is not only the switching time of the controller generation the chatter, but the magnitude value of sliding gain which takes area may exceed 30%.

3 - Third, the proportionate relationship and choose the parameters of sliding mode controller to play a very important role in refining chatter. Therefore it's necessary to balance between the selected values of SMC, for example (the slope of the sliding line, gain values for controlled and time constant of the switching element).

4 - Fourth conclusion that the removing of chatter completely and absolutely cannot be achieved in practice, because of that chatter may be generated from the same elements of the system itself. This is supported by the Eqs. (28, 29)

5 - The fifth conclusion that dealing with the eliminated the chatter may lead to loss the stability, of the system, so it is important when we want to be reduced the chattering must be considerations the stability, it is the basis thing for the designer.

6 - Finally, we recommend that you use the best ways to control with modern software and compatibility with a mathematical model of the system to reach the best solution and the right to get rid of this phenomenon.

References

[1] M. T. a. M. A. S. Zahra Mohammadi, "Designing Flexible Neuro-Fuzzy System Based on Sliding Mode Controller for Magnetic Levitation Systems "International Journal of Computer Science Issues, vol. 8, pp. 160-171, 2011
[2] a. Z. Y. ZHANG Wenhui, "Control of Free-floating Space Robotic Manipulators base on Neural Network," International Journal of Computer Science Issues vol. 9, pp. 322-327, 2012 2012.
[3] M. P. Aghababa and M. E. Akbari, "A chattering-free robust adaptive sliding mode controller for synchronization of two different chaotic systems with unknown uncertainties and external disturbances," Applied Mathematics and Computation, vol. 218, pp. 5757-5768, 2012.
[4] A. Ferrara and M. Rubagotti, "Second-order sliding-mode control of a mobile robot based on a harmonic potential field," Control Theory & Applications, IET, vol. 2, pp. 807-818, 2008.
[5] A. Abrishamifar, et al., "Fixed Switching Frequency Sliding Mode Control for Single-Phase Unipolar Inverters," Power Electronics, IEEE Transactions on, vol. 27, pp. 2507-2514, 2012.
[6] A. Ferreira, et al., "Robust Control With Exact Uncertainties Compensation: With or Without Chattering?," Control Systems Technology, IEEE Transactions on, vol. 19, pp. 969-975, 2011.
[7] C. N. Onwuchekwa and A. Kwasinski, "Analysis of Boundary Control for Buck Converters With Instantaneous Constant-Power Loads," Power Electronics, IEEE Transactions on, vol. 25, pp. 2018-2032, 2010.
[8] T. C. Lin, et al., "Robust adaptive fuzzy sliding mode control for a class of uncertain discrete-time nonlinear systems," International Journal of Innovative Computing, Information and Control, vol. 8, pp. 347-359, 2012.
[9] V. Utkin and L. Hoon, "Chattering Problem in Sliding Mode Control Systems," in Variable Structure Systems, 2006. VSS'06. International Workshop on, 2006, pp. 346-350.

[10] D. Zhang, et al., "A class of second-order sliding mode controller for servo systems," Journal of Control Theory and Applications, vol. 10, pp. 268-272, 2012.
[11] J. R. Wan, et al., "Sliding mode variable structure control of permanent magnet synchronous machine based on MTPA," Dianji yu Kongzhi Xuebao/ElectricalMachines and Control, vol. 16, pp. 30-35, 2012
[12] K. D. Young, et al., "A control engineer's guide to sliding mode control," in Variable Structure Systems, 1996. VSS '96. Proceedings., 1996 IEEE International Workshop on, 1996, pp. 1-14.
[13] K. D. Young, et al., "A control engineer's guide to sliding mode control," Control Systems Technology, IEEE Transactions on, vol. 7, pp. 328-342, 1999.