Optimization and Application of initial Value of Non-equidistant New Information GM(1,1) Model

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Abstract

Aiming the problem of determining initial value of non-equidistant new information GM(1,1) model, researching modeling mechanism of non-equidistant new information GM(1,1) model which taking the newest component of original data as the initial value of response function of grey differential equation, the cause of the problem was found out, a new method for initiating value of non-equidistant new information GM(1,1) model was proposed to minimize the quadratic sum of its fitting error and the optimum formula of initiating value was detruded. The new non-equidistant new information GM(1,1) model with the proposed formula of initiating values has the characteristic of high precision as well as high adaptability. Examples validate the practicability and reliability of the proposed model.

Keywords: *Initialization, Background value, GM(1,1),New information principle, Non- equidistant, Optimization, Grey system.*

1. Introduction

Grey model as an important part in the grey system theory has been successfully used in many fields. Among the models, GM (1, 1) has been greatly concerned and been widely used because of the research characteristics such as the small sample and the poor information, as well as the advantages which is simple and practical [1-6]. The accuracy problem in modeling has been the research focus in the grey system theory field. Sequence spacing was regarded as a multiplier to establish the non-equidistance GM (1, 1) model which supposed that there is the linear relationship between data difference and time difference [2], but the result from this model can't be ensured to be consistent with the reality. Function transformation method was adopted to reduce the standard deviation coefficient to take the original sequence as new data sequence and estimate the model parameters, and then GM (1, 1) was set up [3], but there is the complicated calculation. In order to improve the accuracy of the fitting and the predict, the best calculation formula for background value was deduced using non-homogeneous exponent function to fit one-time accumulated generating sequence and non-equidistant GM(1,1) model was established [4]. Backward accumulation generation was put forward and GOM (1, 1) based on backward accumulation was established [5]. GRM (1,1) based on reciprocal generation was built after proposing reciprocal generation [6]. GRM (1,1) was improved to establish the improved grey model CGRM (1, 1) based on reciprocal generation with better modeling accuracy[7]. But because the first component of the sequence $\mathbf{x}^{(1)}$ is took as initial condition of grey differential equation in this model, it is inadequate for utilizing new information according to new information priority principle in the grey system where there is more cognitive function in new information than in old one. Equidistant new information GM(1,1) model regarded the nth component of the sequence $\mathbf{x}^{(1)}$ as initial condition of grey differential equation was established [8,9], but the problem of determining initial value is neglected in the course of modeling. Multivariable equidistant new information MGM (1, n) model was built, where the nth component of the sequence $\mathbf{x}^{(0)}$ is regarded as initial condition of grey differential equation and the initial value and the coefficient of the background value are optimized [10, 11]. Homogeneous exponent function fitting one-time accumulated generating sequence was used to establish GM (1, 1) with the nth component of $\mathbf{x}^{(0)}$ regarded as initial condition and the optimization initial value [12], but the calculation is more complicated. In this paper, the ideas improving initial value of equidistant GM (1,1) model in [13] was absorbed. Aiming at the problem of determining initial value of non-equidistant new information GM(1,1) model, the paper researched modeling mechanism of non-equidistant new information GM(1,1) model and found out the cause of this problem. On the view of minimizing the quadratic sum of fitting error of the model, new method to obtain the initial value $\hat{x}^{(0)}(t_m)$ was proposed and the computation formula obtaining the initial value $\hat{x}^{(0)}(t_m)$ was derived. A non-equidistant new information GM(1,1) model with high precision as well as high adaptability was established. Examples validate the practicability and reliability of the proposed model.

2. Modeling of Non-equidistant New Information GM(1,1) Model

Definition 1: Supposed the sequence $X^{(0)}=[x^{(0)}(t_1),\cdots,x^{(0)}(t_m)]$, if $\Delta t_i=t_i-t_{i-1}\neq cons$ where $i=2,\cdots,m$, $X^{(0)}$ is called as non-equidistant sequence.

Definition 2: Supposed the sequence $X^{(1)}=[x^{(1)}(t_1),x^{(1)}(t_2),\cdots,x^{(1)}(t_n)]$, if $x^{(1)}(t_1)=x^{(0)}(t_1)$ and $x^{(1)}(t_{k+1})=x^{(1)}(t_k)+x^{(0)}(t_{k+1})\cdot\Delta t_{k+1}$ where $k=1,\cdots,m-1$, $X^{(1)}$ is one-time accumulated generation of non-equidistant sequence $X^{(0)}$, and it is denoted by 1-AG0.

Supposed the original data sequence $\mathbf{X}^{(0)}=[x^{(0)}(t_1),\cdots,^{(0)}(t_m)]$, where $x^{(0)}(t_j)(j=1,2,\cdots m)$ is the observation value at t_j, m is the data number, and the sequence $[x^{(0)}(t_1),x^{(0)}(t_2),\cdots,x^{(0)}(t_m)]$ is non-equidistant, that is, the spacing t_j-t_{j-1} is not constant.

In order to establish the model, firstly the original data is accumulated one time to generate a new sequence as:

$$\mathbf{X}^{(1)}=[x^{(1)}(t_1),x^{(1)}(t_2),\cdots,x^{(1)}(t_m)] \tag{1}$$

where, $x^{(1)}(t_j)(j=1,2,\cdots,m)$ meets the conditions in the Definition 2, that is,

$$x^{(1)}(t_k)=\begin{cases}\sum_{j=1}^{k}x^{(1)}(t_j)(t_j-t_{j-1}) & (k=2,\cdots,m)\\ x^{(0)}(t_1) & (k=1)\end{cases} \tag{2}$$

Accounting to one-time accumulated generation, a non-equidistant GM(1,1) model is established as a first-order grey differential equation as: $\frac{dx^{(1)}}{dt}+a\zeta^{(1)}=b$,where $\zeta^{(1)}$ is the background value. Its albino differential equation is as:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b$$

As the result form of the albino differential equation is exponential, $x^{(1)}$ can be fitted by the equation as $x^{(1)}(t)=Ae^{B\cdot t}+C$. Supposed that $x^{(1)}$ runs through $[t_{k-1},x^{(1)}(t_{k-1})]$, $[t_k,x^{(1)}(t_k)]$ and $[t_{k+1},x^{(1)}(t_{k+1})]$, it can be obtained as:

$$B=\ln\left(\frac{x^{(0)}(t_k)\cdot\Delta t_k}{x^{(0)}(t_{k-1})\cdot\Delta t_{k-1}}\right),k=3,4,\cdots,m \tag{3}$$

$$A=\frac{x^{(0)}(t_k)\cdot\Delta t_k}{e^{B\cdot t_k}-e^{B\cdot t_{k-1}}},\ C=x^{(1)}(t_k)-Ae^{B\cdot t_k} \tag{4}$$

As the result form of the albino differential equation is: $x^{(1)}(t)=Ae^{B\cdot t}+C$, it is derivated to obtain as: $(x^{(1)}(t))'=ABe^{B\cdot t}$, and then $x^{(1)}(t)=\frac{(x^{(1)}(t))'}{B}+C$.

Supposed that $z^{(1)}(t_k)$ is the background value of $x^{(1)}(t_k)$, when $(x^{(1)}(t_k))'=x^{(0)}(t_k)$,

$$z^{(1)}(t_k)=\frac{x^{(0)}(t_k)}{B}+C \tag{5}$$

When $(x^{(1)}(t_k))'=x^{(0)}(t_k)$, the albino differential equation is discredited to get the following form as:

$$x^{(0)}(t_k)+az^{(1)}(t_k)=b \tag{6}$$

The matrix is expressed as follows.

$$\begin{bmatrix}x^{(0)}(t_3)\\x^{(0)}(t_4)\\\cdots\\x^{(0)}(t_m)\end{bmatrix}=\begin{bmatrix}-z^{(0)}(t_3) & 1\\-z^{(1)}(t_4) & 1\\\cdots & \cdots\\-z^{(1)}(t_m) & 1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}$$

Supposed

$$\mathbf{Y}=\begin{bmatrix}x^{(0)}(t_3)\\x^{(0)}(t_4)\\\cdots\\x^{(0)}(t_m)\end{bmatrix},\mathbf{B}=\begin{bmatrix}-z^{(0)}(t_3) & 1\\-z^{(1)}(t_4) & 1\\\cdots & \cdots\\-z^{(1)}(t_m) & 1\end{bmatrix},\Phi=\begin{bmatrix}a\\b\end{bmatrix}$$

Where, Φ is parameter vector to be identified, a and b are the constant to be identified. The most least-squares estimation of Φ is:

$$\hat{\Phi} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Y} \tag{7}$$

According to the new information priority principle of the grey system, after making full use of new information and taking an initial value $x^{(0)}(t_m)$, the time response equation of grey differential equation is:

$$\hat{x}^{(1)}(t_k) = \frac{b}{a} + (x^{(0)}(t_m) - \frac{b}{a})e^{-a(t_k - t_m)} (k = 1, 2, \cdots, m) \tag{8}$$

In Eq.(8), $\hat{x}^{(1)}(t_m) = x^{(0)}(t_m)$ when $t_k = t_m$. After restoring the fitting value of the original data is:

$$\hat{x}^{(0)}(t_k) = \begin{cases} \lim\limits_{\Delta t \to 0} \dfrac{x^{(1)}(t_1) - x^{(1)}(t_1 - \Delta t)}{\Delta t} & (k = 1) \\ \dfrac{(x^{(0)}(t_m) - \frac{b}{a})(1 - e^{a\Delta t_k})e^{-a(t_k - t_m)}}{\Delta t_k} & (k = 2, 3, \cdots, m) \end{cases} \tag{9}$$

The absolute error of the fitting data:

$$q(t_k) = \hat{x}^{(0)}(t_k) - x^{(0)}(t_k) \tag{10}$$

The relative error of the fitting data（%）:

$$e(t_k) = \frac{\hat{x}^{(0)}(t_k) - x^{(0)}(t_k)}{x_i^{(0)(0)}(t_k)} * 100 \tag{11}$$

The mean of the relative error of the fitting data column:

$$f = \frac{1}{m}\sum_{k=1}^{m}|e_i(k)|$$

In non-equidistant GM(1,1) model, the initial value of the differential equation is taken as $x^{(0)}(1) = \hat{x}^{(0)}(1)$. The smallest error of the initial point is zero, and the model parameter $\hat{\mathbf{a}} = [a, b]^{\mathrm{T}}$ and the model fitting value have nothing with the initial value [13]. In non-equidistant new information model, the initial value is took as $x^{(1)}(t_m) = \hat{x}^{(0)}(t_m)$. After modeling and calculating, it is found that the model parameter $\hat{\mathbf{a}} = [a, b]^{\mathrm{T}}$ and the model fitting value have related to the initial value, but the error of the latest point is the smallest.

According to the selection rules for the model estimation equation from the metrology econometric, this method is not the best [13]. If improving initial value to $x^{(1)}(t_m) \neq \hat{x}^{(0)}(t_m)$, Eq.(8) will be transformed into Eq.(12). Considering the smallest error of the overall data, the error square sum of the model can obtain the minimum by using the least square method that determine the initial conditions $\hat{x}^{(0)}(t_m)$ when the error of the latest fitting data points is not equal to zero.

$$\hat{x}^{(1)}(t_k) = \frac{b}{a} + (\hat{x}^{(0)}(t_m) - \frac{b}{a})e^{-a(t_k - t_m)} (k = 1, 2, \cdots, m) \tag{12}$$

While Eq.(9) is transformed into the following equation:

$$\hat{x}^{(0)}(t_k) = \begin{cases} \lim\limits_{\Delta t \to 0} \dfrac{x^{(1)}(t_1) - x^{(1)}(t_1 - \Delta t)}{\Delta t} & (k = 1) \\ \dfrac{(\hat{x}^{(0)}(t_m) - \frac{b}{a})(1 - e^{a\Delta t_k})e^{-a(t_k - t_m)}}{\Delta t_k} & (k = 2, 3, \cdots, m) \end{cases}$$

Supposed $c_k = \dfrac{(1 - e^{a\Delta t_k})e^{-a(t_k - t_m)}}{\Delta t_k}$, $\hat{x}^0(t_k) = c_k(\hat{x}^{(0)}(t_m) - \dfrac{b}{a})$

$\hat{x}^0(t_1)$ can also be expressed as $\hat{x}^0(t_1) = c_1(\hat{x}^{(0)}(t_m) - \dfrac{b}{a})$, where $c_1 = \lim\limits_{\Delta t_k \to 0} \dfrac{(1 - e^{a\Delta t_k})e^{-a(t_1 - t_m)}}{\Delta t_k}$.Considering the effect of computer numerical accuracy it is generally taken as $\Delta t_k = (t_2 - t_1) * \max(\frac{1}{t_1}, 0.01 - 0.001)$, so the fitting value of the original data is obtained:

$$\hat{x}^0(t_k) = c_k(\hat{x}^{(0)}(t_m) - \frac{b}{a}) \tag{13}$$

If $\hat{x}^{(0)}(t_m)$ is known, some data such as the simulation value, the predicted value and the error in non-equidistant new information GM(1,1) model can be obtain by Eq.(13), and then the model is tested [1,14,15].

3. Optimization of the Initial Conditions in Non-equidistant New Information

Theorem 1: In the sense of error square sum in non-equidistant new information GM(1,1) model, the optimal initial condition of this model is:

$$\hat{x}^{(0)}(t_m) = \frac{\sum_{k=1}^{m}(c_k x^{(0)}(t_k) + c_k^2 b/a)}{\sum_{k=1}^{m} c_k^2} \tag{14}$$

where, $c_k = \dfrac{1-e^{a\Delta t_k}}{\Delta t_k} e^{-a(t_k - t_m)}$,

$\Delta t_k = t_k - t_{k-1}, k = 2,3,\cdots,m$, $c_1 = \lim\limits_{\Delta t_k \to 0} \dfrac{(1-e^{a\Delta t_k})e^{-a(t_1-t_m)}}{\Delta t_k}$.

Proof:

Because $\hat{x}^{(0)}(t_k) = c_k(\hat{x}^{(0)}(t_m) - \dfrac{b}{a})(k=1,2,3,\cdots,m)$, supposed S indicates the error square sum of the model, we can obtain the following equation:

$$S = \sum_{k=1}^{m}[x^{(0)}(t_k) - \hat{x}^{(0)}(t_k)]^2 = \sum_{k=1}^{m}[x^{(0)}(t_k) - (c_k(\hat{x}^{(0)}(t_m) - \frac{b}{a})]^2$$

Assumed $\dfrac{dS}{d\hat{x}^{(0)}(t_m)} = 0$, we can obtain the minimum point $\hat{x}^{(0)}(t_m)$.

This proof is completed.

$\hat{x}^{(0)}(t_m)$ is obtained in the case of the smallest error square sum of the model, so it is called as the optimal initial conditions.

4. Example

P. G. Foleiss researched that there is the influence of the temperature on fatigue strength under the long life symmetry cycle of many materials. Table 1 shows the experimental data of the change relation of Ti alloy fatigue strength along with temperature, which is a sequence of non-equidistant spacing. The data in [2,3] were modeled by using the method proposed in this paper and we obtained the following result:

a=0.00094344, b=561.5864, $\hat{x}^{(0)}(t_m)$ =137656.6701

$\hat{x}^{(1)}(t_k) = -457597.9859e^{-0.00094344(t-380)} + 595254.656$

Table 1 Change relation of Ti alloy fatigue strength along with temperature （Mpa）

T / °C (t_k)	100	130	170	210	240	270	310	340	380
$\sigma_{-1}(x^{(0)}(t_k))$	560	557.54	536.10	516.10	505.60	486.1	467.4	453.8	436.4

The fitting value of the original data:

$\hat{x}^{(0)}(t_k)$ =[562.3204, 554.3587, 536.3664, 516.5025, 499.7129, 485.7677, 470.0016, 454.7236, 439.9651]

The absolute error of the fitting data:

$q(t_k)$=[2.3204, -3.1813, 0.26644, 0.40245, -5.8871, -0.33229, 2.6016, 0.92363, 3.5651]

The relative error of the fitting data（%）:

$e(t_k)$ = [0.41435, -0.57059, 0.0497, 0.077979, -1.1644, -0.068358, 0.55662, 0.20353, 0.81694]

The mean of the relative error of the fitting data is 0.43583%.

After the original data were pre-processed by using $t = \dfrac{T-50}{50}$ and $X^{(0)} = \dfrac{\sigma_{-1} - 400}{50}$ in [2], the maximum relative error is 4.86% and the mean relative error is 3.19%. The model was established by using the function transformation method in [3] and the mean relative error is 0.6587%. Homogeneous exponent function fitting one-time accumulated generating sequence was used in [5] and it is 0.9765%. Thus, the example validates the adaptability and the scientific of the proposed model. The combination forecast model [16] can be built with the proposed model instead of traditional GM(1,1) model and the model parameters can also be found the optimum method [17-18].

4. Conclusions

(1) Grey system theory was used and the modeling mechanism of non-equidistant new information GM(1,1) model which taking the newest component of original data as the initial value of response function of grey differential equation was researched. A new method for initiating value $\hat{x}^{(0)}(t_m)$ was proposed to minimize the quadratic sum of its fitting error and the optimum formula of initiating value was detruded. Non-equidistant new information GM(1,1) model was established and the MATLAB program of this model was written.

(2)The model proposed in this paper has the characteristic of high precision as well as high adaptability. Example validates the correctness and validity of the proposed model. There is important practical and theoretical significance and this model should be widely used.

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