Improvement and Application of Initial Value of Non-equidistant GM(1,1) Model

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Abstract

Aiming the problem of determining initial value of non-equidistant GM(1,1) model, in the basis of analyzing model-building mechanism of non-equidistant GM(1,1) model, the cause of the problem was found out and a new method for initiating values of non-equidistant GM(1,1) model was proposed to minimize the quadratic sum of its fitting error. Based on index characteristic of grey model and the definition of integral, the discrete function with non-homogeneous exponential law was used to fit the accumulated sequences, new constructing method of the background value was put forward and formula of background value was given. Combining the proposed formula of background value, the formula of initiating value was detruded. The new non-equidistant GM(1,1) model with the proposed formula of initiating value has the characteristic of high precision as well as high adaptability. Examples validate the practicability and reliability of the proposed model.

Keywords: *Initialization, background value, GM(1,1) model, non-equal interval, precision, grey system.*

1. Introduction

The predictive model is as one important part in the grey system theory and has been demonstrated promising prospect in the agriculture, industry, technology and medical fields. Particularly the GM(1,1) model is one popular predictive grey model but the accuracy is concerned by the researchers[1-3]. The grey system model is based on the equal interval and in fact the actual original data are unequal interval sequence. Establishment of the unequal interval model is a meaningful job. Literature [2] takes the time interval as the multiplier to establish the unequal interval GM(1,1) model, in which the multiplier has the linear relation with the time. In fact it is not accordant with the actual condition. Literature [3] adopts function transformation method to decrease the standard deviation coefficient and form the new sequence, and it also estimates the parameters in the model and establishes the GM(1,1) model, but the calculation is complex. In order to increase the fitting and predictive accuracy of GM(1,1) model literature [4-9] provides the construction method of background values and establishes the unequal interval GM(1,1) model. Literature [4-5] provides the improved method of background values in unequal interval GM(1,1) model and it uses the homogeneous index function to fit the accumulated sequence and get high accuracy, but from the whitening differential equations we know the accumulated sequence has the non-homogeneous index form and the opposite accumulated sequence has the homogenous form, and if the homogenous form instead of the non-homogeneous in the accumulated sequence it will induce great error. Literature [6] uses the non-homogeneous index function to fit the accumulated sequence, deduces the optimization background values and establishes the unequal interval GM(1,1) model. In fact the most method ignores the condition of the initial values in the model and the first point in the original sequence is not used in the establishment of the GM(1,1) model [8]. Literature [9] researches the initial values in the unequal interval GM(1,1) model and points out why the first point is not used and deduces the improved method in the equal GM(1,1) model. Literature [10] adopts the optimization background values λ and revised term in the initial condition to establish the unequal interval GM(1,1) model and the calculation is complex. The paper absorbs the background values construction in literature [7] and uses the non-homogeneous index function to fit the accumulated sequence based on the index and integral character in the grey model. It also provides a integral reconstruction unequal interval GM(1,1) model and gives the construction of background values. For the initial values in the GM(1,1) model the paper researches the mechanism of GM(1,1) model and find out the error term in the initial values. On the condition of the least square error it provides a new way to get the initial values and establishes the unequal interval GM(1,1) model which is combined with the background values. It is with high accuracy and with good theoretic and application values. The example shows it is effective and reliable.

2. The Mechanism of Unequal Interval GM(1,1)

Definition 1 Sequence $X^{(0)}=[x^{(0)}(t_1),\cdots,x^{(0)}(t_m)]$, if $\Delta t_i=t_i-t_{i-1}\neq const$, $i=2,\cdots,m$, $X^{(0)}$ is unequal interval sequence.

Definition 2 Sequence $X^{(1)}=[x^{(1)}(t_1),x^{(1)}(t_2),\cdots,x^{(1)}(t_n)]$, if $x^{(1)}(t_{k+1})=x^{(1)}(t_k)+x^{(0)}(t_{k+1})\cdot\Delta t_{k+1}, k=1,\cdots;m-1$, $x^{(1)}(t_1)=x^{(0)}(t_1)$, $X^{(1)}$ is the unequal interval accumulated sequence of $X^{(0)}$.

Original sequence is $\mathbf{X}^{(0)}=[x^{(0)}(t_1),\cdots,^{(0)}(t_m)]$, where $x^{(0)}(t_j)(j=1,2,\cdots m)$ is the observed value at t_j; m is the number of the sequence column. Sequence $[x^{(0)}(t_1),x^{(0)}(t_2),\cdots,x^{(0)}(t_m)]$ is unequal interval sequence, namely, t_j-t_{j-1} is not the const.

In order to establish the GM(1,1) model we accumulate the original data to get the new sequence firstly.

$$\mathbf{X}^{(1)}=[x^{(1)}(t_1),x^{(1)}(t_2),\cdots,x^{(1)}(t_m)] \tag{1}$$

And $x^{(1)}(t_j)(j=1,2,\cdots,m)$ is satisfied with the condition in definition 2, namely,

$$x^{(1)}(t_k)=\begin{cases}\sum_{j=1}^{k}x^{(1)}(t_j)(t_j-t_{j-1}) & (k=2,\cdots,m)\\ x^{(0)}(t_1) & (k=1)\end{cases} \tag{2}$$

We establish the differential equations $\frac{dx^{(1)}}{dt}+a\zeta^{(1)}=b$ ($\zeta^{(1)}$ is the background value) of $\mathbf{X}^{(1)}$ sequence and the whitening differential equation is:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{3}$$

The difference quotient is

$$x^{(0)}(t_k)+az^{(1)}(t_k)=b \tag{4}$$

Where, $z^{(1)}(t_k)=0.5*(x^{(1)}(t_k)+x^{(1)}(t_{k-1}))$, $z^{(1)}(t_k)$ is the background values of the unequal interval GM(1,1) model and it is the equal operated values.

If $\hat{\mathbf{a}}=[a,b]^{\mathbf{T}}$ are the parameters of unequal interval GM(1,1) model and the estimated values of the least square method in the model are

$$\hat{\mathbf{a}}=(\mathbf{B}^{\mathbf{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{Y} \tag{5}$$

Where

$$\mathbf{B}=\begin{bmatrix}-\frac{1}{2}(x^{(1)}(t_2)+x^{(1)}(t_1)) & 1\\ -\frac{1}{2}(x^{(1)}(t_3)+x^{(1)}(t_2)) & 1\\ \cdots & \cdots\\ -\frac{1}{2}(x^{(1)}(t_m)+x^{(1)}(t_{m-1})) & 1\end{bmatrix},\quad \mathbf{Y}=\begin{bmatrix}x^{(0)}(t_2)\\ x^{(0)}(t_3)\\ \cdots\\ x^{(0)}(t_m)\end{bmatrix}$$

The time respond equations of the grey differential equations (4) are:

$$\hat{x}^{(1)}(t_k)=\frac{b}{a}+(x^{(0)}(t_1)-\frac{b}{a})e^{-a(t_k-t_1)}(k=1,2,\cdots,m) \tag{6}$$

The fitting data after restoring the data are

$$\hat{x}^{(0)}(t_k)=\begin{cases}x^{(0)}(t_1) & (k=1)\\ \frac{x^{(1)}(t_k)-x^{(1)}(t_{k-1})}{\Delta t_k} & (k=2,3,\cdots,m)\end{cases} \tag{7}$$

Absolute error of the fitting data is

$$q(t_k)=\hat{x}^{(0)}(t_k)-x^{(0)}(t_k) \tag{8}$$

Relative error of the fitting data is

$$e(t_k)=\frac{\hat{x}^{(0)}(t_k)-x^{(0)}(t_k)}{x_i^{(0)(0)}(t_k)}*100 \tag{9}$$

The average relative error of the fitting data is

$$f=\frac{1}{m}\sum_{k=1}^{m}\left|e_i(k)\right|.$$

For the equal interval GM(1,1) model $x^{(0)}(1)=\hat{x}^{(0)}(1)$ is taken as the initial value and the error of the initial value is zero, in which it means the parameters $\hat{\mathbf{a}}=[a,b]^{\mathbf{T}}$ and the fitting data have no relation with the initial values $x^{(0)}(1)$ [9]. In the establishment of the unequal

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interval model given $x^{(0)}(t_1)=\hat{x}^{(0)}(t_1)$, $x^{(0)}(t_1)$ is affected on the parameter $\hat{\mathbf{a}}=[a,b]^{\mathrm{T}}$. If we give the error of the initial value is zero it is obviously not the best from econometrics in mathematical economics points[9]. If given $x^{(0)}(t_1)\neq\hat{x}^{(0)}(t_1)$ the equation (6) can be transformed into equation (10). We can use the least square method to make the error for the whole is the least to determine the initial value $\hat{x}^{(0)}(t_1)$. On that condition the error of the first fitting value is not zero.

$$\hat{x}^{(1)}(t_k)=\frac{b}{a}+(\hat{x}^{(0)}(t_1)-\frac{b}{a})e^{-a(t_k-t_1)}(k=1,2,\cdots;m) \qquad (10)$$

Formula (7) can be transformed into formula (11)

$$\hat{x}^{(0)}(t_k)=\begin{cases}\hat{x}^{(0)}(t_1) & (k=1)\\ \dfrac{x^{(1)}(t_k)-x^{(1)}(t_{k-1})}{\Delta t_k} & (k=2,3,\cdots,m)\end{cases} \qquad (11)$$

3. The Optimization Background Values of the Unequal Interval GM(1,1) Model

The meaning of traditional background values is use the trapezium formula to describe the area which is composed of the area between $x^{(1)}(t)$ curve and $[t_k,t_{k+1}]$ interval. When the sequence is changed a little and the background value is fit. When the sequence is changed greatly the construction of the background values will induce big error. So we must research the background values in the unequal interval GM(1,1) model.

Integral operation between interval $[t_k,t_{k+1}]$ for the whitening equation (3) we can obtain

$$x^{(0)}(t_k)\Delta t_k+a\int_{t_{k-1}}^{t_k}x^{(1)}dt=b\Delta t_k \qquad (12)$$

Compared with the grey differential equation $x^{(0)}(t_k)+az^{(1)}(t_k)=b$ the form of background values $\int_{t_{k-1}}^{t_k}x^{(1)}dt$ is more adaptive for the whitening differential equations. From formula (11) we can know the solution of $x^{(0)}(t_k)\Delta t_k+a\int_{t_{k-1}}^{t_k}x^{(1)}dt=b\Delta t_k$ is satisfies the non-homogeneous index form.

Given $x^{(1)}(t_k)=Ge^{-a(t_k-t_1)}+C$, the opposite accumulated sequence is as follows.

$$\begin{aligned}x^{(0)}(t_k)&=\frac{x^{(1)}(t_k)-x^{(1)}(t_{k-1})}{\Delta t_k}\\&=\frac{G(1-e^{a\Delta t_i})}{\Delta t_k}e^{-a(t_k-t_1)}=g_ke^{-a(t_k-t_1)}\end{aligned} \qquad (13)$$

Where

$$g_k=\frac{G(1-e^{a\Delta t_k})}{\Delta t_k}=\frac{G(1-(1+(a\Delta t_k)+\frac{(a\Delta t_k)^2}{2!}+\cdots)}{\Delta t_k}$$

When a and Δt_k is little we expand the first two terms in $e^{a\Delta t_i}$

$$g_k=\frac{G(1-e^{a\Delta t_k})}{\Delta t_k}=\frac{G(-a\Delta t_k)}{\Delta t_k}=-Ga$$

$$\frac{x^{(0)}(t_k)}{x^{(0)}(t_{k-1})}=\frac{e^{-a(t_k-t_1)}}{e^{-a(t_{k-1}-t_1)}}=e^{-a\Delta t_k}$$

So

$$a=-\frac{\ln x^{(0)}(t_k)-\ln x^{(0)}(t_{k-1})}{\Delta t_k}(k=2,3,\cdots,m) \qquad (14)$$

Substituting formula (14) into formula (13)

$$\begin{cases}g_k=\dfrac{x^{(0)}(t_k)}{e^{-a(t_k-t_1)}}=\dfrac{x^{(0)}(t_k)}{[x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_1-t_k}{\Delta t_k}}}\\ G=\dfrac{x^{(0)}(t_k)\Delta t_k[x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_1-t_k}{\Delta t_k}}}{1-\dfrac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}}\end{cases} \qquad (15)$$

For the initial condition $x^{(1)}(t_1)=Ge^{-a(t_1-t_1)}+C=G+C=\hat{x}^{(0)}(t_1)$ we can obtain

$$\begin{aligned}C&=\hat{x}^{(0)}(t_1)-G\\&=\hat{x}^{(0)}(t_1)-\frac{x^{(0)}(t_k)\Delta t_k[x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_1-t_k}{\Delta t_k}}}{1-\dfrac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}}\end{aligned} \qquad (16)$$

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Putting formula (14) and (16) into the background values formula $\int_{t_{k-1}}^{t_k} x^{(1)} dt$, we obtain

$$\begin{aligned} z^{(1)}(t_k) &= \int_{t_{k-1}}^{t_k} x^{(1)} dt = -\frac{\Delta t_k x^{(0)}(t_k)}{a} + C\Delta t_k \\ &= \frac{(\Delta t_k)^2 x^{(0)}(t_k)}{\ln x^{(0)}(t_k) - \ln x^{(0)}(t_{k-1})} + \hat{x}^{(0)}(t_1)\Delta t_k \\ &\quad - \frac{x^{(0)}(t_k)(\Delta t_k)^2 [x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_1-t_k}{\Delta t_k}}}{1-\dfrac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \end{aligned} \tag{17}$$

The estimated parameters of the least square method in the grey differential equation $x^{(0)}(t_k)\Delta t_k + a\int_{t_{k-1}}^{t_k} x^{(1)} dt = b\Delta t_k$ are

$$\hat{\mathbf{a}} = (\mathbf{B}^{\mathbf{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{Y} \tag{18}$$

Where

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(t_2) & \Delta t_2 \\ -z^{(1)}(t_2) & \Delta t_3 \\ \cdots & \cdots \\ -z^{(1)}(t_m) & \Delta t_m \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} x^{(0)}(t_2)\Delta t_2 \\ x^{(0)}(t_3)\Delta t_3 \\ \cdots \\ x^{(0)}(t_m)\Delta t_m \end{bmatrix}$$

If we get $\hat{x}^{(0)}(t_1)$ we can put formula (18) into formula (10) and (11) and we can get the fitting data, predictive data and error term in the unequal interval GM(1,1) model. At last we can check the model [1,11,12].

4. The Improved Initial Condition of the Unequal Interval GM(1,1) Model

Theorem 1 On the condition of the error square sum the optimization initial condition is:

$$\hat{x}^{(0)}(t_1) = \frac{x^{(0)}(t_1) + \sum_{k=2}^{m}(c_k x^{(0)}(t_k) + c_k^2 b/a)}{1 + \sum_{k=2}^{m} c_k^2} \tag{19}$$

Where

$$c_k = \frac{1-e^{a\Delta t_k}}{\Delta t_k} e^{-a(t_k - t_1)}, \Delta t_k = t_k - t_{k-1}, k = 2,3,\cdots,m$$

Prove:
Because

$$\hat{x}^{(1)}(t_k) = \frac{b}{a} + (\hat{x}^{(0)}(t_1) - \frac{b}{a})e^{-a(t_k - t_1)}(k=1,2,\cdots;m),$$

$$\hat{x}^{(0)}(t_k) = \frac{x^{(1)}(t_k) - x^{(1)}(t_{k-1})}{\Delta t_k} = c_k(\hat{x}^{(0)}(t_1) - \frac{b}{a})(k=2,3,\cdots,m),$$

S is the error square sum of the model

$$\begin{aligned} S &= \sum_{k=1}^{m}[x^{(0)}(t_k) - \hat{x}^{(0)}(t_k)]^2 \\ &= [x^0(t_1) - \hat{x}^{(0)}(t_1)]^2 + \sum_{k=2}^{m}[x^{(0)}(t_k) - \hat{x}^{(0)}(t_k)]^2 \\ &= [x^0(t_1) - \hat{x}^{(0)}(t_1)]^2 + \sum_{k=2}^{m}[x^{(0)}(t_k) - (c_k(\hat{x}^{(0)}(t_1) - \frac{b}{a})]^2 \end{aligned}$$

Given $\frac{dS}{d\hat{x}^{(0)}(t_1)} = 0$, we can get the values that make S be the least and $\hat{x}^{(0)}(t_1)$ is

$$\hat{x}^{(0)}(t_1) = \frac{x^{(0)}(t_1) + \sum_{k=2}^{m}(c_k x^{(0)}(t_k) + c_k^2 b/a)}{1 + \sum_{k=2}^{m} c_k^2}$$

Because $\hat{x}^{(0)}(t_1)$ is the values that make the error square sum of the model be the least so we can call it the optimized initial values.

Given $\hat{x}^{(0)}(t_1) = x^{(0)}(t_1)$ we can get $z^{(1)}(t_k)$ from formula (17). Putting $z^{(1)}(t_k)$ into formula (18) the evaluated values of parameters $\hat{\mathbf{a}} = [a,b]^{\mathbf{T}}$ can be got. Using formula (19) we can get $\hat{x}^{(0)}(t_1)$ and putting $\hat{x}^{(0)}(t_1)$ into formula (17) we can get $z^{(1)}(t_k)$ again. Putting $z^{(1)}(t_k)$ into formula (18) we can get the parameters $\hat{\mathbf{a}} = [a,b]^{\mathbf{T}}$ .Usually by 2-3 iterative times we can get the accurate $\hat{x}^{(0)}(t_1)$ and the parameters $\hat{\mathbf{a}} = [a,b]^{\mathbf{T}}$. Putting them into formula (10) and (11) we can get the fitting data, predictive data and error term of the unequal interval GM(1,1) model. Pay attention in order to get $z^{(1)}(t_k)$ we use $\hat{x}^{(0)}(t_1)$, but $\hat{x}^{(0)}(t_1)$ and $x^{(0)}(t_1)$ is almost the same (otherwise the error of the original fitting point is big and the model is not correct). So when we calculate $z^{(1)}(t_k)$ we adopt $x^{(0)}(t_1)$ instead of $\hat{x}^{(0)}(t_1)$ and the error of parameters $\hat{\mathbf{a}}=[a,b]^{\mathrm{T}}$ is little. After getting the parameters we can get $\hat{x}^{(0)}(t_1)$.

5. Examples

P.G. Forest has researched the affection of the temperature to the fatigue strength. Tab 1 is the data of the relation of the temperature and fatigue strength of Ti metal and it is the unequal interval sequence. The paper uses the data in literature [2,3] and establishes the model by the method in the paper, the parameters are

a=0.00098493, b=565.5984, $\hat{x}^{(0)}(t_1)$=559.9968

$$\hat{x}^{(1)}(t_k)=-573689.6393\,e^{-0.00098493\,(t-100)}+574249.6361$$

Tab.1 The relation of Ti alloy fatigue strength along with temperature (Mpa)

T / °C (t_k)	100	130	170	210	240
$\sigma_{-1}(x^{(0)}(t_k))$	560	557.54	536.10	516.10	505.60
T / °C (t_k)	270	310	340	380	
$\sigma_{-1}(x^{(0)}(t_k))$	486.1	467.4	453.8	436.4	

The fitting data of the original data

$\hat{x}^{(0)}(t_k)=$ [559.9968, 556.7804, 537.929, 517.148, 499.6102, 485.0636, 468.6404, 452.7475, 437.41844]

The absolute error of the fitting data

$q(t_k)=$ [-0.003211,-0.75958, 1.829, 1.048, -5.9898, -1.0364, 1.2404,-1.0525, 1.0184]

The relative error of the fitting data(%):

$e(t_k)=$ [-0.00057338,-0.13624, 0.34116, 0.20307, -1.1847,-0.2132, 0.26538, -0.23192, 0.23337]

The average relative error of the fitting sequence is 0.31218%.

Literature [2] takes t = (T–50) / 50 as the pre-processing and the largest error is 4.86% and the average relative error is 3.19%. Literature [3] adopts the function-transformation to establish the model and the relative error is 0.6587%. Literature [5] uses the homogeneous index function to fit the accumulated sequence and the relative error is 0.9765%. So we can see the method in the paper is more adaptively and scientific. The combination forecast model [13] can be built with the proposed model instead of traditional GM(1,1) model and the model parameters can also be found the optimum method [14-15].

4. Conclusions

(1) Using the grey theory on the condition of the fitting error is the least it provides a new way to determine the initial values and deduces an optimized initial values method. According to the index and integral character the paper uses the non-homogeneous index function to fit the accumulated sequence and gives an integral reconstruction background values in the GM(1,1) model. Combined with the integral reconstruction background values it provides the construction of initial values and establish the unequal interval GM(1,1) model and compile the Matlab program.

(2) The model in the paper is characteristic with high accuracy and good adaptive ability. The examples show the method is correct and adaptive. It is meaningful to expand the grey system and is deserved to be applied into the other fields.

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