# Fuzzy Logic Method for Evaluation of Difficulty Level of Exam and Student Graduation 

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#### Abstract

Application of fuzzy logic in processing student evaluation, are expected to represent the mechanisms of human thought processes capable of resolving the problem of evaluation of students, which can be monitored by the teacher directly. With a system of evaluation of student test results by using fuzzy logic will be able to support the needs of teachers as well as those related to monitor student progress so that it can support the success of students. In the Fuzzy Logic method for each criterion are defined into 4 fuzzy set, low, medium, high and very high. item about the difficulty level, the level of difficulty of exams and graduation rates of students and participants ranked the Fuzzy Logic method is the output of the system. Fuzzy Logic will consider both the value of the criteria used, if the difficulty level is very difficult problems and low student scores in a fuzzy set criterion is high, then the student is graduating. This means more equitable Fuzzy Logic in reaching a decision and determine graduation.


Keywords: Fuzzy Logic, student evaluation, Inference engine.

## 1. Introduction

Fuzzy logic has the advantage of modeling the qualitative aspects of human knowledge, and decision making as done by human beings by applying the rule base. Modern information management systems enable the recording and the management of data using sophisticated data models and a rich set of management tools [1]. Application of fuzzy logic in the processing of student test evaluation, expected to represent the mechanism of human thinking processes to solve problems of student exams. With a system of evaluation of students exam results by using fuzzy logic will be able to support the needs of teachers as well as those related tomonitor student progress so as to support its students success.

Fuzzy set theory was proposed in 1965 by Zadeh to help computers reason with uncertain and ambiguous information. Zadeh proposed fuzzy technology as a
means to model the uncertainty of natural language [1],[2]. He reasoned that many difficult problems can be expressed much more easily in terms of linguistic variables. Linguistic variables are words and attributes which are used to describe certain aspects of the real world. One important feature of linguistic variables is the notion of their utility as an expression of data compression. Zadeh describes this as compression granulation. He argues that this is important because it is more general than use of discrete values. This point means that an agent using linguistic variables may be able to deal with more continuous and robust descriptions of reality and problem spaces. Our approach is to design a fuzzy rule base system to control training process.

Fuzzy logic is powerful problem solving methodology with a myriad of applications in embedded control and information processing. Fuzzy provides a remarkably simple way to draw definite conclusions from vague, ambiguous or imprecise information. In a sense, fuzzy logic resembles human decision making with its ability to work from approximate data and find precise solutions.

Unlike classical logic which requires a deep understanding of a system, exact equations, and precise numeric values, fuzzy logic incorporates an alternative way of thinking, which allows modeling complex systems using a higher level of abstraction originating from our knowledge and experience. Fuzzy logic allows expressing this knowledge with subjective concepts such as very hot, bright red, and a long time which are mapped into exact numeric ranges [3].

Fuzzy logic has the advantage of modeling the qualitative aspects of human knowledge, as well as decisions made by humans by applying the rules of the rule base or bases. Application of fuzzy logic in processing student evaluation, are expected to represent the mechanisms of human thought processes
capable of resolving the problem of evaluation of students, which can be monitored by the teacher directly.

With a system of evaluation of student test results by using fuzzy logic will be able to support the needs of teachers as well as those related to monitor student progress so that it can support the success of students. Modern management information system allows in terms of recording and management of data using sophisticated data models and advanced management. In the context of the education system, the information usually includes details about the learning materials, tasks associated with student assignments, exams, and other notes. With the expert system is expected to reduce underarm accuracy of information and simplify the access to information systems, in terms of fuzzy modeling. Fuzzy rule-based system can be considered as a good reference for evaluating the test and quality assurance of an organization for students.

## 2. Methodology

This system is designed for evaluating and teaching the students so that the resulting control system will reliably and safely achieve high performance operation. A block diagram of this research is shown in Fig.1. Basically in fuzzy control system, there are four major stages to accomplish the control process: [1],[4]

- Fuzzy input and output variables \& their fuzzy value
- Fuzzy rule base
- Fuzzy inference engine
- Fuzzification and defuzzification modules


Fig. 1 General Overview System

### 2.1 Difficulty level exam

About the level of difficulty is an opportunity to answer correctly a question at a certain skill level, usually expressed in the form of an index. Difficulty level of the index is generally expressed as a proportion of the size range from 0.00 to 1.00 . The greater the difficulty level of the index obtained from the calculation, then the easier about it. The formula to calculate the level of difficulty (TK) is as follows:

$$
\begin{equation*}
T K=\frac{p}{n} \tag{1}
\end{equation*}
$$

where:
TK = difficulty of item
$\mathrm{p}=$ number of examinees who answered the item correctly
n = number of examinees
Difficulty levels result using the above formula describes the level of difficulty about it. The difficulty level classification problem can be illustrated as follows:

Table 1 : difficulty exam

| great value | Criteria |
| :---: | :---: |
| $0,00-0,45$ | Difficult |
| $0,46-0,75$ | Medium |
| $0,76-1,00$ | Easy |

Point about the difficulty level has two functions, namely usability for the educators and usability testing/teaching. Usefulness for educators include: the re-introduction of the concept of the learning, provide feedback to students about their learning, gain information about the curriculum emphasis, suspect items about the bias. While usability for process of testing and teaching, among others: introduction of the concepts needed to re taught, the signs the strengths and weaknesses of the school curriculum, and weave the test have data on accuracy [5].

### 2.2 Fuzzy Logic Method

This model comprises of four components fuzzy inference engine, fuzzy rules, fuzzifier, and a defuzzifier. The four steps processes are: [6]

## Step 1 Fuzzification

Of the input parameters total dissipated energy and node centrality. Now to resolve the level to which the inputs are belonging to the appropriate fuzzy sets or rule the inputs are analyzed. This study used two phases are carried out to evaluate the exam average grade and evaluate students using the Fuzzy Logic.
a. Evaluating an online exam

We consider two fuzzy input variables as exam average grades ( z 1 ) and difficulty level of exam ( y 1 ) and the output will be the exam level (z1). Membership function of $\mathrm{z} 1, \mathrm{y} 1$ and z 2 shold be as follows ( $0 \leq \mu \leq 1$ ). [7]

1. Exam average grades (z1)

Written test assessment variable is divided into three parts: low, medium, high and very high.


Fig. 2 Fuzzy membership functions for exam average grade (z1)

Based on the picture looks fuzzy set membership degree one owned low value range of 0 to 40 . Fuzzy Fuzzy region with the set being located in the range of 20 to 70. Fuzzy set Fuzzy regions with high lies in the range of 50 to 90 . Fuzzy set Fuzzy regions with extremely high located in the range of 80 to 100 . The formula of variable membership function on the exam average grade ( z 1 ) assessment as follows :

$$
\begin{align*}
& \mu_{r}(a)=\left\{\begin{array}{cc}
1 ; & a \leq 20 \\
\frac{40-a}{20} ; & 20 \leq a \leq 40 \\
0 ; & a \geq 40
\end{array}\right.  \tag{2}\\
& \mu_{s}(a)=\left\{\begin{array}{cc}
0 ; & a \leq 20 \text { or } a \geq 70 \\
\frac{a-20}{20} ; & 20 \leq a \leq 40 \\
1 ; & 40 \leq a \leq 50 \\
\frac{70-a}{20} ; & 50 \leq a \leq 70
\end{array}\right.  \tag{3}\\
& \mu_{t}(a)= \begin{cases}0 ; & a \leq 500 r a \geq 90 \\
\frac{a-50}{20} ; & 50 \leq a \leq 70 \\
1 ; & 70 \leq a \leq 80 \\
\frac{90-a}{20} ; & 80 \leq a \leq 90\end{cases}  \tag{4}\\
& \mu_{s t}(a)= \begin{cases}0 ; & a \leq 80 \\
\frac{a-80}{10} ; & 80 \leq a \leq 90 \\
1 ; & 90 \leq a \leq 100\end{cases} \tag{5}
\end{align*}
$$

2. Difficulty level of exam (y1)

Written test assessment variable is divided into three parts: low, medium, high and very high.


Fig. 3 Fuzzy membership functions for difficulty level of exam (y1)

The formula of variable membership function on the level of exam (y1) assessment as follows :

$$
\begin{align*}
& \mu_{r}(a)=\left\{\begin{array}{cc}
1 ; & a \leq 20 \\
\frac{40-a}{20} ; & 20 \leq a \leq 40 \\
0 ; & a \geq 40
\end{array}\right.  \tag{6}\\
& \mu_{s}(a)=\left\{\begin{array}{cc}
0 ; & a \leq 20 \text { or } a \geq 70 \\
\frac{a-20}{20} ; & 20 \leq a \leq 40 \\
1 ; & 40 \leq a \leq 50 \\
\frac{70-a}{20} ; & 50 \leq a \leq 70
\end{array}\right.  \tag{7}\\
& \mu_{t}(a)=\left\{\begin{array}{lc}
0 ; & a \leq 500 r a \geq 90 \\
\frac{a-50}{20} ; & 50 \leq a \leq 70 \\
1 ; & 70 \leq a \leq 80 \\
\frac{90-a}{20} ; & 80 \leq a \leq 90
\end{array}\right.  \tag{8}\\
& \mu_{s t}(a)=\left\{\begin{array}{cc} 
& 0 ;
\end{array}\right] \begin{array}{ll}
\frac{a-80}{10} ; & 80 \leq a \leq 90 \\
1 ; & 90 \leq a \leq 100
\end{array} \tag{9}
\end{align*}
$$

b. Evaluating students

We consider two fuzzy input variables as exam grade (x1) and difficulty level of exam (y1) and the output will be the student level (x2). Membership function of $\mathrm{x} 1, \mathrm{y} 1$ and x 2 should be as follows $(0 \leq \mu \leq$ $1)$.

1. Student grades ( x 1 )

Written test assessment variable is divided into three parts: low, medium, high and very high.


Fig. 4 Fuzzy membership functions for student grade (x1)

The formula of variable membership function on the level of exam ( y 1 ) assessment as follows

$$
\begin{align*}
& \mu_{r}(a)=\left\{\begin{array}{cc}
1 ; & a \leq 20 \\
\frac{40-a}{20} ; & 20 \leq a \leq 40 \\
0 ; & a \geq 40
\end{array}\right.  \tag{10}\\
& \mu_{s}(a)=\left\{\begin{array}{cc}
0 ; & a \leq 20 \text { or } a \geq 70 \\
\frac{a-20}{20} ; & 20 \leq a \leq 40 \\
1 ; & 40 \leq a \leq 50 \\
\frac{70-a}{20} ; & 50 \leq a \leq 70
\end{array}\right.  \tag{11}\\
& \mu_{t}(a)=\left\{\begin{array}{cc}
0 ; & a \leq 500 r a \geq 90 \\
\frac{a-50}{20} ; & 50 \leq a \leq 70 \\
1 ; & 70 \leq a \leq 80 \\
\frac{90-a}{20} ; & 80 \leq a \leq 90
\end{array}\right.  \tag{12}\\
& \mu_{s t}(a)=\left\{\begin{array}{cc}
0 ; & a \leq 80 \\
\frac{a-80}{10} ; & 80 \leq a \leq 90 \\
1 ; & 90 \leq a \leq 100
\end{array}\right. \tag{13}
\end{align*}
$$

2. Difficulty level of exam (y1)

Written test assessment variable is divided into three parts: low, medium, high and very high.


Fig. 5 Fuzzy membership functions for difficulty level of exam (y1)

The formula of variable membership function on the level of exam (y1) assessment as follows :

$$
\begin{align*}
& \mu_{r}(a)=\left\{\begin{array}{cc}
1 ; & a \leq 20 \\
\frac{40-a}{20} ; & 20 \leq a \leq 40 \\
0 ; & a \geq 40
\end{array}\right.  \tag{14}\\
& \mu_{s}(a)=\left\{\begin{array}{cc}
0 ; & a \leq 20 \text { or } a \geq 70 \\
\frac{a-20}{20} ; & 20 \leq a \leq 40 \\
1 ; & 40 \leq a \leq 50 \\
\frac{70-a}{20} ; & 50 \leq a \leq 70
\end{array}\right.  \tag{15}\\
& \mu_{t}(a)= \begin{cases}0 ; & a \leq 500 r a \geq 90 \\
\frac{a-50}{20} ; & 50 \leq a \leq 70 \\
1 ; & 70 \leq a \leq 80 \\
\frac{90-a}{20} ; & 80 \leq a \leq 90\end{cases}  \tag{16}\\
& \mu_{s t}(a)=\left\{\begin{array}{cc} 
& 0 ; \\
\frac{a-80}{10} ; & 80 \leq a \leq 90 \\
1 ; & 90 \leq a \leq 100
\end{array}\right. \tag{17}
\end{align*}
$$

## Step 2 Rule evaluation

Rules are qualitative statements apply if later into the form, so clearly understood. Rules of the difficulty level of the exam consists 16 rules. Fuzzy rule base for evaluating level of difficulty of the exam is designed as follows :

R1: If (z1) Low and (y1) Low Then (z2) Difficult
R2: If (z1) Low and (y1) Moderate Then (z2) Difficult
R3: If (z1) Low and (y1) High Then (z2) Moderate
R4: If (z1) Low and (y1) Very High Then (z2) Moderate
R5: If (z1) Moderate and (y1) Low Then (z2) Difficult
R6: If (z1) Moderate and (y1) Moderate Then (z2) Moderate
R7: If (z1) Moderate and (y1) High Then (z2) Easy
R8: If (z1) Moderate and (y1) Very High Then (z2) Easy
R9: If (z1) High and (y1) Low Then (z2) Moderate
R10: If (z1) High and (y1) Moderate Then (z2) Moderate

R11: If (z1) High and (y1) High Then (z2) Easy
R12: If (z1) High and (y1) Very High Then (z2) Very Easy
R13: If (z1) Very High and (y1) Low Then (z2) Moderate
R14: If (z1) Very High and (y1) Moderate Then (z2) Easy
R15: If (z1) Very High and (y1) High Then (z2) Very Easy
R16: If (z1) Very High and (y1) Very High Then (z2) Very Easy

Fuzzy rule base for evaluating student is designed as follows:

R1: If x 1 is Low And y 1 is low Then x 2 is pass
$R 2$ : If $x 1$ is low And $y 1$ is medium Then $x 2$ is pass
R3: If $x 1$ is low And $y 1$ is high Then $x 2$ is fail
R4: If x 1 is low And y 1 is very high Then x 2 is fail
R5: If x 1 is medium And y 1 is low Then x 2 is good
R6: If x 1 is medium And y 1 is medium Then x 2 is good
R7: If $x 1$ is medium And $y 1$ is high Then $x 2$ is pass
R8: If $x 1$ is medium And $y 1$ is very high Then $x 2$ is fail
R9: If x 1 is high And y 1 is low Then x 2 is exellent R10: If x 1 is high And y 1 is medium Then x 2 is good R11 If $x 1$ is high And $y 1$ is high Then $x 2$ is pass
R12: If $x 1$ is high And $y 1$ is very high Then $x 2$ is pass
R13: If $x 1$ is very high And $y 1$ is low Then $x 2$ is exellent
R14: If x 1 is very high And y 1 is medium Then x 2 is exellent
R15: If $x 1$ is very high And y1 is high Then x2 is good
R16: If $x 1$ is very high And $y 1$ is very high Then $x 2$ is pass

Step 3 Implications Functions and Inferences Rule

## Implications Functions

Minimum method used to combine any degree of membership of each if then rules are made and expressed in a degree of truth $(\alpha)$. Examples of the use of minimum to rule 6 , rule 7 , rule 10 , rule 11 can be written as follows:
R6: If (z1) Moderate and (y1) Moderate Then (z2) Moderate

$$
\begin{aligned}
\alpha-\text { predikat } & =\mu z 1_{s} \cap \mu y 1_{s} \\
& =\min \left(\mu z 1_{s}[62], \mu y 1_{s}[62]\right) \\
& =\min (0.4 ; 0.4) \\
& =0.4
\end{aligned}
$$

R7: If (z1) Moderate and (y1) High Then (z2) Easy $\alpha-$ predikat $_{2}=\mu z 1_{s} \cap \mu y 1_{t}$

$$
=\min \left(\mu z 1_{s}[62], \mu y 1_{t}[62]\right)
$$

$$
\begin{aligned}
& =\min (0.4 ; 0.6) \\
& =0.4
\end{aligned}
$$

R10: If (z1) High and (y1) Moderate Then (z2) Moderate
$\alpha-$ predikat $_{2}=\mu z 1_{s} \cap \mu y 1_{t}$

$$
=\min \left(\mu z 1_{t}[62], \mu y 1_{s}[62]\right)
$$

$$
=\min (0.6 ; 0.4)
$$

$$
=0.4
$$

R11: If (z1) High and (y1) High Then (z2) Easy $\alpha-$ predikat $_{2}=\mu z 1_{s} \cap \mu y 1_{t}$

$$
\begin{aligned}
& =\min \left(\mu z 1_{t}[62], \mu y 1_{t}[62]\right) \\
& =\min (0.6 ; 0.6) \\
& =0.6
\end{aligned}
$$

The Inference Rules
The method of determining the maximum graduation of FIS is used to evaluate the results of the rules that have been made. Solution output fuzzy set is obtained by taking the maximum value of the rule is appropriate, then use it to modify the area and applying it to the output fuzzy.

## Step 4 Deffuzification

Defuzzification is a process of converting output fuzzy variable into a unique number. Defuzzification process has the capability to reduce a fuzzy set into a crisp single-valued quality or into a crisp set; to convert a fuzzy matrix into a crisp matrix; or to convert a fuzzy number into a crisp number. [8]
In the process of using the Weighted Average, the calculations can be seen below:

$$
\begin{equation*}
W A=\frac{\sum(\mu(x)) *(z)}{\sum(\mu(x))} \tag{18}
\end{equation*}
$$

$z \quad$ : Output score
$W A$ : Weighted Average
$\mu(x)$ : Membership function of fuzzy output area
The example of deffuzification

$$
\begin{aligned}
& \text { WA } \\
& =\frac{(0.25 * 75)+(0.25 * 50)+(0.25 * 50)+(0.75 * 50)}{0.25+0.25+0.25+0.75} \\
& =54.1666=54.17
\end{aligned}
$$

## 3. Axperiments and Results

We can classify the test in accordance with our expert system for the 4 levels:. Easy, Medium, Hard and very
hard [7] Then we use math test scores as an example, so that the results are as follows in figure 6 [6]:

| SOAL | NO. SOAL | JML. <br> PESERTA | JML. BENAR | POINT | KETERANGAN |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Bahasa Indonesia | 1 | 106 | 84 | 79.25 | Mudah |
| Bahasa Indonesia | 2 | 106 | 90 | 84.91 | Mudah |
| Bahasa Indonesia | 3 | 106 | 73 | 68.87 | Menengah |
| Bahasa Indonesia | 4 | 106 | 35 | 33.02 | Sulit |
| Bahasa Indonesia | 5 | 106 | 48 | 45.28 | Sulit |
| Bahasa Indonesia | 6 | 106 | 68 | 64.15 | Menengah |
| Bahasa Indonesia | 7 | 106 | 68 | 64.15 | Menengah |
| Bahasa Indonesia | 8 | 106 | 57 | 53.77 | Menengah |
| Bahasa Indonesia | 9 | 106 | 88 | 83.02 | Mudah |
| Bahasa Indonesia | 10 | 106 | 79 | 74.53 | Menengah |

Fig. 6 Results for difficulty level of each item of the test

After getting the value of the degree of difficulty of the test the next step dalah find difficulty value test. The authors grouped into 4 levels: very easy, easy, moderate and hard. The difficulty level exam using two criteria: the value of difficulty of questions and average student grade.


Fig. 7 Results for difficulty level exam

Step 1: an average grade and value kesulutan exams.
Step 2: an overall degree of membership values of the average student and the difficulty level of the exam grade.
Step 3: a rule or fuzzy criteria.
Step 4: is the value defuzzyfikasi and fuzzy decision. From Figure 7 we can see that the value of the defuzzyfikasi is 63.88 so the level of difficulty of the test was EASY.


Fig. 8 Results for level student

The first column contains the name of the student, the student points and the value of the degree of difficulty of the selected subjects. In column 2 value will be processed to produce value students' graduation and degree completion. We classify student in to 4 levels: fail, pass, good
and excellent. [11] So according to "Bahasa Indonesia" Course the students level are as shown in figure 9.

|  | id | $i d$ jenis_soal | nama | skor | fuzz__index | ket |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | 1 | 20 | Adianus Mei Chi. | 80.00 | 83.80 | A |  |
|  | 2 | 20 | Agus Calvario Da... | 70.00 | 71.30 | B |  |
|  | 3 | 20 | I Gede Agus Les... | 60.00 | 71.30 | B |  |
|  | 4 | 20 | Bagus Kisna Yo... | 90.00 | 83.80 | A |  |
|  | 5 | 20 | Candra Suryawan | 80.00 | 83.80 | A |  |
|  | 6 | 20 | Deny Handreawan | 70.00 | 71.30 | B |  |

Fig. 9 Results for student evaluation

## 1. CONCLUSION

Fuzzy logic is very good when used in evaluating student test making it easier for teachers to assess students according to the level of difficulty of the test. It is also regarded as a good reference for teachers to evaluate the level of the exam is the benefit of this evaluation.

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