Improved delay-range-dependent stability criteria for systems with interval time-varying delay and nonlinear perturbations

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Abstract

In this paper, we consider the problem of robust delay-dependent stability for a class of linear systems with interval time-varying delay and nonlinear perturbations. Less conservative stability criteria is put forward by using Lyapunov-Krasovskii functional approach. Based on delay-central point approach, introducing some free-weighting matrices and using tighter integral inequality for dealing with the cross-terms that emerge from the time derivative of the Lyapunov-Krasovskii functional, new less conservative stability criteria for the system is formulated in terms of linear matrix inequalities .Numerical examples are given to show the effectiveness of the proposed approach.

Keywords:*Lyapunov-Krasovskii(L-K)functional; Robust stability; Interval time-delay; Integral Inequality; Linear matrix inequality (LMI).*

1. Introduction

In control systems, time delay is always one of the sources of instability and poor performance. The system analysis and synthesis with time delayed have received considerable attention in recent years [1-14]. Stability analysis of time-delay systems can be classified into two categories: the delay-independent stability and the delay-dependent stability. Generally speaking, the delay-dependent stability criterion is less conservative than delay-independent stability when the time-delay is small. To derive the delay-dependent stability conditions, many methods have been proposed based on linear matrix inequality (LMI) approach, such as descriptor system approach, bounding techniques, and free weighting matrix approach. An important index of measuring the conservativeness of the obtained conditions is the maximum upper bound on the delay. Finding some less conservative stability conditions motivates the present study.

In some practical systems, time delay may be time-varying and the delay may vary in a range for which the lower bound is not restricted to being zero, such systems are referred to as interval time-varying delay systems [2]. In recently years, many significant results have been reported for this problem [3-14]. For example, The free-weighting matrix method was proposed in [3-5] to investigate the delay-dependent stability of continuous time systems with time-varying delay. Jensen's integral inequality approach was employed in [6-11], where the authors use different integral inequality for dealing the cross-terms that emerge from the time derivative of the L-K functional and obtain different conservative results. A new technique called delay-central point method was proposed in [12]. Based on the delay-central point method and decomposition technique, In [14], the author proposes less conservative stability criteria for computing the maximum allowable bound of the delay range.

In practice, the systems almost contain some uncertainties because it is very difficult to obtain an exact mathematical model due to environment noise, uncertain or slowly varying parameters, etc. Therefore, the stability problem of time-delay systems with nonlinear perturbations has received increasing attention [15-18]. An important issue in this field is to enlarge the feasible region of stability criteria, so how to reduce the conservative is still the topic for the research. A model transformation method was used in [15], A descriptor model transformation together with decomposition technique using the delay term matrix was employed in [16]. A less conservative delay-dependent stability criterion was provided in [17] by using a candidate L-K functional, and bounding the cross terms using free-weighting matrices. Recently, a less conservative delay-dependent stability criterion was provided in [18] by partitioning the delay-interval into two segments of equal length, and evaluating the time-derivative of a candidate L-K functional in each segment of the delay-interval. Nevertheless, there is further scope for reduction in conservatism in the delay-range bound.

In this paper, we deal with the delay-dependent stability problem for a class of linear systems with nonlinear perturbations and interval time-varying delay. Based on delay-central point approach, introducing some free-weighting matrices and using more tighter integral inequality for dealing the cross-terms that emerge from the time derivative of the L-K functional, A new delay-dependent stability criteria for the system is formulated in terms of linear matrix inequalities, which can be easily calculated by using matlab LMI control toolbox, Numerical examples are given to illustrate the effectiveness and less conservatism of the proposed method.

Notations. Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidian space, $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices, The notation $X>0$, for $X\in\mathbb{R}^{n\times n}$, means that the matrix X is a real symmetric positive definite. For an arbitrary matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Problem description and preliminaries

Consider the following system with a time-varying state delay and nonlinear perturbations:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + f(x(t),t) + \\ \qquad g(x(t-h(t)),t) \\ x(t) = \varphi(t), t\in[-h_2, 0] \end{cases} \tag{1}$$

Where, $x(t)\in\mathbb{R}^n$ is the state vector, A , B are constant matrices with appropriate dimensions, $h(t)$ is a time-varying delay satisfying

$$0 \le h_1 \le h(t) \le h_2, \dot{h}(t) \le \mu, \forall t \ge 0 \tag{2}$$

Where, h_1 and h_2 represent the lower and upper bounds of the time-varying delay $h(t)$, respectively, μ is the bound on the delay-derivative, and initial condition $\varphi(t)$ is a continuous vector-valued function. The functions $f(x(t),t)$ and $g(x(t-h(t)),t)$ are unknown nonlinear perturbations with respect to the current state $x(t)$ and in the delay state $x(t-h(t))$, respectively. They satisfy that $f(0,t)=0$, $g(0,t)=0$ and

$$\begin{cases} f^T(x(t),t)f(x(t),t) \le \alpha^2 x^T(t)F^TFx(t) \\ g^T(x(t-h(t)),t)g(x(t-h(t)),t) \le \beta^2 x^T(t-h(t))G^TGx(t-h(t)) \end{cases} \tag{3}$$

Where $\alpha\ge 0$, $\beta \ge 0$ are known scalars, F and G are known constant matrices. For simplicity we denote $f = f(x(t),t)$, $g = g(x(t-h(t)),t)$.

In this paper, we investigate the stability problem of system (1) with the interval time-varying delay satisfying (2) and the nonlinear perturbations f and g satisfying (3).Our main objective is to derive new delay-range-dependent stability conditions under which system (1) is asymptotically stable. The following lemma is introduced which has an important role in the derivation of the main results.

Lemma 1 [11]. For any scalar $h(t)\ge 0$ and any constant matrix $Q\in R^{n\times n}$, $Q=Q^T>0$, the integration $-\int_{t-h(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)ds$ is well defined ,then the following inequality holds :

$$-\int_{t-h(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)ds \le$$
$$h(t)\zeta^T(t)VQ^{-1}V^T\zeta(t) + 2\zeta^T(t)V\left[x^T(t) - x^T(t-h(t))\right]$$

Where,

$$\zeta^T(t) = \left[x^T(t) \quad x^T(t-\frac{h_a}{2}) \quad x^T(t-h_a) \quad x^T(t-h(t)) \quad x^T(t-h_2) \right.,$$
$$\left. x^T(t-\frac{h_2}{2}) \quad \dot{x}^T(t) \quad f^T \quad g^T \right]$$

V is free weighting matrix with appropriate dimensions.

Lemma 2 [19]**.** Suppose $\gamma_1\le\gamma(t)\le\gamma_2$, Where $\gamma(.)$: $\mathbb{R}_+(or\mathbb{Z}_+)\to\mathbb{R}_+(or\mathbb{Z}_+)$ .Then, for any constant matrices Ξ_1, Ξ_2 and Ω with proper dimensions, the following matrix inequality

$$\Omega + (\gamma(t)-\gamma_1)\Xi_1 + (\gamma_2-\gamma(t))\Xi_2 < 0$$

holds, if and only if

$$\Omega + (\gamma_2-\gamma_1)\Xi_1 < 0, \Omega + (\gamma_2-\gamma_1)\Xi_2 < 0.$$

3.Main results

In this section, we present new delay-range-dependent stability conditions for system (1) with the delay satisfying (2) and the perturbations satisfying (3).

Theorem 1 System (1) subject to (2)-(3) is asymptotically stable for a given $0\le h_1\le h_2$ and μ , if there exist scalars $\varepsilon_1\ge 0$, $\varepsilon_2 \ge 0$ and matrices

$P_1>0$, $P_2>0$, $Z_1>0$, $Z_2>0$, $Q=\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}>0$,

$S=\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}>0$,and L_j, N_j, V_j, T_j, $j=1,2$, with appropriate dimensions such that the following LMIs hold,

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$$\begin{bmatrix} \Phi & \sqrt{\frac{h_a}{2}}L & \sqrt{h_\delta}N \\ * & -Z_1 & 0 \\ * & * & -Z_2 \end{bmatrix} < 0 \tag{4}$$

$$\begin{bmatrix} \Phi & \sqrt{\frac{h_a}{2}}L & \sqrt{h_\delta}V \\ * & -Z_1 & 0 \\ * & * & -Z_2 \end{bmatrix} < 0 \tag{5}$$

Where, $\Phi = (\Phi_{i,j})_{9\times 9}$ with

$\Phi_{11} = P_1A + A^TP_1 + P_2 + Q_{11} + S_{11} + L_1 + L_1^T + T_1A + A^TT_1^T + \varepsilon_1 a^2 F^T F$,

$\Phi_{12} = Q_{12} - L_1 + L_2^T, \Phi_{13} = 0$, $\Phi_{14} = P_1B + T_1B$,

$\Phi_{15} = 0, \Phi_{16} = S_{12}$, $\Phi_{17} = -T_1 + A^TT_2^T$,

$\Phi_{18} = \Phi_{19} = P_1 + T_1$, $\Phi_{22} = Q_{22} - Q_{11} - L_2 - L_2^T$,

$\Phi_{23} = Q_{12}, \Phi_{24} = \cdots = \Phi_{29} = 0, \Phi_{33} = V_1 + V_1^T - Q_{22}$,

$\Phi_{34} = -V_1 + V_2^T, \Phi_{35} = \cdots = \Phi_{39} = 0$,

$\Phi_{44} = -(1-\mu)P_2 - V_2 - V_2^T + N_1 + N_1^T + \varepsilon_2\beta^2G^TG$,

$\Phi_{45} = -N_1 + N_2^T, \Phi_{46} = 0, \Phi_{47} = B^TT_2^T, \Phi_{48} = \Phi_{49} = 0$,

$\Phi_{55} = -S_{22} - N_2 - N_2^T$, $\Phi_{56} = -S_{12}^T$, $\Phi_{57} = \cdots = \Phi_{59} = 0$,

$\Phi_{66} = S_{22} - S_{11}$, $\Phi_{67} = \cdots = \Phi_{69} = 0$, $\Phi_{77} = \frac{h_a}{2}Z_1 + h_\delta Z_2 - T_2 - T_2^T$,

$\Phi_{78} = \Phi_{79} = T_2$, $\Phi_{88} = -\varepsilon_1 I$, $\Phi_{89} = 0$, $\Phi_{99} = -\varepsilon_2 I$,

$h_a = (h_1 + h_2)/2, h_\delta = (h_2 - h_1)/2$,

$L = \begin{bmatrix} L_1^T & L_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,

$V = \begin{bmatrix} 0 & 0 & V_1^T & V_2^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,

$T = \begin{bmatrix} T_1^T & 0 & 0 & 0 & 0 & 0 & T_2^T & 0 & 0 \end{bmatrix}^T$,

$N = \begin{bmatrix} 0 & 0 & 0 & N_1^T & N_2^T & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

Proof: Based on delay-central point approach, we dividing delay interval into two equal subintervals at the midpoint h_a, That is $[h_1, h_a]$ and $[h_a, h_2]$, if we can proof that theorem 1 holds for the two subintervals, then theorem 1 is true.

Case 1: when $h_a \le h(t) \le h_2$, Construct a L-K functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{6}$$

$$V_1(t) = x^T(t)P_1x(t) + \int_{t-h(t)}^{t} x^T(s)P_2x(s)ds,$$

$$V_2(t) = \int_{t-\frac{h_a}{2}}^{t} \xi_1^T(s)Q\xi_1(s)ds + \int_{t-\frac{h_2}{2}}^{t} \xi_2^T(s)S\xi_2(s)ds,$$

$$V_3(t) = \int_{-\frac{h_a}{2}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta + \int_{-h_2}^{-h_a}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta,$$

Where

$$\xi_1(s) = \begin{bmatrix} x^T(s) & x^T(s-\frac{h_a}{2}) \end{bmatrix}^T, \xi_2(s) = \begin{bmatrix} x^T(s) & x^T(s-\frac{h_2}{2}) \end{bmatrix}^T.$$

The time-derivative of the L-K functional along the trajectory of (1) is given by

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \tag{7}$$

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t)P_1\dot{x}(t) + x^T(t)P_2x(t) - \\ &(1-\dot{h}(t))x^T(t-h(t))P_2x(t-h(t)) \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{V}_2(t) &= \xi_1^T(t)Q\xi_1(t) - \xi_1^T(t-\frac{h_a}{2})Q\xi_1(t-\frac{h_a}{2}) \\ &+ \xi_2^T(t)S\xi_2(t) - \xi_2^T(t-\frac{h_2}{2})S\xi_2(t-\frac{h_2}{2}) \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{V}_3(t) &= \frac{h_a}{2}\dot{x}^T(t)Z_1\dot{x}(t) + (h_2 - h_a)\dot{x}^T(t)Z_2\dot{x}(t) \\ &- \int_{t-\frac{h_a}{2}}^{t} \dot{x}^T(s)Z_1\dot{x}(s) - \int_{t-h_2}^{t-h_a} \dot{x}^T(s)Z_2\dot{x}(s) \end{aligned} \tag{10}$$

From the condition (2), one can obtain:

$$\begin{aligned} \dot{V}_1(t) &\le 2x^T(t)P_1\dot{x}(t) + x^T(t)P_2x(t) - \\ &(1-\mu)x^T(t-h(t))P_2x(t-h(t)) \end{aligned} \tag{11}$$

Note that

$$-\int_{t-h_2}^{t-h_a} \dot{x}^T(s)Z_2\dot{x}(s)ds = -\int_{t-h_2}^{t-h(t)} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-h(t)}^{t-h_a} \dot{x}^T(s)Z_2\dot{x}(s)ds$$

Using Lemma1, one can obtain:

$$\begin{aligned} -\int_{t-\frac{h_a}{2}}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds &\le \frac{h_a}{2}\zeta^T(t)LZ_1^{-1}L^T\zeta(t) + \\ &2\zeta^T(t)L\left[x(t) - x(t-\frac{h_a}{2})\right] \end{aligned} \tag{12}$$

$$\begin{aligned} -\int_{t-h(t)}^{t-h_a} \dot{x}^T(s)Z_2\dot{x}(s)ds &\le (h(t)-h_a)\zeta^T(t)VZ_2^{-1}V^T\zeta(t) \\ &+ 2\zeta^T(t)V\left[x(t-h_a) - x(t-h(t))\right] \end{aligned} \tag{13}$$

$$\begin{aligned} -\int_{t-h_2}^{t-h(t)} \dot{x}^T(s)Z_2\dot{x}(s)ds &\le (h_2-h(t))\zeta^T(t)NZ_2^{-1}N^T\zeta(t) \\ &+ 2\zeta^T(t)N\left[x(t-h(t)) - x(t-h_2)\right] \end{aligned} \tag{14}$$

On the other hand, for any scalars $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$,it follows from (3) that

$$0 \le \varepsilon_1[\alpha^2 x^T(t)F^T Fx(t) - f^T f] \quad (15)$$

$$0 \le \varepsilon_2[\beta^2 x^T(t-h(t))G^T Gx(t-h(t)) - g^T g] \quad (16)$$

Moreover, for any matrices T with appropriate dimensions, From the system (1), we have

$$0 = 2\zeta^T(t)T[Ax(t) + Bx(t-h(t)) - \dot{x}(t)] \quad (17)$$

Substituting (8)～(17) in (7), the time derivative $\dot{V}(t)$ can be expressed as follows:

$$\dot{V}(t) \le \zeta_1^T(t)(\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_2 - h(t))NZ_2^{-1}N^T + (h(t) - h_a)VZ_2^{-1}V^T)\zeta_1(t)$$

Where

$$\zeta_1^T(t) = \Big[x^T(t) \;\; x^T(t-\frac{h_a}{2}) \;\; x^T(t-h_a) \;\; x^T(t-h(t)) \;\; x^T(t-h_2) \;\; x^T(t-\frac{h_2}{2}) \;\; \dot{x}^T(t) \;\; f^T \;\; g^T\Big]$$

Case2 : when $h_1 \le h(t) \le h_a$, consider a L-K functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (18)$$

$$V_1(t) = x^T(t)P_1x(t) + \int_{t-h(t)}^{t} x^T(s)P_2x(s)ds,$$

$$V_2(t) = \int_{t-\frac{h_a}{2}}^{t} \xi_1^T(s)Q\xi_1(s)ds + \int_{t-\frac{h_1}{2}}^{t} \xi_2^T(s)S\xi_2(s)ds,$$

$$V_3(t) = \int_{-\frac{h_a}{2}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta + \int_{-h_a}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta,$$

Where P_1, P_2, Z_1, Z_2, Q, S are the same matrices used in the L-K functional (6).

Since V , N are slack matrices used in the stability analysis, in the similar manner ,we can obtain

$$\dot{V}(t) \le \zeta_2^T(t)(\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_a - h(t))NZ_2^{-1}N^T + (h(t) - h_1)VZ_2^{-1}V^T)\zeta_2(t),$$

Where

$$\zeta_2^T(t) = \Big[x^T(t) \;\; x^T(t-\frac{h_a}{2}) \;\; x^T(t-h_a) \;\; x^T(t-h(t)) \;\; x^T(t-h_1) \;\; x^T(t-\frac{h_1}{2}) \;\; \dot{x}^T(t) \;\; f^T \;\; g^T\Big],$$

One can see that if $\forall h(t) \in [h_a, h_2]$,

$$\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_2 - h(t))NZ_2^{-1}N^T + (h(t) - h_a)VZ_2^{-1}V^T < 0 \quad (19)$$

and $\forall h(t) \in [h_1, h_a]$

$$\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_a - h(t))NZ_2^{-1}N^T + (h(t) - h_1)VZ_2^{-1}V^T < 0 \quad (20)$$

Then , $\dot{V}(t) < -\varepsilon_i \|x(t)\|^2$ for some scalar $\varepsilon_i > 0, i = 1,2$, from which we conclude that system (1) is asymptotically stable according to L-K stability theory [1].

Applying Lemma 3 to (19) yields the follows:

$$\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_2 - h_a)NZ_2^{-1}N^T < 0 \quad (21)$$

$$\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_2 - h_a)VZ_2^{-1}V^T < 0 \quad (22)$$

Similarly, the convex LMI condition of (20) can be solved as:

$$\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_a - h_1)NZ_2^{-1}N^T < 0 \quad (23)$$

$$\Phi + \frac{h_a}{2}LZ_1^{-1}L^T + (h_a - h_1)VZ_2^{-1}V^T < 0 \quad (24)$$

Since $h_2 - h_a = h_a - h_1 = h_\delta$, the equation pairs (21),(22) are equivalent to (23), (24), Applying Schur complement on (21),(22), completes the proof.

Remark 1 Less conservatism of the proposed stability criteria is attributed to two aspects. On the one hand, based on the delay-central point method of stability analysis, the delay interval is partitioned into two subintervals of equal length, and time-derivative of a candidate L-K functional is evaluated in the respective segments. On the other hand, when deal with the time derivative of L-K functional, we using a more tightly integral inequality (Lemma 1) for bounding the cross terms, hence yields less conservative delay-range bounds.

Remark 2 When the information of the time derivative $h(t)$ is unknown by choosing $P_2 = 0$,we can get delay-dependent and rate-independent stability criterion from Theorem 1.

Remark 3 If there is no perturbation, that is $f = 0$, $g = 0$,then the stability problem of system (1) is reduced to analyzing the stability of the system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) \\ x(t) = \varphi(t), t \in [-h_2, 0] \end{cases} \quad (25)$$

This problem has been widely studied in the recent literature (see, e.g.,[5,8,9,12,13,14]) and the stability criterion for the deterministic system is stated below.

Theorem 2 For given values of h_1, h_2 and μ, System (24) is asymptotically stable, if there exist matrices $P_1>0$, $P_2>0$, $Z_1>0$, $Z_2>0$, $Q=\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}>0$, $S=\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}>0$, and $\tilde{L}_j$, $\tilde{N}_j$, $\tilde{V}_j$, $\tilde{T}_j$, $j=1,2$, with appropriate dimensions such that the following LMIs hold,

$$\begin{bmatrix} \tilde{\Phi} & \sqrt{\frac{h_a}{2}}\tilde{L} & \sqrt{h_\delta}\tilde{N} \\ * & -Z_1 & 0 \\ * & * & -Z_2 \end{bmatrix}<0 \tag{26}$$

$$\begin{bmatrix} \tilde{\Phi} & \sqrt{\frac{h_a}{2}}\tilde{L} & \sqrt{h_\delta}\tilde{V} \\ * & -Z_1 & 0 \\ * & * & -Z_2 \end{bmatrix}<0 \tag{27}$$

Where $\tilde{\Phi}=(\tilde{\Phi}_{i,j})_{7\times 7}$ with

$\tilde{\Phi}_{11}=P_1A+A^TP_1+P_2+Q_{11}+S_{11}+\tilde{L}_1+\tilde{L}_1^T+\tilde{T}_1A+A^T\tilde{T}_1^T$,

$\tilde{\Phi}_{12}=Q_{12}-\tilde{L}_1+\tilde{L}_2^T$, $\tilde{\Phi}_{13}=0$, $\tilde{\Phi}_{14}=P_1B+\tilde{T}_1B$,

$\tilde{\Phi}_{15}=0$, $\tilde{\Phi}_{16}=S_{12}$, $\tilde{\Phi}_{17}=-\tilde{T}_1+A^T\tilde{T}_2^T$,

$\tilde{\Phi}_{22}=Q_{22}-Q_{11}-\tilde{L}_2-\tilde{L}_2^T$, $\tilde{\Phi}_{23}=Q_{12}$,

$\tilde{\Phi}_{24}=\cdots=\tilde{\Phi}_{27}=0$, $\tilde{\Phi}_{33}=\tilde{V}_1+\tilde{V}_1^T-Q_{22}$,

$\tilde{\Phi}_{34}=-\tilde{V}_1+\tilde{V}_2^T$, $\tilde{\Phi}_{35}=\tilde{\Phi}_{36}=\tilde{\Phi}_{37}=0$,

$\tilde{\Phi}_{44}=-(1-\mu)P_2-\tilde{V}_2-\tilde{V}_2^T+\tilde{N}_1+\tilde{N}_1^T$, $\tilde{\Phi}_{45}=-\tilde{N}_1+\tilde{N}_2^T$,

$\tilde{\Phi}_{46}=0$, $\tilde{\Phi}_{47}=B^T\tilde{T}_2^T$, $\tilde{\Phi}_{55}=-S_{22}-\tilde{N}_2-\tilde{N}_2^T$,

$\tilde{\Phi}_{56}=-S_{12}^T$, $\tilde{\Phi}_{57}=0$, $\tilde{\Phi}_{66}=S_{22}-S_{11}$, $\tilde{\Phi}_{67}=0$,

$\tilde{\Phi}_{77}=\frac{h_a}{2}Z_1+h_\delta Z_2-\tilde{T}_2-\tilde{T}_2^T$,

$h_a=(h_1+h_2)/2$, $h_\delta=(h_2-h_1)/2$,

$\tilde{L}=\begin{bmatrix}\tilde{L}_1^T & \tilde{L}_2^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$, $\tilde{V}=\begin{bmatrix}0 & 0 & \tilde{V}_1^T & \tilde{V}_2^T & 0 & 0 & 0\end{bmatrix}^T$,

$\tilde{T}=\begin{bmatrix}\tilde{T}_1^T & 0 & 0 & 0 & 0 & 0 & \tilde{T}_2^T\end{bmatrix}^T$, $\tilde{N}=\begin{bmatrix}0 & 0 & 0 & \tilde{N}_1^T & \tilde{N}_2^T & 0 & 0\end{bmatrix}^T$.

4.Numerical examples

In this section, we use two numerical examples to show that the proposed results are improvements over some exiting ones.

Example 1 Consider system (1) satisfying (2),(3) with the following parameter:

$$A=\begin{bmatrix}-1.2 & 0.1 \\ -0.1 & -1\end{bmatrix}, B=\begin{bmatrix}-0.6 & 0.7 \\ -1 & -0.8\end{bmatrix}, F=G=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}.$$

For given values of α,β and μ, we apply Theorem 1 to calculate the maximal allowable value h_2 that guarantees the asymptotical stability of the system are listed in Table 1. From the table, it is easy to see that our proposed stability criterion gives a much less conservative results than those in [17, 18] since the proposed analysis uses delay-central point method as well as tighter bounding on the time-derivative of L-K functional.

Example 2 Consider system (25) with following matrices:

$$A=\begin{bmatrix}-2 & 0 \\ 0 & -0.9\end{bmatrix}, B=\begin{bmatrix}-1 & 0 \\ -1 & -1\end{bmatrix}.$$

The purpose is to calculate the allowable upper bound of h_2 that guarantee the asymptotic stability of the above system for given lower bound h_1. Table 2 lists the comparison results for $\mu=0.5$ and $\mu=0.9$, Table 3 lists the results for unknown μ. From the tables, it is clear that the proposed stability criterion is less conservative than those in [5,8,12,13,14]. Especially, when $h_1=5$, the result in [5,8,12] are not feasible while the MUBD obtained using our method is 5.1713.

Table 1 Admissible upper bounds h_2 for various μ and $h_1=0.5,1$

h_1	α,β / μ	$\alpha=0,\beta=0.1$			$\alpha=0.1,\beta=0.1$		
		0.5	0.9	1.1	0.5	0.9	1.1
0.5	[17]	1.442	1.338	1.338	1.284	1.245	1.245
	[18]	1.558	1.558	1.558	1.384	1.384	1.384
	Theorem 1	1.5636	1.5636	1.5636	1.3858	1.3858	1.3858
1	[17]	1.543	1.543	1.543	1.408	1.408	1.408
	[18]	1.760	1.760	1.760	1.532	1.532	1.532
	Theorem 1	1.7897	1.7897	1.7897	1.5647	1.5647	1.5647

Table 2 Admissible upper bounds h_2 for given h_1

μ	Method	$h_1=0$	$h_1=1$	$h_1=2$	$h_1=3$	$h_1=4$	$h_1=5$
0.5	[5]	2.0439	2.0764	2.4328	3.2234	4.0643	--
	[8]	2.0723	2.1276	2.5048	3.2591	4.0744	--
	[12]	2.0801	2.1513	2.7113	3.3839	4.1136	--
	[13]	2.1484	2.3239	2.8630	3.5729	4.3343	5.1306
	[14] (N=2)	2.2022	2.3912	2.9578	3.6384	4.3736	5.1463
	Theorem 2	2.1471	2.5652	3.1124	3.7448	4.4369	5.1713
0.9	[5]	1.3789	1.7424	2.4328	3.2234	4.0643	--
	[8]	1.5304	1.8737	2.5048	3.2591	4.0744	--
	[12]	1.6654	2.1251	2.7113	3.3839	4.1136	--
	[13]	1.7157	2.2302	2.8630	3.5729	4.3343	5.1306
	[14] (N=2)	1.8828	2.3585	2.9578	3.6384	4.3736	5.1463
	Theorem 2	2.1377	2.5627	3.1085	3.7408	4.4340	5.1703

Table 3 Admissible upper bounds h_2 for various h_1 and unknown μ

μ	Method	$h_1=0$	$h_1=1$	$h_1=2$	$h_1=3$	$h_1=4$	$h_1=5$
Any μ	[5]	1.3454	1.7424	2.4328	3.2234	4.0643	--
	[8]	1.5296	1.8737	2.5049	3.2591	4.0744	--
	[12]	1.6654	2.1251	2.7113	3.3839	4.1136	--
	[13]	1.7157	2.2302	2.8630	3.5729	4.3343	5.1306
	[14] (N=2)	1.8828	2.3585	2.9578	3.6384	4.3736	5.1463
	Theorem 2	2.1377	2.5627	3.1085	3.7408	4.4340	5.1703

5.Conclusions

This paper studies the problem of robust delay-dependent stability for a class of linear systems with interval time-varying delay and nonlinear perturbations, based on the delay-central point approach, appropriate free-weighting matrices and convex combination technique, less conservative robust stability criteria were proposed. The reduction in the conservatism of the proposed stability criteria is mainly attributed due to the use of new bounding condition for dealing with the cross-terms. Numerical examples have illustrated the effectiveness of the proposed method.

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