Residual Stress Calculation of Swage Autofrettage Gun Barrel

Liezhen Chang[1], Yutian Pan[2] and Xinmou Ma[2]

[1] College of Science, North University of China, Taiyuan, 030051, P.R. China.

[2] College of Mechatronic Engineering, North University of China, Taiyuan, 030051, P.R. China

Abstract

To calculate the residual stresses of swage autofrettage thick-walled cylinder, some simulations were run for different friction coefficients. It was found that the friction coefficient did not influence the residual stress and plastic radius. Thus, it is unnecessary to consider the effect of the friction coefficient on plastic radius and residual stress. Also utilized all equations of the spacial axisymmetric problem and boundary condition and continuity condition of swaging, the formula of plasic radius and the stress and strain of thick-walled cylinder and swage were deduced. Eventually by using the results of elastic analysis of unloading gun barrel, the formula of residual stress of gun barrel was derived. In oder to verify the accuracy, the theoretical value and the results of experiment were compared. The theoretical value shows good agreement with results of experiment.

Keywords: *swage autofrettage; gun barrel; excess; plastic radius.*

Nomenclature

a	Inner radius of the tube
b	Outer radius of the tube
r_m	Mandrel radius(to parallel portion)
δ	Mandrel-Tube interference
E	Young's modulus of the tube
E_1	Young's modulus of the mandrel
ν	Poisson's ratio of the tube
ν_1	Poisson's ratio of the mandrel
σ_y	Yield stress
ρ_1	Theory value of plastic radius
ρ_2	Experimental value of plastic radius
w	Axial displacement of the tube
u_1	Radial displacement of the mandrel
r	Radial distance
μ	Friction coefficient mandrel and tube
u	Radial displacement of the gun barrel
ε	Relative error
$\varepsilon^p_{r,\theta,z}$	Radial, hoop and axial total strain
$\varepsilon^e_{r,\theta,z}$	Radial, hoop and axial elastic strain of tube
$\varepsilon^p_{r,\theta,z}$	Radial, hoop and axial plastic strain of tube
$\sigma_{r,\theta,z}$	Radial, hoop and axial stress of tube
$\sigma^e_{r,\theta,z}$	Radial, hoop and axial stress of tube during unloading
$\sigma^R_{r,\theta,z}$	Residual radial , hoop and axial stress of tube
$\varepsilon_{r1,\theta1,z1}$	Radial , hoop and axial total strain of mandrel
$\sigma_{r1,\theta1,z1}$	Radial, hoop and axial stress of mandrel
$C_1, C_2, \cdots, C_7$	constants
$D_1, D_2, \cdots, D_4$	constants

1. Introduction

For an elastic gun barrel under internal pressure, the largest tensile tangential stress occurs at the inside diameter. To counteract this large tensile tangential stress at the bore, autofrettage techniques will produce compressive tangential residual stresses in the material near the bore have been commonly used. The tangential compressive residual stresses at the inside diameter due to the autofrettage process counteract this large tensile tangential stress and retard crack formation and growth at the bore, so autofrettage techniques can improve strength of gun barrel and extend the lifetime of gun barrel. Swage autofrettage process is used for internal forming and sizing of gun barrels designed to withstand high internal pressures. In this process, a mandrel with a diameter slightly larger than that of the tube is pressed inside the cylinder.

Residual stresses can have a significant influence on the fatigue lives of engineering components[1-9]. For the accurate assessment of fatigue lifetimes a detailed knowledge of the residual stress profiles generated is required together with methods for incorporating these into fatigue lifetime calculation procedures. Significant advances have been made in recent years for obtaining accurate and reliable determinations of residual stress distributions. These include both experimental [10] and numerical methods [11-16].

In this paper a brief outline of these is provided. To study the effection of friction coefficient， a series of swaging simulations were run between which was varied. On the basis of simulations, we utilize all equations of the spacial axisymmetric problem and boundary condition and continuity condition of swaging, and deduce the plastic radius and residual stresses of swage autofrettaged gun barrel.

2. Friction Coefficient Investigation

2.1 Numerical modeling

The parameter to be investigated was the coefficient of friction , between the mandrel and the tube, for which a value of 0.05~0.1 had been used until this stage[15]. According to the Coulomb model of friction used, the shear stress (τ_{rz}) at the surface when sliding varies. This constitutes a boundary value for the shear stress field within the tube during the swaging procedure, hence the shear stress field will vary as friction coefficient is changed, in turn influencing the other components, such as displacement , residual stresses, residual strains and plastic radius of swage autofrettage tube. To achieve these, a series of swaging simulations were run between which was varied whilst all other parameters remained static. The following values of μ .

This paper a numerical study on the swage autofrettage process. The process is modeled axi-symmetric using finite element analysis, and the effects of various friction coefficient are studied. The process is modeled as an iso- numerical study on the swage autofrettage process. The thermal process, because due to the relatively small. plastic deformation in this process, the heat rise due to plastic deformation is not expected to significantly change the material behavior.

The material of gun tube used in our model is 32CrNi3MoVE and mandrel is WC. The gun tube is assumed to be bilinear kinematic hardening material and the mandrel to be elastic，with properties given in table 1.

Table 1: Material Properties

Property	WC	32CrNi3MoVE
E /GPa	707	211
ν	0.285	0.28
σ_y /MPa	—	1190
H /GPa	—	2.06

The numerical study is carried out using the ANSYS10.0 finite element software. It is studied by a two dimensional axi-symmetric analysis.

The ANSYS model was dimensioned to the same ratios as that used by PAN [16]; specifically, this meant that the values of wall ratio, K, equalled 2.2. Inside diameter $2a$ = 101.85mm, external diameter $2b$ = 224.07 mm, a relatively short length of steel tube l_z =260mm.

Mandrels typically consist of two conical sections joined by a short length of constant diameter; the forward conical section has a shallower slope than the rear section. The conical sections not only aid alignment of the mandrel, but also help control the initial deformation (forward cone) and subsequent unloading (rear cone) of the tube. The length of parallel section of mandrel l , l =6.4mm, the mandrel's forward slopes θ_{MF} , equalled 1.5° and the rear slope θ_{MR} , equalled 3° . The lengths of the sloped sections are not stated by PAN, they are terminated when the radius reduces to r_a , and r_a less than the inner radius of gun tube. Mandrel-Tube Interference δ = 1.8mm (r_m - $a = \delta$).

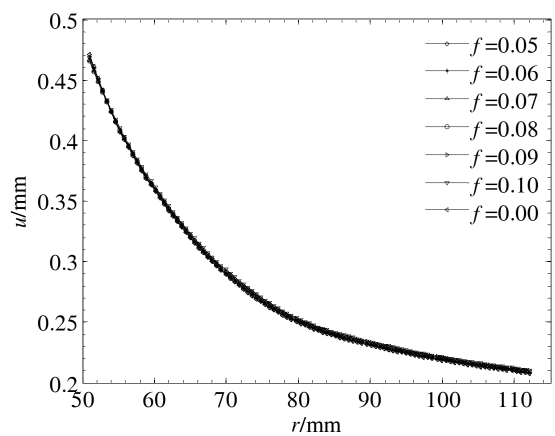

Fig. 1 Radial displacement

Fig. 1 shows the mandrel and gun tube dimensions used. For comparison, in the case of a lubricated sliding contact between two surfaces of hardened steel, $\mu \approx$ 0.05~0.10. a series of swaging simulations were run between which μ was varied, whilst all other parameters remained static. The following values of μ were used: 0, 0.05, 0.06, 0.07, 0.08, 0.09, and 0.10.

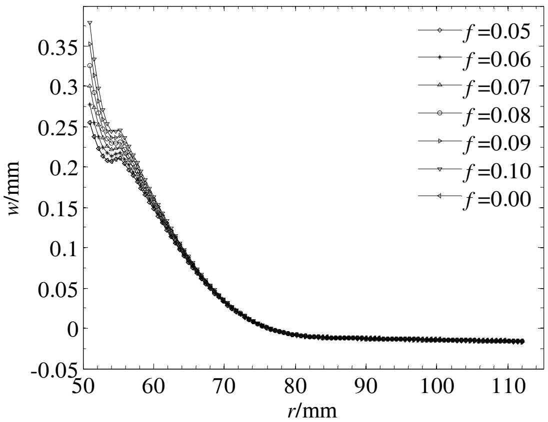

Fig. 2 Axial displacement

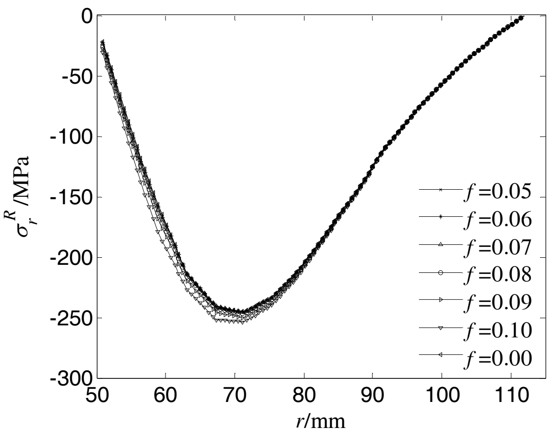

Fig. 3 Residual radial stress

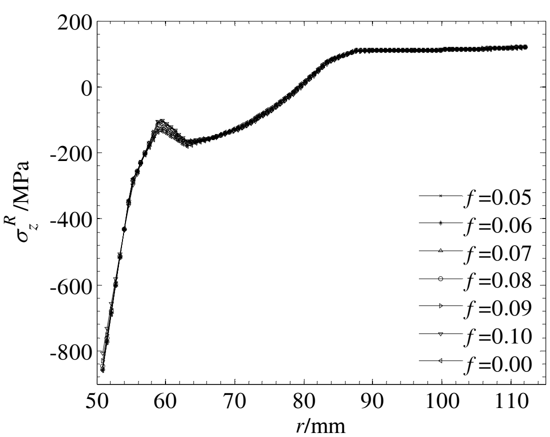

Fig. 4 Residual axial stress

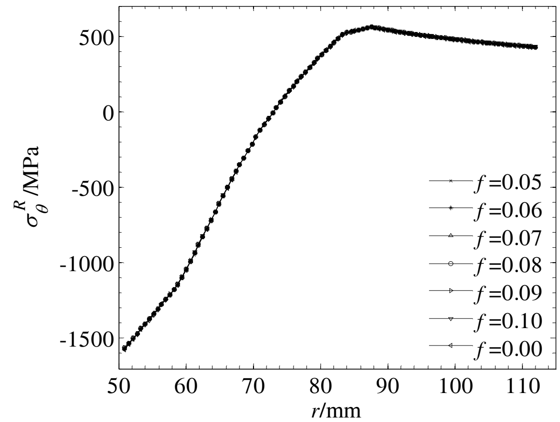

Fig. 5 Residual hoop stress

2.2 Numerical results and discussion

Radial displacement Fig. 1 remains stable while friction coefficient varies. Axial displacement Fig. 2 increases slightly when friction coefficient is increased, most clearly seen as the radial position of the onset of constant axial displacement in Fig. 2, in the region 80mm $\leqslant r \leqslant$ 112.035mm. This occurs because frictional force do not directly cause outward deflection of the tube and only significantly vary near the inner diameter; Fig. 2 shows such variation effectively limited to 50.925mm$\leqslant r \leqslant$ 80mm. Residual stresses (Fig.3~Fig.5}) show only small variation with change in the friction coefficient. Particularly, Residual radial stress and residual hoop stress remains relatively stable while friction coefficient varies. Residual axial stresses increases slightly as friction coefficient increases, but follow a similar trend.

Above all, the plastic radius remain stable while friction coefficient varies.

3. Stresses of thick-walled tube and mandrel during loading

3.1 Assumption

Assessment of the transient and residual stress-strain and displacement distributions in a swage-like procedure. Swage autofrettage is a spatial axsymetric elasto-plasticity moving contact problem. To simplify the problem, we assume:

(1) The length of the parallel section of the mandrel is very short, so it can be considered that the stresses and plastic radius of gun barrel within this section will not vary along axial direction.

(2) According to the Coulomb friction model, make the friction force proportional to the contact pressure, via a coefficient, μ, of constant value; this is defined by $\tau_{rz} \approx \mu\sigma_r |_{r=a}$, in the case of a lubricated sliding contact between two surfaces of hardened steel, $\mu \approx 0.05 \sim 0.10$. Comparing σ_r and τ_{rz}, $\tau_{rz} \ll \sigma_r$, so it was set as zero. Then σ_r and σ_z are approximate principle stresses.

(3) The axial strain of gun barrel is zero，that is to say the gun barrel is plane strain.

3.2 Plastic region of gun barrel

Compatibility:

$$\begin{cases} \varepsilon_r = \dfrac{\partial}{\partial r}[r\varepsilon_\theta], \\ \dfrac{\partial \gamma_{rz}}{\partial z} = \dfrac{\partial^2}{\partial z^2}[r\varepsilon_\theta] + \dfrac{\partial \varepsilon_z}{\partial r}. \end{cases} \tag{3.1}$$

Equilibrium equations:

$$\begin{cases} \dfrac{\partial \sigma_r}{\partial r} + \dfrac{\partial \tau_{rz}}{\partial z} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0, \\ \dfrac{\partial \tau_{rz}}{\partial r} + \dfrac{\partial \sigma_z}{\partial z} + \dfrac{\tau_{rz}}{r} = 0. \end{cases} \tag{3.2}$$

Strains:

$$\begin{cases} \varepsilon_r = \varepsilon_r^e + \varepsilon_r^p, \\ \varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p, \\ \varepsilon_z = \varepsilon_z^e + \varepsilon_z^p. \end{cases} \tag{3.3}$$

Base on Generalized Hook's law, elastic strains:

$$\begin{cases} \varepsilon_r^e = E^{-1}[\sigma_r - \nu(\sigma_\theta + \sigma_z)], \\ \varepsilon_\theta^e = E^{-1}[\sigma_\theta - \nu(\sigma_z + \sigma_r)], \\ \varepsilon_z^e = E^{-1}[\sigma_z - \nu(\sigma_r + \sigma_\theta)]. \end{cases} \tag{3.4}$$

According to Tresca's yield criterion, we get plastic potential function:

$$f = \sigma_\theta - \sigma_r - \sigma_y \tag{3.5}$$

From Drucker postulate $d\varepsilon_{ij}^p = \dfrac{\partial f}{\partial \sigma_{ij}} d\lambda$, we have

$$\begin{cases} d\varepsilon_r^p = \dfrac{\partial f}{\partial \sigma_r} d\lambda, \\ d\varepsilon_\theta^p = \dfrac{\partial f}{\partial \sigma_\theta} d\lambda, \\ d\varepsilon_z^p = \dfrac{\partial f}{\partial \sigma_z} d\lambda. \end{cases} \tag{3.6}$$

Substituting Eq.(3.5) into Eq.(3.6), then

$$\begin{cases} d\varepsilon_r^p = -d\lambda, \\ d\varepsilon_\theta^p = d\lambda, \\ d\varepsilon_z^p = 0. \end{cases} \tag{3.7}$$

According to Eq.(3.7), set

$$\begin{cases} \varepsilon_r^p = -\lambda(r), \\ \varepsilon_\theta^p = \lambda(r), \\ \varepsilon_z^p = 0. \end{cases} \tag{3.8}$$

Based on Eq.(3.3), we get the strains of gun barrel's plastic region

$$\begin{cases} \varepsilon_r = E^{-1}[\sigma_r - \nu(\sigma_\theta + \sigma_z)] - \lambda(r), \\ \varepsilon_\theta = E^{-1}[\sigma_\theta - \nu(\sigma_z + \sigma_r)] + \lambda(r), \\ \varepsilon_z = E^{-1}[\sigma_z - \nu(\sigma_r + \sigma_\theta)]. \end{cases} \tag{3.9}$$

Based on assumption 3，that is $\varepsilon_z = 0$, yield

$$\sigma_z = \nu(\sigma_r + \sigma_\theta). \tag{3.10}$$

Based on assumption 1 and 2, Eqs.(3.1) and (3.3) can be simplified:

$$\frac{d\varepsilon_\theta}{dr} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0, \tag{3.11}$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{3.12}$$

On the basis Tresca's yield criterion，the hoop stress and the radial stress of gun barrel(the axial stress is intermediate stress) satisfy:

$$\sigma_\theta - \sigma_r = \sigma_y. \tag{3.13}$$

According to Eqs.(3.10)~(3.13), yield

$$\lambda(r) = -C_1/r^2 - E^{-1}(1-\nu^2)\sigma_y. \tag{3.14}$$

Autofrettage stresses(in plastic region):

$$\begin{cases} \sigma_r = C_2 + \sigma_y \ln r, \\ \sigma_\theta = C_2 + \sigma_y \ln r + \sigma_y, \\ \sigma_z = \nu(2C_2 + 2\sigma_y \ln r + \sigma_y). \end{cases} \tag{3.15}$$

Since $\varepsilon_r^p |_{r=\rho} = \varepsilon_\theta^p |_{r=\rho} = 0$, that is $\lambda(\rho) = 0$, and from Eq.(3.14), we have

$$C_1 = E^{-1}(\nu^2 - 1)\rho^2\sigma_y. \tag{3.16}$$

Substituting Eq.\eqref{Eq16} into Eq.\eqref{Eq14}, then

$$\lambda_r = -E^{-1}(1-\nu^2)\rho^2\sigma_y(1-\rho^2/r^2). \tag{3.17}$$

3.3 Elastic region of gun barrel

Strains:

$$\begin{cases} \varepsilon_r = \dfrac{du}{dr}, \\ \varepsilon_\theta = \dfrac{u}{r}, \\ \varepsilon_z = 0. \end{cases} \tag{3.18}$$

According to Generalized Hook's law , stresses can be expressed as

$$\begin{cases} \sigma_r = \dfrac{E}{1+\nu}\varepsilon_r + \dfrac{E\nu}{(1+\nu)(1-2\nu)}\Theta, \\ \sigma_\theta = \dfrac{E}{1+\nu}\varepsilon_\theta + \dfrac{E\nu}{(1+\nu)(1-2\nu)}\Theta, \\ \sigma_z = \dfrac{E}{1+\nu}\varepsilon_z + \dfrac{E\nu}{(1+\nu)(1-2\nu)}\Theta. \end{cases} \tag{3.19}$$

where $\Theta = \varepsilon_r + \varepsilon_\theta + \varepsilon_z = \dfrac{du}{dr} + \dfrac{u}{r}$.

Substituting Eq.(3.18) into Eq.(3.19), then

$$\begin{cases} \sigma_r = \dfrac{E}{1+\nu}\dfrac{du}{dr} + \dfrac{E\nu}{(1+\nu)(1-2\nu)}(\dfrac{du}{dr}+\dfrac{u}{r}), \\ \sigma_\theta = \dfrac{E}{1+\nu}\dfrac{u}{r} + \dfrac{E\nu}{(1+\nu)(1-2\nu)}(\dfrac{du}{dr}+\dfrac{u}{r}), \\ \sigma_z = \dfrac{E\nu}{(1+\nu)(1-2\nu)}(\dfrac{du}{dr}+\dfrac{u}{r}). \end{cases} \tag{3.20}$$

Substituting Eq.(3.20) into Eq.(3.12), we have

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = 0. \tag{3.21}$$

Solve Eq.(3.21), we have

$$u = D_1/r + D_2 r. \tag{3.22}$$

Substituting Eq.(3.22) into Eq.(3.18), thus

$$\begin{cases} \varepsilon_r = -D_1/r^2 + D_2, \\ \varepsilon_\theta = D_1/r^2 + D_2, \\ \varepsilon_z = 0. \end{cases} \tag{3.23}$$

Substituting Eqs.(3.22) and (3.23) into Eq.(3.20), autofrettage stresses(in elastic region):

$$\begin{cases} \sigma_r = \dfrac{E}{1+\nu}(-\dfrac{D_1}{r^2}+D_2) + \dfrac{E\nu}{(1+\nu)(1-2\nu)}(2D_2), \\ \sigma_\theta = \dfrac{E}{1+\nu}(\dfrac{D_1}{r^2}+D_2) + \dfrac{E\nu}{(1+\nu)(1-2\nu)}(2D_2), \\ \sigma_z = \nu[\dfrac{2ED_2}{(1+\nu)} + \dfrac{4E\nu D_2}{(1+\nu)(1-2\nu)}], \end{cases} \quad (r > \rho). \tag{3.24}$$

For convenience, we set $C_3 = ED_2/(1+\nu) + E\nu D_2/[(1+\nu)(1-2\nu)]$, $C_4 = -ED_1/(1+\nu)$ and Eq. (3.24) can be written

$$\begin{cases} \sigma_r = C_3 + C_4/r^2, \\ \sigma_\theta = C_3 - C_4/r^2, \\ \sigma_z = 2\nu C_3, \end{cases} \quad (r > \rho). \tag{3.25}$$

3.4 Elastic region of mandrel

The mandrel is assumed to be an elastically behaving tube, of inner radius zero. As it is assumed to behave elastically, the stresses within the mandrel may be assessed using Lamé's case, Eq.(3.26).

The stress of mandrel is given by Lame equation:

$$\begin{cases} \sigma_{r1} = D_3 + D_4/r^2, \\ \sigma_{\theta 1} = D_3 - D_4/r^2. \end{cases} \tag{3.26}$$

However, as the mandrel is solid r decreases to zero at its centre; hence, the constant must be set to zero to avoid singularities. This means:

$$\begin{cases} D_4 = 0, \\ \sigma_{r1} = \sigma_{\theta 1} = D_3. \end{cases} \tag{3.27}$$

Generalized Hook's law:

$$\begin{cases} \varepsilon_{r1} = E_1^{-1}[\sigma_{r1} - \nu_1(\sigma_{\theta 1} + \sigma_{z1})], \\ \varepsilon_{\theta 1} = E_1^{-1}[\sigma_{\theta 1} - \nu_1(\sigma_{z1} + \sigma_{r1})], \\ \varepsilon_{z1} = E_1^{-1}[\sigma_{z1} - \nu_1(\sigma_{r1} + \sigma_{\theta 1})]. \end{cases} \tag{3.28}$$

Assume mandrel satisfy volume incompressibility:

$$\varepsilon_{r1} + \varepsilon_{\theta 1} + \varepsilon_{z1} = 0. \tag{3.29}$$

Substituting Eq. (3.28) into Eq. (3.29), thus

$$(1-2\nu_1)(\sigma_{r1} + \sigma_{\theta 1} + \sigma_{z1})E_1^{-1} = 0. \tag{3.30}$$

Since $\nu_1 \neq 0.5$, so

$$\sigma_{z1} = -(\sigma_{r1} + \sigma_\theta) = -2D_3. \tag{3.31}$$

as $\varepsilon_{\theta_1} = u_1 / r$, so

$$u_1 = r\varepsilon_{\theta 1}. \tag{3.32}$$

Substituting Eqs. (3.27) and (3.31) into Eq. (3.28), we have

$$\varepsilon_{\theta 1} = E_1^{-1}(1+\nu_1)D_3. \tag{3.33}$$

Substituting Eq. (3.33) into Eq. (3.32), then

$$u_1 = E_1^{-1}(1+\nu_1)D_3 r. \tag{3.34}$$

Set $D_3 = C_5$, yield

$$\sigma_{r1} = \sigma_{\theta 1} = D_3 = C_5, \tag{3.35}$$

$$u_1 = E_1^{-1}(1+\nu_1)D_3 r = E_1^{-1}(1+\nu_1)C_5 r. \tag{3.36}$$

3.5 Determine integration constants

Determine integration constants according to the boundary condition and continuity condition.

Boundary condition:

(1) The external surface of gun barrel is free，so the radial stress of the external surface is zero, that is $\sigma_r |_{r=b} = 0$;

(2) The radial stress at the inner surface of gun barrel equals the radial stress at the outer surface of the parallel section of the mandrel, that is $\sigma_r |_{r=a} = \sigma_{r1} |_{r=r_m}$;

(3) The radial displace of at the inner surface of gun barrel subtract the radial displace at the outer surface of the parallel section of the mandrel equals half magnitude of interference, that is $u|_{r=a} - u_1|_{r=r_m} = \delta$.

Continuity condition:

The stresses of elasto-plastic demarcation point are continuous.

From the boundary conditions and continuity conditions, we can obtain the following equations:

$$C_3 + C_4/b^2 = 0, \tag{3.37}$$

$$C_3 + C_4/a^2 = C_5, \tag{3.38}$$

$$\begin{aligned} &aE^{-1}[(1-\nu^2)\sigma_y + (1-\nu-2\nu^2)lna\sigma_y + \\ &(1-\nu-2\nu^2)C_2] - E^{-1}(1-\nu^2)\rho^2\sigma_y(a- \\ &\rho^2/a)n - E^{-1}(1+\nu_1)C_5 r_m = \delta, \end{aligned} \tag{3.39}$$

$$C_2 + \sigma_y \ln\rho = C_3 + C_4/\rho^2, \tag{3.40}$$

$$C_2 + \sigma_y \ln\rho + \sigma_y = C_3 - C_4/\rho^2. \tag{3.41}$$

Solve the above five Eqs. (3.37)- (3.41), we have

$$\begin{aligned} \delta = \sigma_y[&E^{-1}(1-\nu-2\nu^2)(4a\ln(a/\rho)) + \\ &E_1^{-1}2r_m(1+\nu_1)\rho^2(1/a^2 - 1/b^2) + \\ &4E^{-1}a^{-1}(1-\nu^2)\rho^2 + 2E^{-1}a\rho^2 b^{-2}(1 \\ &-\nu n - 2\nu^2)], \end{aligned} \tag{3.42}$$

$$C_5 = -0.5\rho^2\sigma_y(1/a^2 - 1/b^2), \tag{3.43}$$

$$C_4 = -0.5\rho^2\sigma_y, \tag{3.44}$$

$$C_3 = \rho^2\sigma_y/(2b^2), \tag{3.45}$$

$$C_2 = 0.5(-\sigma_y - 2\sigma_y ln\rho + \rho^2\sigma_y/b^2). \tag{3.46}$$

Solving plastic radius ρ from Eq.(3.42),substitute ρ into Eqs.(3.43)~(3.46) and then the four constants C_2 ~ C_5 can be determined. The stresses and strains of gun barrel and mandrel during loading can also be determined.

4 Residual stresses

From Eq.(3.25), obtain the elastic solution of gun barrel during unloading:

$$\begin{cases} \sigma_r^e = C_6 + C_7/r^2, \\ \sigma_\theta^e = C_6 - C_7/r^2, \quad (r > \rho). \\ \sigma_z^e = 2\nu C_6, \end{cases} \tag{4.1}$$

Residual stresses:

$$\begin{cases} \sigma_r^R = \sigma_r - \sigma_r^e, \\ \sigma_\theta^R = \sigma_\theta - \sigma_\theta^e, \quad (r > \rho). \\ \sigma_z^R = \sigma_z - \sigma_z^e, \end{cases} \tag{4.2}$$

According to $\sigma_r^R|_{r=a} = 0$ and $\sigma_r^e|_{r=b} = 0$, we have

$$\begin{cases} C_6 + C_7/a^2 = C_2 + \sigma_y \ln a, \\ C_6 + C_7/b^2 = 0. \end{cases} \tag{4.3}$$

Solve Eq. (4.3) and obtain

$$\begin{cases} C_6 = -a^2(b^2-a^2)(C_2 + \sigma_y \ln a), \\ C_7 = a^2 b^2 (b^2-a^2)(C_2 + \sigma_y \ln a). \end{cases} \tag{4.4}$$

Substituting Eq. (4.4) into Eq. (4.1), thus

$$\begin{cases} \sigma_r^e = \dfrac{-a^2}{b^2-a^2}(C_2 + \sigma_y lna)(1 - \dfrac{b^2}{r^2}), \\ \sigma_\theta^e = \dfrac{-a^2}{b^2-a^2}(C_2 + \sigma_y lna)(1 + \dfrac{b^2}{r^2}), \quad (r > \rho) \\ \sigma_z^e = \dfrac{-2\nu a^2}{b^2-a^2}(C_2 + \sigma_y lna), \end{cases} \tag{4.5}$$

From Eq. (3.15) and Eq.(4.2), we obtain the residual stresses of plastic region of gun barrel:

$$\begin{cases} \sigma_r^R = \sigma_y(-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} + ln\dfrac{r}{\rho}) + \dfrac{a^2\sigma_y}{b^2-a^2} \times \\ \qquad (-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} + ln\dfrac{a}{\rho})(1 - \dfrac{b^2}{r^2}), \\ \sigma_\theta^R = \sigma_y(\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} + ln\dfrac{r}{\rho}) + \dfrac{a^2\sigma_y}{b^2-a^2} \times \\ \qquad (-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} + ln\dfrac{a}{\rho})(1 + \dfrac{b^2}{r^2}), \qquad (r \le \rho). \\ \sigma_z^R = \nu\sigma_y(\dfrac{\rho^2}{b^2} + 2ln\dfrac{r}{\rho}) + \dfrac{2\nu a^2\sigma_y}{b^2-a^2} \times \\ \qquad (-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} + ln\dfrac{a}{\rho}), \end{cases} \tag{4.6}$$

From Eq.(3.25) and Eq.(4.2), we obtain the residual stresses of elastic region of gun barrel:

$$\begin{cases} \sigma_r^R = \dfrac{\sigma_y}{2}(\dfrac{\rho^2}{b^2} - \dfrac{\rho^2}{r^2}) + \dfrac{a^2\sigma_y}{b^2-a^2}(-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} \\ \qquad + \ln\dfrac{a}{\rho})(1 - \dfrac{b^2}{r^2}), \\ \sigma_\theta^R = \dfrac{\sigma_y}{2}(\dfrac{\rho^2}{b^2} + \dfrac{\rho^2}{r^2}) + \dfrac{a^2\sigma_y}{b^2-a^2}(-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} \qquad (r > \rho) \\ \qquad + \ln\dfrac{a}{\rho})(1 + \dfrac{b^2}{r^2}), \\ \sigma_z^R = \dfrac{\nu\sigma_y\rho^2}{b^2} + \dfrac{2\nu a^2\sigma_y}{b^2-a^2}(-\dfrac{1}{2} + \dfrac{\rho^2}{2b^2} + \ln\dfrac{a}{\rho}), \end{cases} \tag{4.7}$$

5 Comparison of theory and experimental value

The radius of the parallel section of the mandrel equal 12.65mm, that is r_m =12.65mm. Material and geometric parameters for the problems are list in Tables 2 and 3.

Table 2: Material Properties

Property	tube	Mandrel
Young's modulus/Gpa	201.88	588
Poisson's ratio	0.27	0.25

Table 3: Material Properties

No.	material	σ_y /MPa	a /mm	b /mm
6	PCrNiMo	1000	12.365	28.730
7	PCrNi3Mo	1009	12.380	27.245
8	PCrNi3Mo	1019	12.465	21.675
9	PCrNi3Mo	1019	12.405	27.240
10	PCrNi3Mo	1019	12.550	18.800
11	PCrNi3Mo	1019	12.410	27.240
12	PCrNiMo	1058	12.485	23.685
13	PCrNiMo	1058	12.425	27.310
14	PCrNiMo	1058	12.370	29.690
No.	δ /mm	ρ_1 /mm	ρ_2 /mm	ε /%
6	0.57	26.67	28.36	5.96
7	0.56	25.91	26.05	0.54
8	0.37	21.74	21.22	-2.45
9	0.49	24.71	24.70	-0.04
10	0.20	16.48	15.72	-4.83
11	0.48	24.49	25.31	3.24
12	0.33	20.36	20.47	0.54
13	0.45	23.40	24.04	2.66
14	0.56	25.79	26.10	1.19

From Eq.(3.42), the plastic radius of all tubes can be solved, and list in table3. The theoretical value and the results of experiment were compared. The theoretical value shows good agreement with results of experiment.

By using Eqs. (4.6) and (4.7), the residual radial stress and hoop stress of the No.4 tube were obtained. Compared the theoretical value and the results of experiment (Fig.6), we find that the theoretical value shows good agreement with results of experiment.

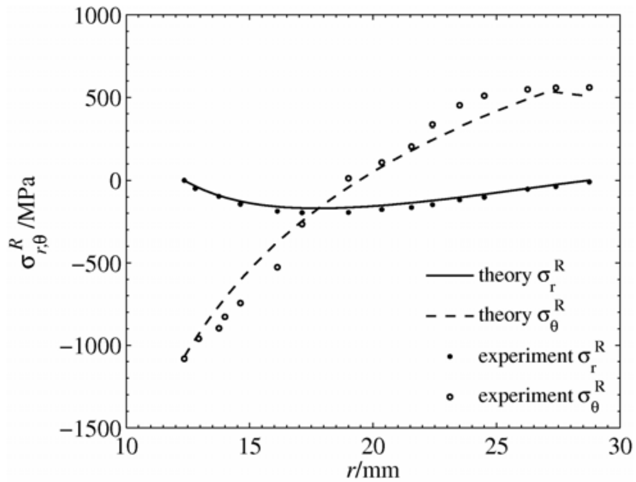

Fig. 6 Comparison of residual stress

6 Conclusions

The influence of the coefficient of friction on residual stresses and plastic radius was investigated by using a finite element method. The following values of μ were used: 0, 0.05, 0.06, 0.07, 0.08, 0.09, and 0.10. It was found that the friction coefficient in the range of 0~0.10 did not influence the residual stress and plastic radius. Thus, it is unnecessary to consider the effect of the friction coefficient on plastic radius and residual stress. Also utilized all equations of the spacial axisymmetric problem and boundary condition and continuity condition of swaging, the formulas of plasic radius and the residual stress of gun barrel was derived. Conclusions are summarized as follows:

(1) The friction coefficient in the range of 0~0.10 have hardly any influence on the residual stress and plastic radius.

(2) By using the formula of plastic radius, the plastic radius of nine different tubes were calculated. It is shown that the theoretical value of plastic radius coincide with the experiment results better. This formula can be used to calculate the plastic radius and over strain according to the excess, or determine the excess according to the overstrain.

(3) By using the formula of the residual stresses, the residual stresses of swage autofrettage tube are calculated. Compared the theoretical value and the results of experiment (Fig. 6) , we found that the theoretical value agreed well with results of experiment.

Acknowledgments

This work is supported by the Science Foundation of North University of China (2010, 2011).

References

[1] J.M. Alegre, P. Bravo, and M. Preciado, “Fatigue behaviour of an autofrettaged high-pressure vessel for the food industry”, Engineering Failure Analysis, Vol.14, 2007, pp. 396-407.

[2] Song-In Lee, and Seung-Kee Koh, “Residual stress effects on the fatigue life of an externally grooved thick-walled pressure vessel”, International Journal of Pressure Vessels and Piping, Vol. 79, 2002, pp. 119-126.

[3] Seung-Kee Koh, “Fatigue analysis of autofrettaged pressure vessels with radial holes”, International Journal of Fatigue, Vol. 22, 2000, pp. 717-726.

[4] W.Y. Chien, J. Pan, D. Close, and S. Ho, “Fatigue analysis of crankshaft sections under bending with consideration of residual stresses”, International Journal of Fatigue, Vol. 27, 2005, pp. 1-19.

[5] Hamid Jahed, Behrooz Farshi, and Mohammad Hosseini, “Fatigue life prediction of autofrettage tubes using actual material behaviour”, International Journal of Pressure Vessels and Piping, Vol. 83, 2006, pp. 749-755.

[6] S.K.Koh, “Fatigue life simulation and estimation of an autofrettaged thick-walled pressure vessel with an external groove”, International Journal of Fatigue,Vol.18, No.127, 1996, pp. 49-56.

[7] L. Zhu, X.Y. Tao, and Liu Cengdian, “Fatigue strength and crack propagation life of inservice high pressure tubular reactor under residual stress”, International Journal of Pressure Vessels and Piping, Vol. 75, 1998, pp. 871-877.

[8] Edward T. H., and U. P. Parker, “Finite Element Investigation of Bauschinger Effect in High-Strength A723 Pressure Vessel Steel”, Journal of Pressure Vessel Technology, Vol. 128, 2006, pp. 185-189.

[9] M. Perl, “The Change in Overstrain Level Resulting From Machining of an Autofrettaged Thick-Walled Cylinder”, Journal of Pressure Vessel Technology, Vol. 122, 2000, pp. 9-14.

[10] G.H. Majzoobi, G.H. Farrahi, M.K. Pipelzadeh, and A. Akbari, “Experimental and finite element prediction of bursting pressure in compound cylinders”, International Journal of Pressure Vessels and Piping, Vol, 81, 2004, pp. 889-896.

[11] R. Bihamta, M.R. Movahhedy, and A.R.Mashreghi, “A numerical study of swage autofrettage of thick-walled tubes”, Materials and Design, Vol. 28, 2007, pp. 804-815.

[12] J.M. Alegre, P. Bravo, M. Preciado, and M. Solaguren-Beascoa. “Simulation procedure of high pressure vessels using the wire winding technique”, Engineering Failure Analysis, Vol. 17, 2010, pp. 61-69.

[13] E. Herz, O. Hertel, and M. Vormwald, “Numerical simulation of plasticity induced fatigue crack opening and closure for autofrettaged intersecting holes”, Engineering Fracture Mechanics, Vol. 78, No. 3, 2011, pp. 559-572.

[14] Michael C. G., “Determination of residual stress distributions in autofrettaged thick-walled cylinders”, PhD thesis, Department of Engineering Systems and Management, Cranfield University, Cranfield, UK, 2008.

[15] RUI Jia-bai, “A new approach of mechanical autofrettage theory and photoplastiicity digital speckle correlation”, PhD thesis, Department of Engineering Mechanics, Tsinghua University, Beijing, P.R. China, 1994. (in Chinese)

[16] Pan Yutian. “The design of gun tube”, Beijing : The publishing house of weapon industry , 2007. (in Chinese),

Liezhen Chang Received the B.C. degree in Engineering Mechnics from Taiyuan Heavy Machinery Institute, China, in 2002. Received the M.S. degree in Engineering Mechanics from North University of China in 2007, She is currently working toward the Ph.D. degree at Gun, automatism weapon and ammunition engineering from North University of China. Now, I worked in North University of China as a lecture. My research interests include FEM, nonlinear dynamics and nonlinear PDE exact solution and approximately solution, Solid mechanics, Autofrettage technology.

Yutian Pan is a professor in North University of China, My research interests weapon design and nonlinear dynamics.

Xinmou Ma Received the B.C. degree in Mechnism design and manufacture from North China Institute of Technology, Taiyuan, China, in 2002. Received the M.S. degree and Ph. D. degree at Gun, automatism weapon and ammunition engineering from North University of China in 2005 and 2012, respectively. Now, I worked in North University of China as a lecture. My research interests include nonlinear dynamics, chaos, nonlinear PDE exact solution, approximately analytical solution and numerical solution.