

Study of Edge Detection Based On 2D Entropy

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Abstract

Edges of an image are considered a type of crucial information that can be extracted by applying detectors with different methodology. It is a main tool in pattern recognition, image segmentation, edge detection and scene analysis. In this paper, we present a new technique of edge detection based on two-dimensional Tsallis entropy. The two-dimensional Tsallis entropy was obtained from the two-dimensional histogram which was determined by using the gray value of the pixels and the local average gray value of the pixels, the work it was applied a generalized entropy formalism that represents a recent development in statistical mechanics. The effectiveness of the proposed method is demonstrated by using examples from the real-world and synthetic images. The performance evaluation of the proposed technique in terms of the quality of the edge images are presented.

Keywords: *Tsallis entropy, Edge detection, Image segmentation, 2D histogram.*

1. Introduction

Edge detection is an important field in image processing. Edges characterize object boundaries and are therefore useful for segmentation, registration, feature extraction, and identification of objects in a scene[3]. An effective edge detector reduces a large amount of data but still keeps most of the important feature of the image. Edge detection refers to the process of locating sharp discontinuities in an image. These discontinuities originate from different scene features such as discontinuities in depth, discontinuities in surface orientation, and changes in material properties and variations in scene illumination [2].

The principal assumption of the use of global thresholding as a segmentation technique is that "objects" and "backgrounds" can be distinguished by inspecting only image gray level values. Segmentation consist in subdividing an image into its constituent part and extracting those of interest. Many techniques for global thresholding have been developed over the years to segment images and recognize patterns (e.g. [1, 2, 3, 7, 8, 10, 12, 13, 14, 16]).

In general, thresholding methods can be classified into parametric and nonparametric methods. For parametric approaches, the gray-level distribution of each group is

assumed to obey a Gaussian distribution, and then the approaches attempt to find an estimate of the parameters of Gaussian distribution that best fits the histogram. Wang et al. [18] integrated the histogram with the Parzen window technique to estimate the spatial probability distribution. Fan et al. [4] approximated the histogram with a mixed Gaussian model, and estimated the parameters with an hybrid algorithm based on particle swarm optimization and expectation maximization. Zahara et al. [21] fitted the Gaussian curve by Nelder-Mead simplex search and particle swarm optimization. To resolve the histogram Gaussian fitting problem, Nakib et al. used an improved variant of simulated annealing adapted to continuous problems [9].

Nonparametric approaches find the thresholds that separate the gray-level regions of an image in an optimal manner based on some discriminating criteria, such as the entropy measures. [6, 19, 5]

Recent developments in statistical mechanics based on Tsallis entropy have intensified the interest of investigating it as an extension of Shannon's entropy [11]. It appears in order to generalize the Boltzmann-Gibbs' traditional entropy to non-extensive physical systems.

Recently M. A. El-Sayed et al [1] proposed an efficient technique based on 2d Tsallis entropy for image thresholding. Our algorithm has been inspired by the ideas from Refs.[1, 15, 20].

This paper is organized as follows: in Section 2 presents some fundamental concepts of the mathematical setting of entropy. In Section 3, we describe the relation between the threshold and 2d Tsallis entropy. Section 4, we introduce the newly proposed edge detection technique based on 2d Tsallis entropy. In Section 5, we report the effectiveness of the proposed method when applied to some real-world and synthetic images. In Section 6, we present some concluding remarks about our method.

2. Preliminaries

The entropy of a discrete source is often obtained from the probability distribution $p=\{p_i\}$. Therefore, $0\leq p_i\leq 1$ and

$\sum_{i=1}^k p_i = 1$, and the Shannon entropy is described as

$S = -\sum_{i=1}^k p_i \ln(p_i)$, being k the total number of states. If we

consider that a physical system can be decomposed in two statistical independent subsystems A and B , the Shannon entropy has the extensive property (additivity) $S(A+B) = S(A) + S(B)$. This formalism has been shown to be restricted to the Boltzmann-Gibbs-Shannon (BGS) statistics. However, for nonextensive physical systems, some kind of extension appears to become necessary. Tsallis [17] has proposed a generalization of the BGS statistics which is useful for describing the thermostistical properties of nonextensive systems. It is based on a generalized entropic form,

$S_q = \frac{1}{q-1} (1 - \sum_{i=1}^k (p_i)^q)$, where the real number q is a

entropic index that characterizes the degree of nonextensivity. This expression recovers to BGS entropy in the limit $q \rightarrow 1$. Tsallis entropy has a nonextensive property for statistical independent systems, defined by the following pseudo additivity entropic rule

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q).S_q(A).S_q(B).$$

Let $f(m, n)$ be the gray value of the pixel located at the point (m, n) . In a digital image $\{f(m, n) | m \in \{1, 2, \dots, M\}, n \in \{1, 2, \dots, N\}\}$ of size $M \times N$, let the histogram be $h(x)$ for $x \in \{0, 1, 2, \dots, 255\}$. For the sake of convenience, we denote the set of all gray levels $\{0, 1, 2, \dots, 255\}$ as G . Global threshold selection methods usually use the gray level histogram of the image. The optimal threshold is determined by optimizing a suitable criterion function obtained from the gray level distribution of the image and some other features of the image.

Let t be a threshold value and $B = \{b_0, b_1\}$ be a pair of binary gray levels with $\{b_0, b_1\} \in G$. Typically b_0 and b_1 are taken to be 0 and 255, respectively. The result of thresholding an image function $f(m, n)$ at gray level t is a binary function $f_t(m, n)$ such that $f_t(m, n) = b_0$ if $f_t(m, n) \leq t$ otherwise, $f_t(m, n) = b_1$. In general, a thresholding method determines the value t^* of t based on a certain criterion function. If t^* is determined solely from the gray level of each pixel, the thresholding method is point dependent [16].

In order to compute the two-dimensional histogram of a given image we proceed as follow. Calculate the average gray value of the neighborhood of each pixel. Let $g(x, y)$ be the average of the neighborhood of the pixel located at the point (x, y) . The average gray value for the 3×3 neighborhood of each pixel is calculated as

$$g(x, y) = \left\lfloor \frac{1}{9} \sum_{a=-1}^1 \sum_{b=-1}^1 f(x+a, y+b) \right\rfloor,$$

where $\lfloor r \rfloor$ denotes the integer part of the number r . While computing the average gray value, disregard the two rows from the top and bottom and two columns from the sides. The pixel's gray value, $f(x, y)$, and the average of its neighborhood, $g(x, y)$, are used to construct a two-dimensional histogram using : $h(k, m) = Prob(f(x, y) = k \text{ and } g(x, y) = m)$, where $\{k, m\} \in G$.

For a given image, there are several methods to estimate this density function. One of the most frequently used methods is the *method of relative frequency*. The normalized histogram $\hat{h}(k, m)$ is approximated by: number of elements in the event ($f(x, y) = k$ and $g(x, y) = m$), divided by number of elements in the sample space. Hence, $\hat{h}(k, m)$ can be calculate as number of pixels with gray value k and average gray value m , divided by number of pixels in the image .

The total number of frequencies (occurrences), $n_{(i, j)}$ of the pair (i, j) , divided by the total number of pixels, $N \times M$, defines a joint probability mass function, $p(i, j)$. Thus

$$p(i, j) = \frac{n_{(i, j)}}{M \times N} \quad \text{for } i, j = 0, 1, \dots, 255$$

3. The Threshold And 2D Entropy

The threshold is obtained through a vector (t, s) where t , for $f(x, y)$, represents the threshold of the gray level of the pixel and s , for $g(x, y)$, represents the threshold of the average gray level of the pixel's neighborhood. The frequency of occurrence of each pair of gray values is calculated. From this a surface can be drawn that will have two peaks and one valley. The object and background correspond to the peaks and can be separated by selecting the vector (t, s) that maximizes the sum of two class entropies. Using this vector (t, s) , the histogram is divided into four quadrants (see Fig. 1). We denote the first quadrant by $[t+1, 255] \times [0, s]$, the second quadrant by $[0, t] \times [0, s]$, the third quadrant by $[0, t] \times [s+1, 255]$, and the fourth quadrant by $[t+1, 255] \times [s+1, 255]$.

Since two of the quadrants, first and third, contain information about edges and noise alone, they are ignored in the calculation. Because the quadrants which contain the object and the background, second and fourth, are considered to be independent distributions, the probability values in each case must be normalized in order for each of the quadrants to have a total probability equal to 1.

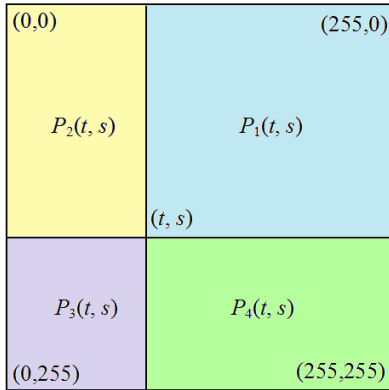


Fig. 1 Quadrants in the 2D histogram due to thresholding at (t, s) .

Our normalization is accomplished by using a posteriori class probabilities, $P_2(t, s)$ and $P_4(t, s)$. We assume that the contribution of the quadrants which contains the edges and noise is negligible, hence we further approximate $P_4(t, s)$ as $P_4(t, s) \approx 1 - P_2(t, s)$. The Tsallis entropy of order q of an image is defined as:

$$S_q = \frac{1}{q-1} [1 - \sum_{i=0}^{255} \sum_{j=0}^{255} p(i, j)^q]$$

where $q \neq 1$ is a positive real parameter. Since $\lim_{q \rightarrow 1} S_q = S$,

Tsallis entropy S_q is a one parameter generalization of the Shannon entropy S , where

$$S = - \sum_{i=0}^{255} \sum_{j=0}^{255} p(i, j) \ln p(i, j)$$

Tsallis entropies associated with object and background distributions are given by

$$S_q^A(t, s) = \frac{1}{q-1} [1 - \sum_{i=0}^t \sum_{j=0}^s (\frac{p(i, j)}{P_2(t, s)})^q]$$

and

$$S_q^B(t, s) = \frac{1}{q-1} [1 - \sum_{i=t+1}^{255} \sum_{j=s+1}^{255} (\frac{p(i, j)}{1 - P_2(t, s)})^q]$$

Here we have assumed that the off-diagonal probabilities are negligible and $S_q^B(t, s)$ is computed by using $1 - P_2(t, s)$ instead of $P_4(t, s)$. We try to maximize the information measure between the two classes (object and background). When $S_q(t, s)$ is maximized, the luminance pair (t, s) that maximizes the function is considered to be the optimum threshold pair (t^*, s^*) [7]

$$(t^*(q), s^*(q)) = \text{Arg max}_{(t, s) \in G \times G} S_q^{A+B}(t, s)$$

$$= \text{Arg max}_{(t, s) \in G \times G} [S_q^A(t, s) + S_q^B(t, s)]$$

$$+ (1 - q) \cdot S_q^A(t, s) \cdot S_q^B(t, s)].$$

For a priori chosen q , we will use only the optimal threshold $t^*(q)$ to threshold an image.

Theorem 1: The threshold value equals to the same value found by Shannon's method when $q \rightarrow 1$.

Proof: The limiting case of the proposed extension is Shannon's method. To see this, compute the limiting value of $S_q^A(t, s)$ and $S_q^B(t, s)$ as $q \rightarrow 1$. Hence,

$$\lim_{q \rightarrow 1} S_q^{A+B}(t, s) = \lim_{q \rightarrow 1} [S_q^A(t, s) + S_q^B(t, s) + (1 - q) \cdot S_q^A(t, s) \cdot S_q^B(t, s)].$$

$$= \lim_{q \rightarrow 1} [S_q^A(t, s) + S_q^B(t, s)].$$

$$= \lim_{q \rightarrow 1} [\frac{1}{q-1} (1 - \sum_{i=0}^t \sum_{j=0}^s (\frac{p(i, j)}{P_2(t, s)})^q) + \lim_{q \rightarrow 1} [\frac{1}{q-1} (1 - \sum_{i=t+1}^{255} \sum_{j=s+1}^{255} (\frac{p(i, j)}{1 - P_2(t, s)})^q)]]$$

$$= - \lim_{q \rightarrow 1} \frac{d}{dq} (\sum_{i=0}^t \sum_{j=0}^s (\frac{p(i, j)}{P_2(t, s)})^q) - \lim_{q \rightarrow 1} \frac{d}{dq} (\sum_{i=t+1}^{255} \sum_{j=s+1}^{255} (\frac{p(i, j)}{1 - P_2(t, s)})^q)$$

$$= - \lim_{q \rightarrow 1} \sum_{i=0}^t \sum_{j=0}^s \frac{d}{dq} (\frac{p(i, j)}{P_2(t, s)})^q - \lim_{q \rightarrow 1} \sum_{i=t+1}^{255} \sum_{j=s+1}^{255} \frac{d}{dq} (\frac{p(i, j)}{1 - P_2(t, s)})^q$$

but, $\frac{d}{dq} (a^q) = e^{q \ln a} \cdot \ln a$,

i.e. $\lim_{q \rightarrow 1} \frac{d}{dq} (a^q) = e^{\ln a} \cdot \ln a = a \cdot \ln a$

hence ,

$$\lim_{q \rightarrow 1} S_q^{A+B}(t, s) = - \sum_{i=0}^t \sum_{j=0}^s (\frac{p(i, j)}{P_2(t, s)}) \ln (\frac{p(i, j)}{P_2(t, s)}) - \sum_{i=t+1}^{255} \sum_{j=s+1}^{255} (\frac{p(i, j)}{1 - P_2(t, s)}) \ln (\frac{p(i, j)}{1 - P_2(t, s)})$$

$$= S^A(t, s) + S^B(t, s)$$

Therefore, when $q \rightarrow 1$, the threshold value equals to the same value found by Shannon's method. Thus this proposed method includes Shannon's method as a special case. The following expression can be used as a criterion function to obtain the optimal threshold at $q \rightarrow 1$.

$$(t^*(1), s^*(1)) = \text{Arg max}_{(t, s) \in G \times G} [S^A(t, s) + S^B(t, s)].$$

In order to reduce the execution time, we take $t=s$, i.e. the value of t is lies on the diagonal of quadrants in the 2D histogram and the calculation on only two square

matrices, P_2 with $t \times t$ and P_4 with $(255-t) \times (255-t)$. See Fig. 2.

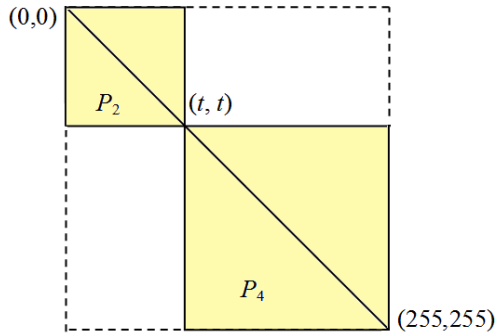


Fig. 2 Quadrants in the 2D histogram due to thresholding at (t, t) .

The complete *Tsallis2DThr* algorithm can now be described as follows:

Algorithm Tsallis2DThr;

Input: A digital grayscale image A of size $M \times N$.

Output: The optimal threshold $t^*(q)$ of A .

Begin

1. Let $f(x, y)$ be the original gray value of the pixel at the point (x, y) , $x=1..M$, $y=1..N$.
2. Calculate the average gray level value $g(x, y)$ in a 3×3 neighborhood around the pixel (x, y) , according to $g(x, y) = \left[\frac{1}{9} \sum_{a=-1}^1 \sum_{b=-1}^1 f(x+a, y+b) \right]$.
3. Calculate the joint probability mass function,

$$p(i, j) = \frac{n_{(i,j)}}{M \times N}, \text{ for } i, j = 0, 1, \dots, 255.$$

4. If $q \neq 1$ Then

$$(t^*(q), t^*(q)) = \text{Arg max}_{(t,s) \in G \times G} [S_q^A(t,t) + S_q^B(t,t) + (1-q) \cdot S_q^A(t,t) \cdot S_q^B(t,t)].$$

$$\text{Else } (t^*(1), t^*(1)) = \text{Arg max}_{(t,s) \in G \times G} [S_q^A(t,t) + S_q^B(t,t)]$$

EndIf.

5. For $x=1..M$, $y=1..N$:

If $f_i(x, y) \leq t^*$ then $f_i(x, y) = 0$ Else $f_i(x, y) = 255$

EndIf.

End.

4. Edge Detection Technique And 2D Tsallis Entropy

We will use the usual masks for detecting the edges. A spatial filter mask may be defined as a matrix w of size $m \times n$. Assume that $m=2\mu+1$ and $n=2\rho+1$, where μ, ρ are nonzero positive integers. For this purpose, smallest meaningful size

of the mask is 3×3 . Such mask coefficients, showing coordinate arrangement as Fig. 3.a. Image region under the above mask is shown as Fig. 3.b.

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

(a)

$f(x-1, y-1)$	$f(x-1, y)$	$f(x-1, y+1)$
$f(x, y-1)$	$f(x, y)$	$f(x, y+1)$
$f(x+1, y-1)$	$f(x+1, y)$	$f(x+1, y+1)$

(b)

Fig. 3 Coordinate arrangement and image region under 3×3 mask.

In order to edge detection, firstly classification of all pixels that satisfy the criterion of homogeneousness, and detection of all pixels on the borders between different homogeneous areas. In the proposed scheme, first create a binary image by choosing a suitable threshold value using Tsallis entropy. Window is applied on the binary image. Set all window coefficients equal to 1 except centre, centre equal to \times as shown in Fig. 4.

1	1	1
1	\times	1
1	1	1

Fig. 4 Window coefficients of 3×3 mask.

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Then, the entropy of each central pixel of image under the window is calculated as $S(CPix) = -p_c \ln(p_c)$.

Where, p_c is the probability of central pixel $CPix$ of binary image under the window. When the probability of central pixel, $p_c = 1$, then the entropy of this pixel is zero. Thus, if the gray level of all pixels under the window homogeneous,

$p_c=1$ and $S=0$. In this case, the central pixel is not an edge pixel. Other possibilities of entropy of central pixel under window are shown in Table 1.

Table 1 p and S of central under window

p	1/9	2/9	3/9	4/9
S	0.2441	0.3342	0.3662	0.3604
p	5/9	6/9	7/9	8/9
S	0.3265	0.2703	0.1955	0.1047

In cases $p_c=8/9$, and $p_c=7/9$, the diversity for gray level of pixels under the window is low. So, in these cases, central pixel is not an edge pixel. In remaining cases, $p_c \leq 6/9$, the diversity for gray level of pixels under the window is high. The complete *2DTSallisEdgeDetection* algorithm can now be described as follows:

Algorithm 2DTSallisEdgeDetection;

Input: A grayscale image $A (M \times N)$.

Output: The edge detection image g .

Begin

1. Select suitable t^* , q , using *2DTSallisThr*.
2. Create a binary image:
 If $f(x, y) \leq t^*(q)$ then $f(x,y)=0$ Else $f(x, y)=1$.
3. Create a mask w , with dimensions $m \times n$:
 Normally,
 $m=n=3$. $\mu=(m-1)/2$ and $\rho=(n-1)/2$.
4. For all $1 \leq x \leq M$ and $1 \leq y \leq N$:
 Find g an output image by set $g=f$.
5. For all $\rho+1 \leq y \leq N-\rho$ and $\mu+1 \leq x \leq M-\mu$,
 checking for edge pixels:
 - i. $sum = 0$;
 - ii. For all $-\rho \leq k \leq \rho$ and $-\mu \leq j \leq \mu$:
 If $(f(x, y) = f(x+j, y+k))$ Then
 $sum = sum+1$.
 - iii. If $(sum > 6)$ Then $g(x,y)=0$ Else $g(x,y)=1$.

End algorithm.

5. Experimental Results:

In this section, we discuss the experimental results obtained using the proposed method. This discussion includes the choice of the optimal threshold and the presentation of the optimal threshold values of some real-world and synthetic images. These images are shown in Figs. 5–18. Our analysis is based on how much information is lost due to thresholding. In this analysis, given edge images of a same original image, we prefer the one which lost the least amount of information. The

optimal threshold value was computed by the proposed method for these images. Table 1 lists the optimal threshold values that are found for these images for q values equal to 1.0, 0.3, 0.5, 0.7, 0.9, 1.0 and 2.0, respectively.

The original images together with their histograms and the edge images obtained by using the optimal threshold of some values t^* are displayed side by side in Figs. 5–18. Using the above twenty images and also some other images, we conclude that when q value lies between 0 and 1, our proposed method produced good optimal threshold values. Moreover, the optimal threshold value does not change very much when the fractional q value changes a little. However, when q was greater than 1, this proposed method did not produce good threshold values. In fact the threshold values produced were unacceptable (see the last column of Table 2).

When the value of q was one, the threshold value produced was not always a good threshold value (see Figs. 5–18). This new method performs better with fractional values of q . In this method of edge detection, we have used in addition to the original gray level function $f(x, y)$, a function $g(x, y)$ that is the average gray level value in a 3×3 neighborhood around the pixel (x, y) .

Table 2 t^* values for various values of q

Image	$t^*(...)$						
	0.1	0.3	0.5	0.7	0.9	1	2
bacteria	102	102	102	102	102	154	6
blood1	141	141	141	141	141	160	141
bonemarr	134	149	149	149	157	216	157
cameraman	102	84	84	84	84	119	84
eight	154	154	154	154	154	228	154
flowers	120	120	120	120	120	70	86
ic	132	150	150	150	150	23	252
moon	132	132	85	85	55	197	55
mri	131	107	96	77	77	101	55
retina	123	119	119	219	219	113	219
rice	131	115	115	115	115	96	115
Saturn	123	119	119	96	45	165	41
shot1	121	165	165	165	165	161	165
Tire	123	116	114	114	114	162	190

This approach can be extended to an image pyramid, where an image on the next higher level is composed of average gray level values computed for disjoint 3×3 squares. From the point of view of computational time and image quality, a neighborhood size of 3×3 with q value around 0.1 would be ideal for thresholding with this proposed method.



Fig. 5. eight image, and its the edge images with proposed method $t^*(0.1)=154$.

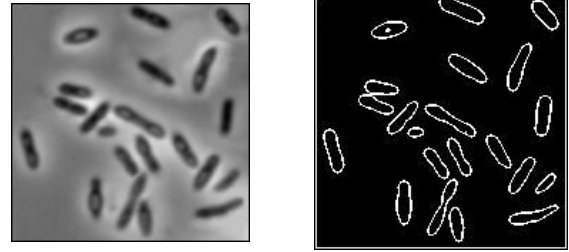


Fig. 10. bacteria image, and its the edge images with proposed method $t^*(0.1)=102$.

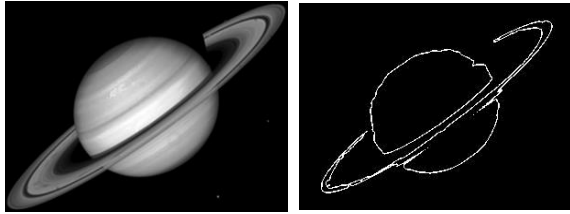


Fig. 6. Saturn image, and its the edge images with proposed method $t^*(0.1)=45$.

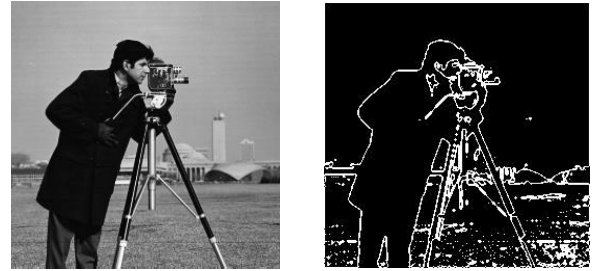


Fig. 11. cameraman image, and its the edge images with proposed method $t^*(0.1)=113$.

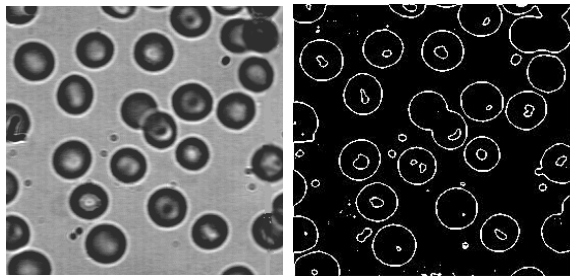


Fig. 7. blood1 image, and its the edge images with proposed method $t^*(0.1)=141$.

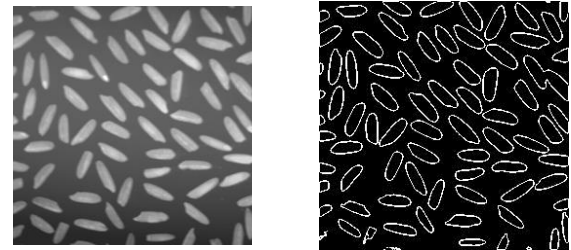


Fig. 12. rice image, and its the edge images with proposed method $t^*(0.1)=115$.

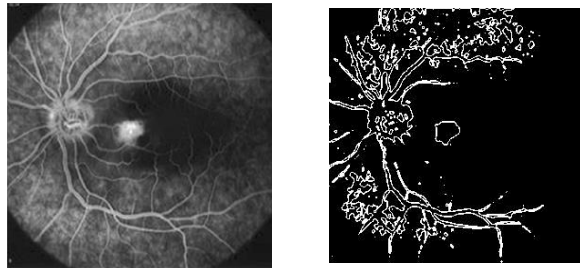


Fig. 8. retina image, and its the edge images with proposed method $t^*(0.1)=123$.

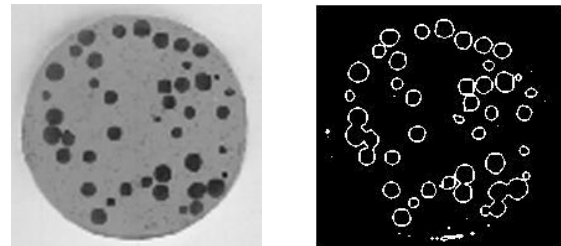


Fig. 13. shot1 image, and its the edge images with proposed method $t^*(0.1)=121$.

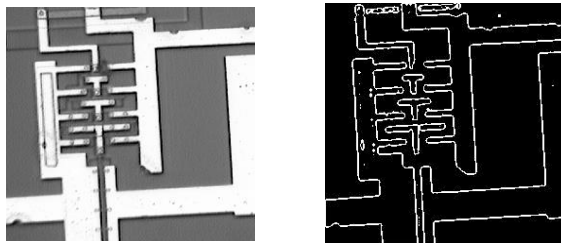


Fig. 9. ic image, and its the edge images with proposed method $t^*(0.1)=132$.



Fig. 14. mri image, and its the edge images with proposed method $t^*(0.1)=131$.

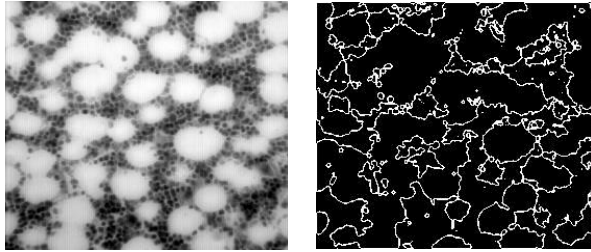


Fig. 15. bonemarr image, and its the edge images with proposed method $t^*(0.1)=134$.



Fig. 16. flowers image, and its the edge images with proposed method $t^*(0.1)=120$.

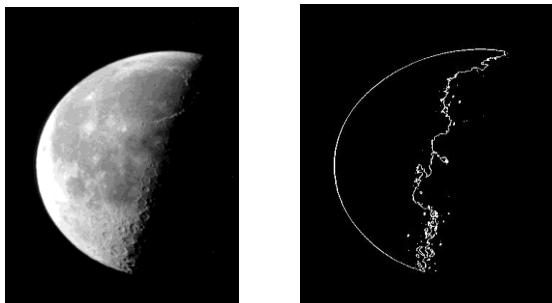


Fig. 17. moon image, and its the edge images with proposed method $t^*(0.1)=132$.

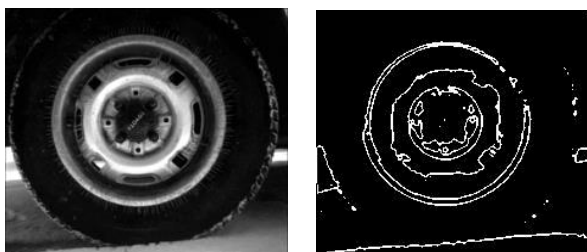


Fig. 18. tire image, and its the edge images with proposed method $t^*(0.1)=114$.

5. Conclusion

In this paper, we introduced the newly proposed technique based on 2D Tsallis entropy of order q . It is a powerful technique for image edge detection. In almost every image

used, the proposed method yielded a good threshold value for fractional q coefficient, that is, when $0 < q < 1$. For $q > 1$, the proposed method did not yield a good threshold value. When $q = 1$, the method yielded, in some cases, good optimal threshold values and, in some other cases, unacceptable threshold values. The Tsallis q coefficient can be used as an adjustable value and can play an important role as a tuning parameter in the image processing chain for the same class of images. This can be an advantage when the image processing tasks depend on an automatic thresholding. It is already pointed out in the introduction that the two-dimensional extension gives rise to the exponential increment of computational time. However, the proposed method is decrease the computation time and this method is easily implemented. The software used to generate the results in this paper was written in the commercial software MATLAB 8 on a computer with 2.1GHz Intel Core 2 Duo CPU laptop with 2 GB of RAM, and thus we have taken the advantage of the vector computation that MATLAB offers. Because of this our method takes few seconds.

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