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# Computationally Efficient Control System Based on Digital Dynamic Pulse Frequency Modulation for Microprocessor Implementation

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**Abstract**

This paper is focused on digital dynamic pulse-frequency modulation (DPFM) control systems that can be implemented on a microcontroller. We describe a structure of a discrete nonlinear closed-loop system that is equivalent to DPFM. A general-case model of a digital modulator of l-th order is obtained and an example of implementation and computational performance is demonstrated.

***Keywords:*** *Pulse-frequency modulation, controlled plant, discrete filter, pulse generator, microcontroller*

## 1. Introduction

Digital control systems based on dynamic pulse-frequency modulation are widely accepted. The main reason for their popularity is a set of features, such as the low susceptibility to noise and simplicity in microprocessor-based implementation.

Theory of deterministic control systems based on dynamic pulse-frequency modulation is well-established and their practical use is becoming more common in the industrial environment. However, many technological processes cannot be controlled in a trivial way, as they are characterized by the presence of transport delays. In addition, it is expected that operation of technological lines remains stable even in the presence of internal and external random variations of both signals and parameters of the system. Hence, these factors must be taken into consideration on the stages of design, implementation, and integration of controls [1] – [3].

## 2. System Overview

DPFM devices are widely used in control systems [4] – [7]. They are based on the principle of dynamic pulse-frequency modulation [8] – [16] and their structure can be illustrated as shown in Fig. 1.

A discrete filter DF performs certain transformation of a pulse signal x[nT] into signal y[nT]. At a time instance nT= nvT,

Digital control system presented below includes a DPFM, a pulse-forming generator (PFG), and a controlled plant.

Functions of both the DPFM and PFG are implemented on a microcontroller that generates a sequence of pulses, each with the width h [16].

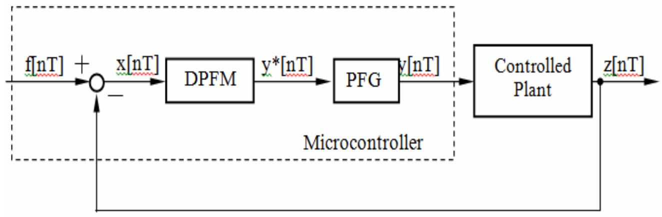


Fig. 1. Dynamic pulse-frequency modulation control system

The DPFM proposed in this paper is digital, hence its implementation requires special treatment of its description. First, we can formulate mathematical definition of a closed-loop digital control system that is equivalent to the DPFM. By using analogy with an analog version of a pulse-frequency modulator [9] we can represent a digital DPFM as a combination of a discrete filter (DF) and a pulse generator (PG), as shown in Fig. 2.

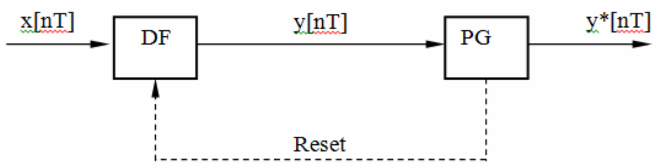


Fig. 2. Equivalent structure of a digital DPFM

when the value of y[nT] is greater or equal than some threshold Δ, a pulse generator PG produces a single pulse that can be described by a delta $\tilde{\delta}[(n-n_v)T]$ with a proper sign, which resets all memory registers of the discrete filter DF as follows

$$\tilde{\delta}[(n-n_v)T]=\begin{cases}1, & \text{if} \quad n=n_v,\\ 0, & \text{if} \quad n\neq n_v.\end{cases} \tag{1}$$

By using the concepts from [9], [14] – [16], a model of a digital DPFM can be formed as a closed-loop system shown in Fig. 3, which has a block of reset (BoR) that includes a nonlinear function A[x,y*,$\gamma_m$] and a relay element (RE).

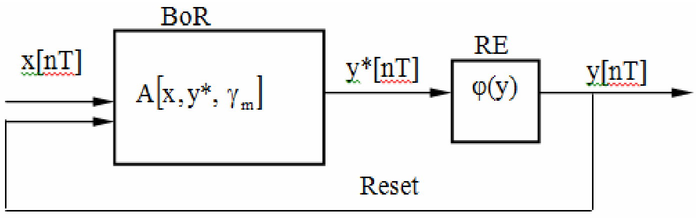


Fig. 3. Configuration of a nonlinear equivalent system

Dynamic equations that describe the above control system and all its components, including the DPFM and controlled plant can be summarized as follows [6], [7]:

$$x[nT]=f[nT]-z[nT], \tag{2}$$

where z[nT] is the output signal of a controlled plant that can be described by a certain operator

$$z[nT]=H[\lambda,\gamma_0,y^*[nT]|0\leq\gamma\leq T] \tag{3}$$

Here $\gamma_0$ is a parameter that denotes transport delay of the plant. In addition,

$$y^*[nT]=\sum_n \lambda_{v+1}\,\delta[(n-n_{v+1})T] \tag{4}$$

where

$$\lambda_{v+1}=\text{sign } y[n_{v+1}T-0] \tag{5}$$

Also we have

$$y[n_{v+1}T-0]=\lambda_{v+1}\cdot\Delta \tag{6}$$

$$y[n_vT+0]=0 \tag{7}$$

and

$$y[nT]=\Phi[y(rT),x(rT),\gamma_mT|\,n_v\leq r\leq n_{v+1}] \tag{8}$$

where $x[nT]=x(t)|_{t=nT}$ is the error signal; y*[nT] is the output of DPFM, y[nT] – is the output of discrete filter DF; $\gamma_m$ is a parameter of the modulator that is used to account for transport delay of the controlled plant such that $\gamma_m\geq\gamma_0$.

## 3. Detailed Analysis of the Control System

One of the main tasks is finding the form of operator A[x,y*,$\gamma_m$] such that the system shown in Fig. 3 generates the same sequence of pulses as a digital DPFM. Let a linear digital filter DF be described by an l-th order difference equation.

$$\begin{aligned}a_0y[(n+1)T]&=a_1y[nT]+a_2y[(n-1)T]+\ldots\\&+a_\ell y[(n-\ell+1)T]+b_1x[nT]+\\&+b_2x[(n-1)T]+..\\&\ldots+b_\ell x[(n-\ell+1)T]\end{aligned} \tag{9}$$

By applying z-transform to (9) we obtain:

$$\begin{aligned}&Y(z)\left(a_0z-a_1-a_2z^{-1}-\ldots-a_\ell z^{-\ell+1}\right)=\\&=X(z)\left(b_1+b_2z^{-1}+\ldots+b_\ell z^{-\ell+1}\right)+a_0\,z\,y[0]+\\&+\left(a_2+a_3z^{-1}+\ldots+a_\ell z^{-\ell+2}\right)y[-T]+\\&+\left(a_3+a_4z^{-1}+\ldots+a_\ell z^{-\ell+1}\right)y[-2T]+\ldots\\&\ldots+a_\ell y[(-\ell+1)T]+(b_2+b_3z^{-1}+\ldots\\&\ldots+b_\ell z^{-\ell+2}x[-T]+\ldots+b_\ell x[(-\ell+1)T])\end{aligned} \tag{10}$$

Let us introduce a new notation

$$G(z)=a_0z-a_1-a_2z^{-1}-\ldots-a_\ell z^{-\ell+1} \tag{11}$$

$$R(z)=b_1+b_2z^{-1}+\ldots+b_\ell z^{-\ell+1} \tag{12}$$

$$Q_p(z)=\sum_{i=p}^{\ell}a_iz^{p-i};\qquad(p=1,2,\ldots,\ell). \tag{13}$$

$$F_p(z)=\sum_{i=p}^{\ell}b_iz^{p-i};\qquad(p=1,2,\ldots,\ell). \tag{14}$$

Then, by taking into consideration expressions (11) – (14), equation (10) can be re-written as follows

$$\mathrm{Y}(z)=\frac{R(z)}{G(z)}\left(X(z)+\frac{1}{R(z)}\sum_{p=1}^{\ell}Q_p(z)y[(-p+1)T]+\right.$$
$$\left.+\frac{1}{R(z)}\sum_{p=2}^{\ell}F_p(z)x[(-p+1)T]\right) \qquad (15)$$

Let the initial time be $n_\nu T$. Let us assume that $y(n_\nu T-0)\neq 0$, then a solution to equations (9) and (10) becomes

$$\tilde{y}[nT]=\sum_{m=0}^{\infty}\omega[(n-m)T]\big(x[mT]+\big(x[mT]+$$
$$+\sum_{p=1}^{\ell}\sum_{k=n_\nu}^{m}g_p[(m-k)T]\tilde{\delta}[(k-n_\nu)T]\,y[(n_\nu-p+1)T]+$$
$$+\sum_{p=2}^{\ell}\sum_{k=n_\nu}^{m}f_p[(m-k)T]\tilde{\delta}[(k-n_\nu)T]x[(n_\nu-p+1)T]\Big),$$
$$n_\nu<n<n_{\nu+1} \qquad (16)$$

where

$$\omega[mT]=Z^{-1}\left\{\frac{R(z)}{G(z)}\right\};$$
$$g_p[mT]=Z^{-1}\left\{\frac{Q_p(z)}{R(z)}\right\};$$
$$f_p[mT]=Z^{-1}\left\{\frac{F_p(z)}{R(z)}\right\};$$

When controlling plants with a transport delay [17], equation (16) can be re-written as

$$\tilde{y}[nT]=\sum_{m=0}^{\infty}\omega[(n-m)T]\big(x[mT]+$$
$$+\sum_{p=1}^{\ell}\sum_{k=n_\nu}^{m}g_p[(m-k)T]\tilde{\delta}\,[(k-n_\nu)T]y[(n_\nu-p+1)T]\Big)-$$
$$-\eta[kT]-\mu[kT]+$$
$$+\sum_{p=2}^{\ell}\sum_{k=n_\nu}^{m}f_p[(m-k)T]\times\tilde{\delta}[(k-n_\nu)T]x[(n_\nu-p+1)T] \qquad (17)$$

By comparing equations (16) and (17), we conclude that $y[n_{\nu+1}T]=\tilde{y}[n_{\nu+1}T]$ if components of the signal are in the following form.

$$\eta[mT]=\sum_{p=1}^{\ell}\sum_{k=n_\nu}^{m}g_p[(m-k)T]*\ldots$$
$$*\,\tilde{\delta}[(k-n_\nu)T]y[(n_\nu-p+1)T]+$$
$$+\sum_{p=2}^{\ell}\sum_{k=n_\nu}^{m}f_p[(m-k)T]*\ldots$$
$$*\,\tilde{\delta}[(k-n_\nu)T]x[(n_\nu-p+1)T] \qquad (18)$$

And

$$\mu[mT]=\begin{cases}x[mT], & \text{если}\quad n_\nu<m<(n_\nu+\gamma_m)\\ 0, & \text{если}\quad (n_\nu+\gamma_m)<m<n_{\nu+1}\end{cases} \qquad (19)$$

To obtain the reset signal $\eta[mT]$ let us consider transformation of the signal $y[nT]$ performed by a relay element RE, which can be given by the following nonlinearity $\varphi(y)$.

$$u[nT]=\varphi(y[nT])=\begin{cases}+1, & if\quad y[nT]\geq\Delta,\\ 0, & if\quad -\Delta<y[nT]<\Delta,\\ -1, & if\quad y[nT]\leq-\Delta,\end{cases} \qquad (20)$$

Let us introduce a new signal

$$s[nT]=u^2[nT]=\begin{cases}1, & if\ \ |y[n]T|\geq\Delta,\\ 0, & if\ \ |y[n]T|<\Delta.\end{cases} \qquad (21)$$

By analyzing expression (17) we conclude that

$$s[nT]=\tilde{\delta}[(n-n_\nu)T], \qquad n_\nu\leq n\leq n-1 \qquad (22)$$

In order to form additional signal $\mu[nT]$, let us use the existing output signal $s[qT]$. Then signal $\mu[nT]$ can be expressed in the form of a product as follows.

$$\mu[nT]=x[nT]\chi[nT] \qquad (23)$$

where

$$\chi[nT]=\sum_{\theta=0}^{r}q_{\gamma_m}[(n-\theta)T]\,s[\theta T] \qquad (24)$$

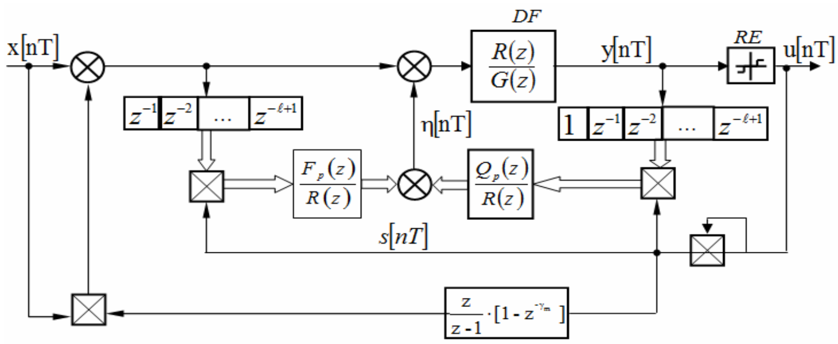


Fig. 4. Structure of an equivalent model of a digital pulse-frequency modulator

Then the impulse response transfer function can be found as follows.

$$q_{\gamma_m}[nT] = Z^{-1}\{G_{\gamma_m}(z)\}$$

$$G_{\gamma_m}(z) = \frac{z}{z-1} \cdot [1 - z^{-\gamma_m}]$$

Equations (17) – (18) and (22) – (24) give a complete description of a nonlinear operator A[x,y*,$\gamma_m$]. Using these equations we can come up with a structural model of a digital DPFM shown in Fig. 4.

By comparing the derived model of a digital DPFM presented in this paper with a similar model discussed in [9] we can see significant differences. First, we are not using a discrete-time integrator block. Also, instead of a bi-stable relay function with hysteresis we are using an unambiguous nonlinear relay-type function that has a sensitivity dead-zone. Both features allow significant simplification of the analysis of the model of the digital DPFM.

## 4. Computational Performance of the Design

It should also be understood that the designed controller can be intended for fast operation that imposes some feasibility constraints on the amount of real-time computations. Many adaptive control techniques used for controlled plants with transport delay perform computationally intensive tasks. The approach presented in this paper relies only on simple operations with individual input/output signals. In order to assess the complexity of the proposed algorithm, consider the number of floating point operations (flops) necessary to perform one step of calculations. For comparison, we also estimated the number of flops for such a well-known computational task as matrix inversion. It is very common that controller complexity depends on the order of the system. Inversion is performed on the matrix that has the size equal to this order. Fig. 3 presents the results of computational complexity evaluation for the systems with the order being varied from 1 to 5.

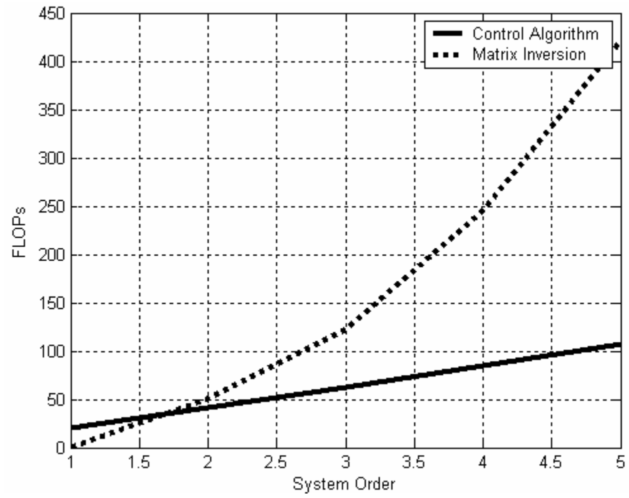


Fig. 5. Computational efficiency of the algorithm

The results shown in the above figure have been obtained using MATLAB. The control law given by the equations in the previous section been implemented for an l-th order dynamic system, as shown in Fig. 4. A standard MATLAB function has been used for matrix inversion. It can be seen from Fig. 5 that as the system order increases, flops number for matrix inversion grows in an exponential fashion. At the same time, for our control procedure one can observe virtually linear relationship. This example shows that the suggested algorithm is advantageous especially for higher order systems.

## 5. Conclusions

In this paper, we presented a control system based on a digital dynamic pulse-frequency modulator. Its behavior is described by difference equations augmented with additional logic relationships. To exclude the logic conditions of the DPFM, we developed an equivalent discrete-time nonlinear control system where parametric relationships were replaced with equivalent signals. Our simulation experiments demonstrate that the nonlinear control system offers a feasible solution for the applications that require good computational efficiency. Since the microcontroller implementation is the primary concern addressed in this paper, its scope does not extend beyond the computational efficiency of the proposed algorithm. At the same time, control system performance of the DPFM has been demonstrated in a different paper of this research group [18].

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