Analysis of a Parallel System with Priority to Repair over Maintenance Subject to Random Shocks

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Abstract

The goal of this paper is to analyze a two unit parallel system subject to random shocks. For this purpose, a reliability model is developed in which two identical units work in parallel. The operative unit is subjected to random shocks with some probabilities. The unit fails completely through normal mode. The maintenance and repair of the unit are conducted by a server who visits the system immediately. The shocked unit undergoes for maintenance whereas repair of the unit is done at its failure due to the reasons other than shocks. Priority is given to repair over maintenance. All random variables are statistically independent. The shock and failure times of the unit are exponentially distributed where as the distributions of maintenance and repair times are taken as arbitrary. Using regenerative point technique and semi- Markov process, several measures of system effectiveness are obtained in steady state. The graphical behavior of MTSF, availability and profit function has been analyzed for particular values of various parameters and costs.

Keywords: *Parallel System, Random Shocks, Maintenance, Repair and Priority.*

1. Introduction

The method of redundancy has widely been adopted by the researchers including Murari and Goyal (1984) and Singh (1989) while analyzing systems of two or more units not only to attain better reliability but also to reduce the down time of the system. But sometimes cold standby redundancy is not suggestive when shocks occur during operation of the system. In such a situation, the units in the system may be may be operated in parallel mode to share the impact of shocks. It is a known fact that cold standby redundancy is better than parallel redundancy so far as reliability is concerned. But, no research paper has been written by the researchers on these types of systems so far in the subject of reliability. However, a little work has been carried out by the authors including Murari and Al-Ali [1988] and Wu and Wu [2011] on the reliability modeling of shock models with the concept of maintenance and repairs. Shocks are the external environmental conditions which cause perturbation to the system, leading to its deterioration and consequent failure. The shocks may be caused by external factors such as fluctuation of unstable electric power, power failure, change in climate conditions, change of operator, etc. or due to internal factors such as stress and strain. Many systems like power generation and automotive industries are vulnerable to damage caused by shock attacking that may occur over the service life. Sometimes, a system may or may not be affected by the impact of shocks and the system may fail due to operation and / or due to random shocks.

Keeping in view the practical applications in mind, here a reliability model is developed in which two identical units work in parallel. The operative unit is subjected to random shocks with some probabilities. The unit fails completely through normal mode. The maintenance and repair of the unit are conducted by a server who visits the system immediately. The shocked unit undergoes for maintenance whereas repair of the unit is done at its failure due to the reasons other than shocks. Priority is given to repair over maintenance. The shock and failure times of the unit are exponentially distributed where as the distributions of maintenance and repair times are taken as arbitrary. Several reliability characteristics such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to repair and maintenance, expected number of maintenance and repair and profit function are evaluated in steady state using semi-Markov process and regenerative point technique. The graphical behavior of MTSF, availability and profit has been observed with respect to shock rate for fixed values of other parameters.

2. Notations

E	:	Set of regenerative states.
O	:	The unit is operative and in normal mode.
p_0	:	The probability that shock is effective.
q_0	:	The probability that shock is not effective.
μ	:	Constant rate of the occurrence of a shock.

λ : Constant failure rate of the unit.
m(t)/M(t) : pdf / cdf of maintenance time of the unit after the effect of a shock.
FUr / FWr /FUR : The Unit is completely failed and under repair / waiting for repair/ under continuous repair from previous state
SUm/SUM : Shocked unit under maintenance and under maintenance continuously from previous state
SWm : Shocked unit waiting for maintenance
g (t) / G (t) : pdf / cdf of repair time of the completely failed unit
$q_{ij}(t)$ / $Q_{ij}(t)$: pdf and cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state
$q_{ij.k}(t)$ / $Q_{ij.k}(t)$: pdf and cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].
$M_i(t)$: Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state.
$W_i(t)$: Probability that the server is busy in state S_i up to time t without making transition to any other regenerative state or returning to the same via one or more non regenerative states.
m_{ij} : Contribution to mean sojourn time in state S_i when system transits directly to state S_j (S_i,$S_j \in E$) so that $\mu_i = \sum_j m_{ij}$ Where $m_{ij} = \int t\, dQ_{ij}(t) = -q_{ij}^{*/}(0)$ and μ_i is the mean sojourn time in state $S_i \in E$

(s) / © : Symbol for Stieltjes convolution / Laplace convolution.
~ / * : Symbol for Laplace Stieltjes Transform (LST) / Laplace Transform (LT).

The following are the possible transition states of the system
$S_0 = (O, O)$, $S_1 = (SUm, O)$, $S_2 = (SUM, SWm)$, $S_3=(O,FUr)$, $S_4 =(FUR, SWm)$,$S_5=(FUR,FWr)$, $S_6=(SUM,FWr)$,
The transition states S_0, S_1, S_3 are regenerative and states S_2, S_4, S_5, S_6 are non regenerative as shown in figure 1.

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$ as

$$p_{00} = \frac{2q_0^2\mu}{2q_0^2\mu + 2\lambda + 4p_0\mu}, \quad p_{01} = \frac{4p_0\mu}{2q_0^2\mu + 2\lambda + 4p_0\mu},$$

$$p_{03} = \frac{2\lambda}{2q_0^2\mu + 2\lambda + 4p_0\mu}, \quad p_{53} = g^*(0),$$

$$p_{10} = m^*(\lambda+\mu), p_{11} = \frac{q_0\mu}{\lambda+\mu}\left[1 - m^*(\lambda+\mu)\right],$$

$$p_{21} = m^*(0), \quad p_{41} = g^*(0), \quad p_{61} = g^*(0)$$

$$p_{12} = \frac{p_0\mu}{\lambda+\mu}\left[1 - m^*(\lambda+\mu)\right]$$

$$p_{16} = \frac{\lambda}{\lambda+\mu}\left[1 - m^*(\lambda+\mu)\right], p_{30} = g^*(\lambda+\mu),$$

$$p_{33} = \frac{q_0\mu}{\lambda+\mu}\left[1 - g^*(\lambda+\mu)\right],$$

$$p_{11.2} = \frac{p_0\mu}{\lambda+\mu}\left[1 - m^*(\lambda+\mu)\right]$$

$$p_{31.4} = \frac{p_0\mu}{\lambda+\mu}\left[1 - m^*(\lambda+\mu)\right]$$

$$p_{34} = \frac{p_0\mu}{\lambda+\mu}\left[1 - g^*(\lambda+\mu)\right],$$

$$p_{33.5} = p_{35} = \frac{\lambda}{\lambda+\mu}\left[1 - g^*(\lambda+\mu)\right] \quad (1)$$

It can be easily verified that

$p_{00}+p_{01}+p_{03} = p_{10}+p_{11}+p_{12}+p_{16}+p_{11.2} = p_{21} = p_{30}+p_{33}+p_{34}+p_{35} = p_{30}+p_{33}+p_{33.5}+p_{31.4} = p_{31} = p_{41} = p_{43} = p_{53} = p_{61} = p_{63} = p_{10}+p_{11.3}+p_{14} = p_{20}+p_{21.6}+p_{22.5} = 1$ (2)

The mean sojourn times (μ_i) in the state S_i for m(t) = $\theta e^{-\theta t}$ and g(t) = $\gamma e^{-\gamma t}$ are follows

$$\mu_0 = \frac{1}{2q_0^2\mu + 2\lambda + 4p_0\mu}, \quad \mu_1 = \frac{1}{\lambda+\mu+\theta},$$

$$\mu_3 = \frac{1}{\lambda+\mu+\gamma},$$

$$\mu_4 = \frac{1}{\theta}, \quad \mu_6 = \frac{1}{\gamma}, \quad \mu_1^1 = \frac{\lambda + p_0\mu + \theta}{\theta(\lambda+\mu+\theta)},$$

$$\mu_3^1 = \frac{\lambda+\gamma}{(\lambda+\mu+\gamma)} \quad (3)$$

4. Reliability and Mean Time To System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ_i (t):

$$\varphi_i(t)=\sum_j Q_{i,j}(t)®\varphi_j(t)+\sum_k Q_{i,k}(t) \tag{4}$$

where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly. Taking LST of above relation (4) and solving for $\tilde{\phi}_0(s)$.We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \tag{5}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (5).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad \text{where} \tag{6}$$

$N_1 = \mu_0(1-p_{11})(1-p_{33}) + \mu_1 p_{01}(1-p_{33}) + \mu_3 p_{03}(1-p_{11})$ and

$D_1 = (1-p_{00})(1-p_{11})(1-p_{33})-p_{01}\ p_{10}(1-p_{33})-p_{03}\ p_{30}(1-p_{11})$ (7)

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t)=M_i(t)+\sum_j q^n{}_{i,j}(t)©A_j(t) \tag{8}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state, we have

$M_0(t)=e^{-(\lambda+\mu)t}$, $M_1(t)=e^{-(\lambda+\mu)t}\overline{M(t)}$, $M_3(t)=e^{-(\lambda+\mu)t}\overline{G(t)}$

Taking LT of above relations (8) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$$

where

N_2 = [(1-$p_{11.2}$-p_{11}-p_{16} p_{61})(1- p_{33}- $p_{33.5}$)] μ_0 - [(-p_{01})(1-p_{33}-$p_{33.5}$)-$p_{03}p_{31.4}$] μ_1 + [p_{03}(1-$p_{11.2}$-p_{11}-p_{16} p_{61})] μ_3

and

D_2 = [(1-$p_{11.2}$-p_{11}-p_{16} p_{61})(1- p_{33}- $p_{33.5}$)] μ_0 - [(-p_{01})(1-p_{33}-$p_{33.5}$)-$p_{03}p_{31.4}$](μ_1' +p_{16} μ_6)+ [p_{03}(1-$p_{11.2}$-p_{11}-p_{16} p_{61})] μ_3'

6. Busy Period Analysis of The Server

6.1 Due to repair

Let $B_i^R(t)$ be the probability that the server is busy in repair of the system (unit) at an instant't' given that the system entered state i at t = 0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j^R(t) \tag{10}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to repair up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3(t)= e^{-(\lambda+\mu)t}\,\overline{G(t)}+(\lambda e^{-(\lambda+\mu)t}©1)\overline{G(t)}+(p_0\mu e^{-(\lambda+\mu)t}©1)\overline{G(t)}+(q_0\mu e^{-(\lambda+\mu)t}©1)\overline{G(t)}$$

$$W_6(t)= \overline{G(t)}$$

Now taking L.T. of relations (10) and obtain the value of $B^R{}_0{}^*(s)$and by using this, the time for which server is busy in steady state is given by

$B^R{}_0 = \lim_{s\to 0} B^{R*}{}_0(s)$ = N_3/D_2,

where

N_3 = [p_{03}(1-$p_{11.2}$-p_{11}- p_{16} p_{61})] w_3^* (s)+ [(-p_{01})(1-p_{33}-$p_{33.5}$)-$p_{03}p_{31.4}$]W_6 p_{16} and D_2 is already defined.

6.2 Due to Maintenance

Let $B_i^M(t)$ be the probability that the server is busy in maintenance of the system (unit) at an instant't' given that the system entered state i at t = 0. The recursive relations for $B_i^M(t)$ are as follows:

$$B_i^M(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j^M(t) \tag{11}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1(t)= e^{-(\lambda+\mu)t}\,\overline{M(t)}+(\lambda e^{-(\lambda+\mu)t}©1)\overline{M(t)}+(p_0\mu e^{-(\lambda+\mu)t}©1)\overline{M(t)}+(q_0\mu e^{-(\lambda+\mu)t}©1)\overline{M(t)}$$

Now taking L.T. of relations (11) and obtain the value of $B^M{}_0{}^*(s)$and by using this, the time for which server is busy in steady state is given by

$$B^M_0 = \lim_{s\to 0} sB^{M*}_0(s) = N_4/D_2$$

Where

$N_4 = -[(-p_{01})(1-p_{33}-p_{33.5})-p_{03}p_{31.4}]\, w_1^*(s)$ and D_2 is already defined.

7. Expected Number of Maintenance

Let $N_i^M(t)$ be the expected number of maintenances conducted by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i^M(t)$ are given as

$$N_i^M(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_j^M(t)\right] \qquad (12)$$

where j is any regenerative state to which the given regenerative state i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of relations (12) and solving for $\tilde{R}_0^p(s)$. The expected numbers of maintenances per unit time are given by

$$N_0^M(\infty) = \lim_{s\to 0} sN_0^M(s) = \frac{N_5}{D_2}$$

where D_2 is already mentioned.

$N_5 = -(p_{10}+p_{11.2})\ [(-p_{01})(1-p_{33}-p_{33.5})-p_{03}p_{31.4}]$ and D_2 is already defined.

8. Expected Number of Repairs

Let $N_i^R(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i^r(t)$ are given as

$$N_i^R(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_j^R(t)\right] \qquad (13)$$

Where j is any regenerative state to which the given regenerative state i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of relations (13) and solving for $\tilde{R}_0^r(s)$. The expected numbers of repairs per unit time are given by

$$N^R_0 = \lim_{s\to 0} \frac{N^{R*}_0(s)}{s} = N_6/D_2$$

where

$N_6 = (p_{30}+p_{31.4}+p_{33.5})[p_{03}(1-p_{11}-p_{11.2})-p_{16}p_{61}]$ and D_2 is already defined.

9. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = \begin{matrix} K_0A_0 - K_1B_0^M - K_2B_0^R - K_3N_0^M \\ -K_4N_0^R - K_5 \end{matrix} \qquad (14)$$

where

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to maintenance

K_2 = Cost per unit time for which server is busy due to repair (12)

K_3 = Maintenance cost per unit

K_4 = Repair cost per unit

K_5 = fixed cost of the server and $A_0, B_0^M, B_0^R, N_0^M, N_0^R$ are already defined.

10. Conclusion

In this paper, a shock model of a parallel system with repair and maintenance has been analyzed for a particular case $g(t) = \gamma e^{-\gamma t}$, $m(t) = \theta e^{-\theta t}$ giving priority to repair of the failed unit over maintenance of the shocked unit. Some important reliability measures are obtained giving particular values to various parameters (13) and costs. Graphs are drawn to depict the behavior of MTSF, availability and profit with respect to shock rate (μ) as shown respectively in figures 2, 3 and 4. From figure 2, it is observed that MTSF declines rapidly with the increase of shock rate (μ) while it decreases with the increase of failure rate (λ). But, if we increase repair and maintenance rates, the value of MTSF becomes more. It is interesting to note that MTSF increases rapidly if values of p_0 and q_0 are interchanged, i.e. p_0=.4 and q_0=.6. Figure 3 and 4 highlight that availability and profit go on decreasing with the increase of shock rate (μ) and failure rate (λ). However, their values become more with the increase of maintenance (14) and repair rates as well as by interchanging the values of p_0 and q_0.

11. References:

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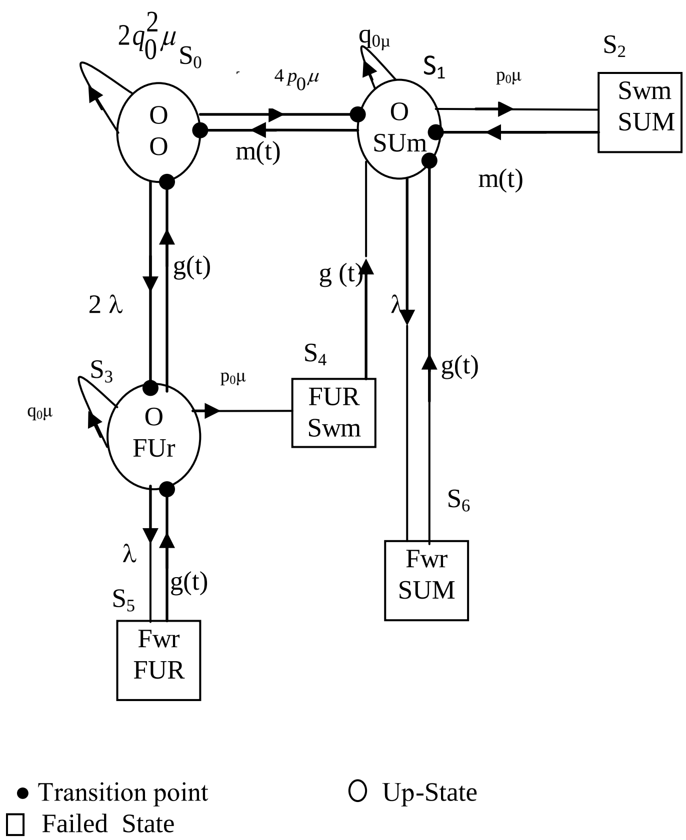

Fig. 1: State Transition Diagram

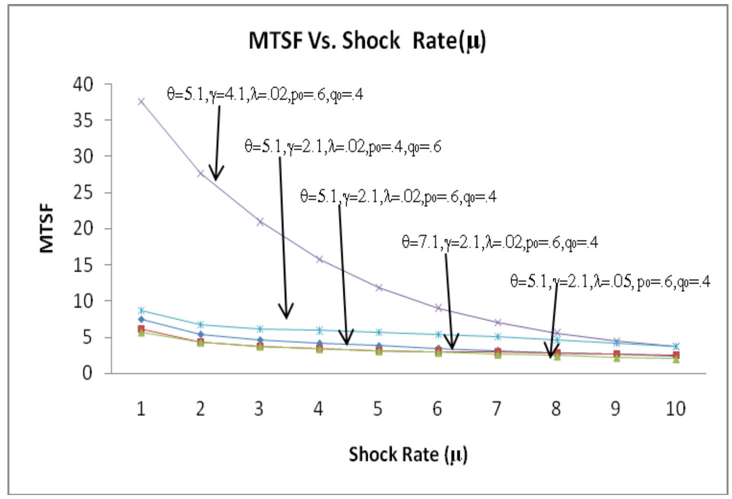

Fig. 2: MTSF Vs. Shock Rate

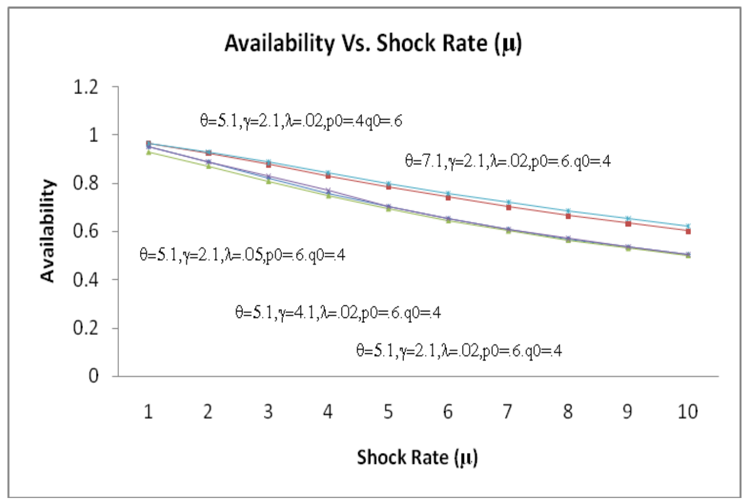

Fig. 3: Availability Vs. Shock Rate

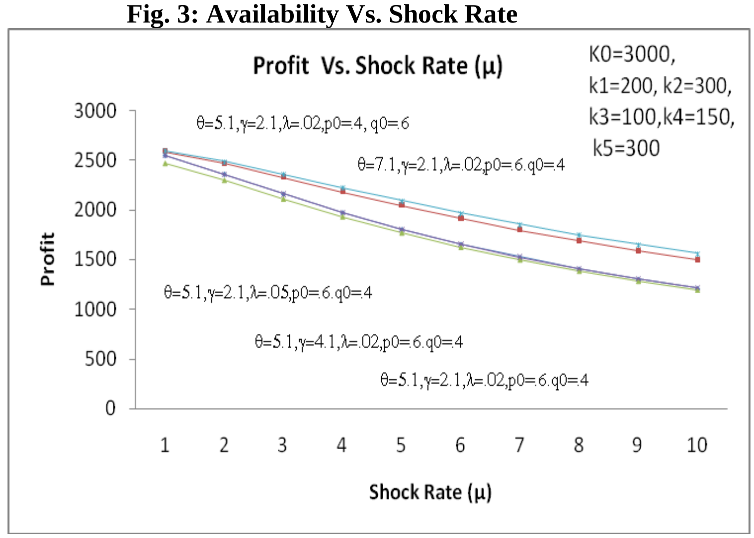

Fig. 4: Profit Vs. Shock Rate

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