A Model of Image Denoising Based on the Fusion of Anisotropic Diffusion and Total Variation Models*

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Abstract

In this paper, a new denoising technique for images corrupted with additive salt and pepper noise and white Gaussian noise is proposed. The technique used here is to combine the anisotropic diffusion (PM) model and total variation (TV) model. The new technique utilizes both advantages of PM model and TV model, while avoiding the disadvantages of both of them. To evaluate our algorithm several experiments have been conducted. The experimental results affirm the high performance of our model.

Keywords: PM model, TV model, Image features, Partial Differential Equation (PDE).

1. Introduction

Since the noise is related to high frequencies, it is difficult to remove the noise, while preserving the important features [7]. The most efficient algorithm is the one that has the ability to solve this problem. Image de-noising has many applications that push people to look for better algorithms to overcome the drawbacks of the existing ones. There are many algorithms of image de-noising, for instance, multi-resolution geometry analysis, which is based on wavelet theory [15]-[17], has attracted a lot of attention. Recently, partial differential equation (PDE) becomes an important approach of image de-noising, such as total variation (TV) [18], anisotropic diffusion (PM), and so on [1]. In 1990 Perona and Malik (PM) [4] proposed the anisotropic diffusion model, which is useful tool for multi-scale description of images, image segmentation, edge detection and image enhancement [8]. The basic idea of PM is to evolve a family of smoothed image $u(x,y)$ from the initial image $u_0(x,y)$, using the following partial differential equation:

$$\frac{\partial u}{\partial t} = div(g(|\nabla u|)\nabla u) = g(|\nabla u|)\Delta u + \nabla g \cdot \nabla u,$$

where $g(|\nabla u|)$ is designed to preferably smooth pixels inside a region rather than pixels near the boundary. However, the disadvantage of the PM model is tending to impair textures and details of image so that de-noising is not sufficient in the whole process. Another traditional approach to partial differential equation based image processing techniques was proposed by Rudin, Osher, and Fatemi (ROF) [5]. The authors proposed to minimize the total variation of the noisy image subject to constrains involving the statistics of the noise. Total variation (TV) minimization is a successful approach to recover images with sharp edges. Nevertheless, TV model can cause Gibbs-type artifacts. These Gibbs-type artifacts cannot be acceptable for applications like image feature, object detection. In order to reduce the Gibbs-type artifacts produced by PM, and TV models, we can use $\int |\nabla u|^2$ as a measure of image smoothness. This can reduce the Gibbs-type artifacts, but unfortunately penalizes too much the gradients corresponding to edges [9]. In order to simultaneously reduce the Gibbs-type artifacts without causing any damage in the image, we use a weighted function combining the PM model and TV model to get our new model.

The rest of this paper is organized as follows: In Section 2, we briefly describe the anisotropic diffusion and total variation models. The proposed method is described in Section 3. In Section 4, we present our experimental results that confirm the efficiency of proposed model. Some concluding remarks are presented in Section 5.

2. PM Model and TV Model

2.1 PM model

For the images of anisotropic diffusion, let us consider the energy functional of the image as follows:

$$E(u)=\iint_{\Omega}\frac{k^2}{2}\ln(1+\frac{|\nabla u|^2}{k^2})dxdy. \quad (1)$$

Let

$$E(u)=\iint_{\Omega}F(x,y,u,u_x,u_y)dxdx \quad (2)$$

The partial derivatives of the integrand

$$E(u)=F(x,y,u,u_x,u_y)=\frac{k^2}{2}\ln(1+\frac{|\nabla u|^2}{k^2}) \quad (3)$$

are: $F_u=0$, $F_{u_x}=\frac{u_x}{1+\frac{|\nabla u|^2}{k^2}}$, and $F_{u_y}=\frac{u_y}{1+\frac{|\nabla u|^2}{k^2}}$.

Therefore the corresponding Euler-Lagrange equation to (1) is:

$$0=F_u+\frac{\partial}{\partial x}F_{u_x}+\frac{\partial}{\partial y}F_{u_y}=(\frac{\partial}{\partial x},\frac{\partial}{\partial y})\cdot(\frac{u_x}{1+\frac{|\nabla u|^2}{k^2}},\frac{u_y}{1+\frac{|\nabla u|^2}{k^2}})$$

$$=\nabla\cdot(\frac{\nabla u}{1+(\frac{|\nabla u|}{k})^2})=\nabla\cdot(g(|\nabla u|)\nabla u), \quad (4)$$

where $g(s)$ is the diffusion function defined as:

$$g(s)=\frac{1}{1+(\frac{s}{k})^2}.$$

The authors in [4] considered the diffusion process:

$$\frac{\partial u}{\partial t}=\nabla\cdot(g(|\nabla u|)\nabla u), \quad (5)$$

with the given noisy signal u_0 as the initial condition $u(x,y,t)=u_0(x,y)$. Here the time acts as a scale parameter for filtering. Typically, $g(s)$ is a non-negative decreasing function, such that $g(s)$ tends to zero as s approaches infinity. One of the serious problems in the diffusion model is that it is very sensitive to noise. To obtain reconstruction u of a degraded image u_0, Nordström et al.[2] has suggested the energy functions as follows:

$$E=\iint_{\Omega}(\lambda(u-u_0)^2+g(|\nabla u|)\cdot|\nabla u|^2+k^2(g(|\nabla u|)-\ln g(|\nabla u|)))dxdy, \quad (6)$$

where $\nabla u=(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y})$ is gradient operator, Ω is the domain of the image and λ is the Lagrange multiplier.

The corresponding Euler-Lagrange equations to this energy function are given by:

$$-\nabla\cdot(g(|\nabla u|)\nabla u)+2\lambda(u-u_0)=0, \quad (7)$$

$$0=\lambda^2(1-\frac{1}{g(|\nabla u|)})+|\nabla u|^2. \quad (8)$$

Joachim Weickert [3] regarded the energy function as follows:

$$E(u,g(|\nabla u|)\nabla u)=\iint_{\Omega}[\lambda(u-u_0)^2+k^2\cdot\ln(\frac{1}{g(|\nabla u|)})]dxdy, \quad (9)$$

and the corresponding Euler Lagrange equation is:

$$-\nabla\cdot(g(|\nabla u|)\nabla u)+2\lambda(u-u_0)=0. \quad (10)$$

A general expression of the anisotropic diffusion equation first proposed by Perona and Malik [4] can be written as:

$$\begin{cases}\frac{\partial u}{\partial t}=\nabla\cdot(g(|\nabla u|)\nabla u)-2\lambda(u-u_0),\\ u(x,y,t)=u_0(x,y),\end{cases} \quad (11)$$

here $\lambda(u-u_0)$ is additional bias term.

PM model is an ill-posed problem, and the results will fall into the local optimal solution. When the noise intensity is large, the gradient of the noise and the gradient of the edge are similar, so the equation (11) cannot be used for de-noising. This is the reason why the PM model causes Gibbs-type artifacts.

2.2 TV model

A classical way to overcome ill-posed minimization problems is to add a regularization term to the energy. This idea was introduced in 1977 by Tikhonov and Arsenin [14]. The authors proposed to consider the following minimization problem:

$$E(u)=\iint_{\Omega}(|\nabla u|^2+\lambda(u-u_0)^2)dxdy, \quad (12)$$

where the first term of (12) is the smoothing term. The L^p norm with $p=2$ of the gradient allows us to remove the noise but unfortunately penalizes to much the gradient corresponding to edges [9]. Rudin, Osher and Fatemi (ROF) [5] in 1992 proposed to use the L^1 norm of the gradient of u, instead of the L^2 norm, that is, minimizing the total variations:

$$TV(u)=\iint_{\Omega}|\nabla u|dxdy, \quad (13)$$

subject to

$$\iint_\Omega u(x,y)dxdy = \iint_\Omega u_0(x,y)dxdy \ , \quad (14)$$

$$\frac{1}{\Omega}\iint_\Omega (u(x,y)-u_0(x,y))dxdy = \sigma^2 \ . \quad (15)$$

Here the additive noise $n(x,y)$ is of zero mean and has known variance σ^2.

For the images of total variation, let us consider the energy functional of the image as follows:

$$E(u) = \int\int_\Omega |\nabla u| \, dxdy \, . \quad (16)$$

Let

$$E(u) = \int\int_\Omega F(x,y,u,u_x,u_y)dxdy \, . \quad (17)$$

The partial derivatives of the integrand

$$F(x,y,u,u_x,u_y) = \sqrt{u_x^2+u_y^2} \quad (18)$$

are:

$$F_u = 0 \, , \ F_{u_x} = \frac{u_x}{(u_x^2+u_y^2)^{\frac{1}{2}}} \ \text{, and } F_{u_y} = \frac{u_y}{(u_x^2+u_y^2)^{\frac{1}{2}}} \ .$$

The corresponding Euler-Lagrange equation to (16) is:

$$0 = F_u + \frac{\partial}{\partial x}F_{u_x} + \frac{\partial}{\partial y}F_{u_y} = (\frac{\partial}{\partial x},\frac{\partial}{\partial y})\cdot\left(\frac{u_x}{(u_x^2+u_y^2)^{\frac{1}{2}}},\frac{u_y}{(u_x^2+u_y^2)^{\frac{1}{2}}}\right) = \nabla\cdot(\frac{\nabla u}{|\nabla u|}) \cdot \quad (19)$$

By introducing Lagrange multiplier λ , the energy functional of the image can be redefined as:

$$E(u) = \iint_\Omega (|\nabla u| + \lambda(u-u_0)^2)dxdy \quad (20)$$

To obtain the minimum, the energy functional needs to satisfy:

$$-\nabla\cdot(\frac{\nabla u}{|\nabla u|}) + 2\lambda(u-u_0) = 0 \quad (21)$$

By the gradient descent method, we get the TV de-noising model as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nabla\cdot(\dfrac{\nabla u}{|\nabla u|}) - 2\lambda(u-u_0), \\ u(x,y,t) = u_0(x,y). \end{cases} \quad (22)$$

the TV model preserves the edges better than anisotropic diffusion (PM) model. But the disadvantage of TV model is that the block effect is produced when dealing with the flat areas, and consequently the local details characteristics of the original image are lost [10,11]. The weighted method proposed in this paper, which is different from those in [12,13], establishes the energy functional model by weighting the anisotropic diffusion (PM) model and the total variation (TV) model, reducing the noise by optimizing the energy functional. The new model avoids the edge blurring and eliminates the block effect as de-noising.

Recently the authors in [6] multiplied TV model, and PM model by $\varphi = e^{-\frac{1}{|\nabla u|}} - h$, and $\psi = 2(1-\varphi)$ respectively and summed the two terms to get an new model, for more details see [6].

3. New Model

In image processing, removal of noise without blurring the image edges is a difficult problem. Typically noise is characterized by high spatial frequencies in an image, and the details of the image, such as edge and texture, principally appear in the high frequency region. The task of image filtering is to remove the noise and preserve the details simultaneously, namely, to have possibly least diffusion in the regions which contain more image features, and possibly most diffusion in the regions which contain less image features. In this paper we use a weighted function complying the TV model and PM model to get a new de-noising model. Considering the characteristics of the anisotropic diffusion de-noising model and the total variation model, we use a weighted function combining the two models to get a new de-noising model, which provides a new approach for solving the contradiction in the image restoration. Now we restore the original image u by the degraded image u_0, taking the energy functional of the image as follows:

$$E(u) = \iint_\Omega (\mu|\nabla u| + (1-\mu)(k^2\cdot\ln(\frac{1}{g|\nabla u|})) + \lambda(u-u_0)^2)dxdy, \quad (23)$$

where the weight function $\mu \in [0,1]$.

From (10), (21) the corresponding Euler-Lagrange equation is:

$$-\mu\nabla\cdot(\frac{\nabla u}{|\nabla u|}) - (1-\mu)\nabla\cdot(g(|\nabla u|)\nabla u) + 2\lambda(u-u_0) = 0 \ \cdot \quad (24)$$

By using the gradient descent method, the new model can be expressed as:

$$\begin{cases} \frac{\partial u}{\partial t} = \mu \nabla \cdot (\frac{\nabla u}{|\nabla u|}) + (1-\mu)\nabla \cdot (g(|\nabla u|)\nabla u) - 2\lambda(u-u_0), \\ u(x,y,t) = u_0(x,y). \end{cases} \quad (25)$$

From the above new model we can expect that:

1- In the region which contains more image features (such as edges, etc.), the new model will play good role to preserve the edges of the image, namely this model will highlight the total variation (TV) model, therefore μ should be close to one.

2- In the flat areas of image, which contains less image features, the new model will highlight the role of the anisotropic diffusion (PM) model, therefore μ should be close to zero.

For our model we select the following weight function:

$$\begin{cases} 1 \quad |\nabla u| \geq \xi, \\ \sin^2(\frac{\pi|\nabla u|}{2\xi}) \quad 0 \leq |\nabla u| < \xi. \end{cases} \quad (26)$$

Here ξ is a threshold setting by the specific circumstances. To solve problem (25) by using the finite difference method, we let

$$T = \nabla \cdot (\frac{\nabla u}{|\nabla u|}) = \frac{u_x^2 u_{yy} - 2u_x u_y u_{xy} + u_y^2 u_{xx}}{(u_x^2 + u_y^2)^{\frac{3}{2}}}, \quad (27)$$

$$P = \nabla \cdot (g(|\nabla u|)\nabla u) = \frac{k^2 u_{xx} + k^2 u_{yy} - u_x^2 u_{xx} + u_y^2 u_{xx} - 4u_x u_y u_{xy} + u_x^2 u_{yy} - u_y^2 u_{yy}}{(\frac{k^2 + u_x^2 + u_y^2}{k})^2}. \quad (28)$$

Here $g(|\nabla u|) = \frac{1}{1+(\frac{|\nabla u|}{k})^2}$, $k = k_0 e^{-\Delta t(n-1)}$, Where k_0 is the initial value, n is the number of iterations. Replacing the first order derivatives by central divided differences and the second order derivatives by forward divided differences, we can rewrite the new model as the discrete form as follows:

$$\frac{u^{n+1} - u^n}{\Delta u} = \mu T^n + (1-\mu)P^n - 2\lambda(u^n - u_0), \quad (29)$$

where, $n = 0,1,2,.....,$ is the time level.

Introduce the space discrete sign:

$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t},$$

$$T_{i,j}^n = \nabla^n \cdot (\frac{\nabla u_{i,j}^n}{|\nabla u_{i,j}^n|}) = \frac{(u_x^2)_{i,j}^n (u_{yy})_{i,j}^n - 2(u_x)_{i,j}^n (u_y)_{i,j}^n (u_{xy})_{i,j}^n + (u_y^2)_{i,j}^n (u_{xx})_{i,j}^n}{((u_x^2)_{i,j}^n + (u_y^2)_{i,j}^n)^{\frac{3}{2}}},$$

$$P_{i,j}^n = \frac{k^2 (u_{xx})_{i,j}^n + k^2 (u_{yy})_{i,j}^n - (u_x^2)_{i,j}^n (u_{xx})_{i,j}^n + (u_y^2)_{i,j}^n (u_{xx})_{i,j}^n - 4(u_x)_{i,j}^n (u_y)_{i,j}^n (u_{xy})_{i,j}^n + (u_x^2)_{i,j}^n (u_{yy})_{i,j}^n - (u_y^2)_{i,j}^n (u_{yy})_{i,j}^n}{(\frac{k^2 + (u_x^2)_{i,j}^n + (u_y^2)_{i,j}^n}{k})^2}.$$

We have

$$u_{i,j}^{n+1} = u_{i,j}^n + \Delta t(\mu T_{i,j}^n + (1-\mu)P_{i,j}^n - 2\lambda(u_{i,j}^n - u_0)). \quad (30)$$

Where

$$u_x \approx \frac{u_{i+h,j} - u_{i-h,j}}{2h}, \quad (31)$$

$$u_y \approx \frac{u_{i,j+h} - u_{i,j-h}}{2h}, \quad (32)$$

$$u_{xx} \approx \frac{u_{i+h,j} + u_{i-h,j} - 2u_{i,j}}{h^2}, \quad (33)$$

$$u_{yy} \approx \frac{u_{i,j+h} + u_{i,j-h} - 2u_{i,j}}{h^2}, \quad (34)$$

$$u_{xy} \approx \frac{u_{i+h,j+h} - u_{i-h,j+h} - u_{i+h,j-h} + u_{i-h,j-h}}{4h^2} \quad (35)$$

4. Experimental Results and Analysis

In this section we compare the proposed approach with other methods in terms of the visual quality of de-noising images, Mean Square Error (MSE) according to (36), and Peak Signal to Noise Ratio (PSNR) according to (37).

$$MSE = \frac{1}{N_x \times N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} [I_{de}(i,j) - I_{or}(i,j)]^2 \quad (36)$$

$$PSNR_{dB} = 10\lg(\frac{255 \times 255}{MSE}) \quad (37)$$

Where N_x and N_y are the number of pixels horizontally and vertically respectively, and $I_{de}(i,j)$, $I_{or}(i,j)$ are the de-noised image and original image, respectively. 255 is the peak signal with an 8-bit resolution.

We take the commonly used 256×256 bit standard Lena, Cameraman and Boat images processed by the different de-noising methods as examples. The experimental results are shown in Figures 1 and 2. In these two cases we select the parameters as follows: the time step size $\Delta t = 0.1$, the grid step size $h = 1$, $\lambda = 0.01$, $\xi = 0.2$, $k_0 = 30$ and the number of iteration $n = 50$.

In Figure 1 the image is corrupted with additive salt and pepper noise of different variances and the different de-noising methods are applied. In Figure 2 the image is corrupted with additive Gaussian noise with standard deviation $\sigma = 20$. Figures 1 and 2 show that the new model has a very obvious de-noising effect; it not only maintains the advantages of the PM model and TV model, but also overcomes the disadvantage of the two models. The quantitative results are presented in Tables 1, 2, 3. It can be seen from Tables 1 and 2 that the PSNR of the new model is the maximum, and from Table 3 we can see that the MSE of our model is the minimum, that means the de-noising effect of the proposed model is the best. From Table 2 we find that increasing the variance of the noise will decrease the PSNRs of the four algorithms, which means that the de-noising effect is worse. However the PSNR of the new model is the largest among the four algorithms for the same variance, which means the de-noising effect of the new algorithm, is the best in terms of PSNR. From Table 3 we find that MSEs of the four models increase as the variance of the noise increases, which means that the de-nosing effect is worse. Nevertheless the MSE of the proposed model is the lowest among the four models for same variance, this means the de-noising effect of the new model, is the best in terms of MSE. From Figure 3, we can see that, the histograms of TV, PM, and Ref. [6] models indicate poor contrast, unlike the new model that indicates a good one. From Figure 4 we can see that the proposed method in all noise power has the highest PSNR. From Figure 5 we can see that the proposed method in all noise power has the lowest MSE.

Table 1: PSNRs of TV, PM, Ref. [6] and the proposed algorithms. Input images are polluted by white Gaussian noise with standard deviation $\sigma = 20$

Images	TV model	PM model	Ref.[6] model	New model
Lena	26.1118	21.1110	26.3169	26.8563
Cameraman	25.9651	20.9268	26.1402	26.5882
Boat	26.2158	21.1099	26.4259	26.9844

Table 2: The PSNRs of different algorithms with different variances of salt and pepper noise.

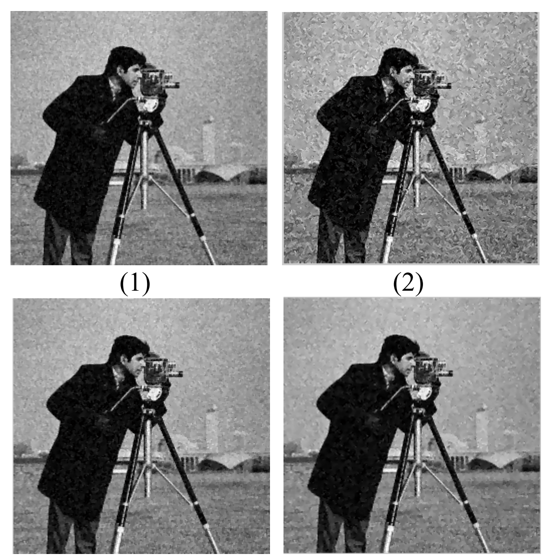
Variance	0.02	0.03	0.04	0.05	0.06	0.07
TV	28.5527	26.5138	25.6617	23.9651	22.7291	21.9510
PM	24.5940	23.4454	22.6831	21.4331	20.2884	19.5865
Ref.[6]	28.5065	26.6232	25.8379	24.1758	22.8750	22.1054
New	28.7249	26.9657	26.1118	24.4722	23.1470	22.4304

Table 3: The MSEs of different algorithms with different variances of salt and pepper noise.

Variance	0.02	0.03	0.04	0.05	0.06	0.07
TV	9.9297	11.7124	13.6880	15.6827	18.2597	20.3861
PM	15.2512	17.0483	19.0745	21.1954	24.4319	26.6479
Ref.[6]	9.9659	11.5849	13.3847	15.3942	17.9597	20.0574
New	9.6443	11.2226	12.8299	14.9440	17.3382	19.2486

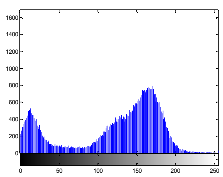
(1) (2) (3) (4)

Figure 1: Results of de-noising obtained with Lena image (variance of the salt and pepper noise=0.02), (1) Result of TV algorithm, (2) Result of PM algorithm, (3) Result of reference [6] algorithm, (4) Result of the new algorithm.

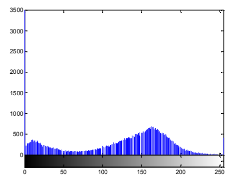


(1) (2) (3) (4)

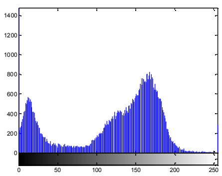
Figure 2: Results of de-noising obtained with cameraman image (standard deviation of the noise is σ =20), (1) Result of TV algorithm, (2) Result of PM algorithm, (3) Result of reference [6] algorithm, (4) Result of the new algorithm.



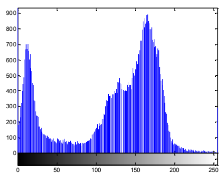
(1)



(2)



(3)



(4)

Figure 3: Histograms of Fig.2: (1) Histogram of image (1), (2) Histogram of image (2), (3) Histogram of image (3), and (4) Histogram of image (4).

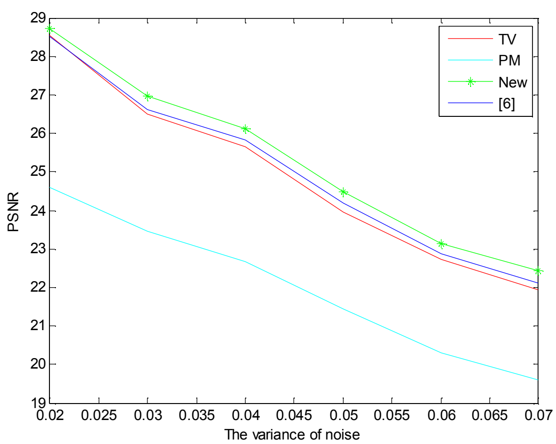


Figure 4 ：PSNR(dB) graph of the TV, PM, Ref. [6] and new algorithms for various salt and pepper noise levels for Lena image.

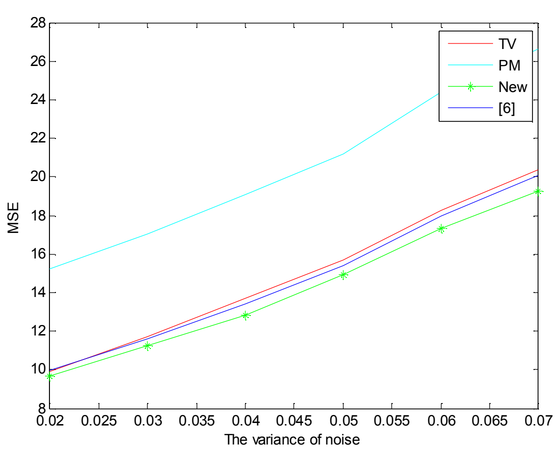


Figure 5 ：MSE graph of the TV, PM, Ref. [6] and new algorithms for various salt and pepper noise levels for Lena image.

5. Conclusions

In this paper we propose a new approach for image de-noising based on the combination of PM model and TV model. In our model we reduced the noise by optimizing the energy functional. From the performance of the simulations, our model has more de-noising ability in terms of MSE, PSNR, and visual quality compared with the anisotropic diffusion (PM) model, the total variation (TV) model and reference [6] model. To evaluate the proposed algorithm, several experiments were presented. Experimental results confirmed the high performance of the proposed algorithm compared with some well-known algorithms. The proposed algorithm can also be extended to other types of noises. If other methods for solving the partial differential equations are applied to new model, the convergence speed may be improved. In future research we will focus on constructing better algorithms in smoothing images and preserving image features.

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References

[1] F. Li, F. Gao, and N. Cai, "A new algorithm for removing salt and pepper noise from images", Journal of Information and Computational Science, Vol. 8, No. 16, 2011, pp. 3819-3825.
[2] K. N. Nordstrom, "Biased anisotropic diffusion - a unified regularization and diffusion approach to edge detection", Image Vision Computing, Vol. 8, Iss. 4, 1990, pp. 318-327.
[3] J. Weickert, Anisotropic Diffusion in Image Processing, Germany, B.G. Teubner Stuttgart, 1998.
[4] P. Perona, and J. Malik, "Scale space and edge detection using anisotropic diffusion", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 12, No. 7, 1990, pp. 629-639.
[5] L. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms", Phisica D, Vol. 60, Iss. 1-4 , 1992, pp. 259-268.
[6] X. Zhange, R. Wang, and L. Jiao, "Partial differential equation model method based on image feature for denoising.", International Conference on Multi-Platform/Multi-Sensor Remote Sensing and Mapping , 11-12 Jan., 2011, pp. 1-4.
[7] P. Chatterjee, and P. Milanfar, "Is denoising dead?", IEEE Transactions on Image Processing , Vol. 19, No. 4, 2010, pp. 895-911.

[8] Y. You, W. Xu, A. Tannenbaum, and M. Kaveh, "Behavioral analysis of anisotropic diffusion in image processing", IEEE Trans Image Processing, Vol. 5, No. 11, 1996, pp. 1539-1553.

[9] G. Aubert, and P. Kornprobst, Mathematical Problems in Image Processing PDEs and the Calculus of Variations (second edition), New York, USA, Springer, 2006.

[10] M. Lysaker, A. Lundervold, and X. C. Tai, "Noise removal using fourth-order partial differential equation with application to medical magnetic resonance images in space and time", IEEE Transactions on Image Processing, Vol. 12, No. 12, 2003, pp. 1579-1590.

[11] M. Lysaker, S. Osher, and X. C. Tai, "Noise removal using smoothed normals and surface fitting", IEEE Transactions on Image Processing, Vol. 13, Iss. 10, 2004, pp. 1345-1357.

[12] W. Ling-hua, and Y. Jia-hong, "Image de-noising model of combining curvelet and LLT model", Computer Engineering and Applications, Vol. 46, Iss.25, 2010, pp. 168-170.

[13] D. Hong-wei, "Combined model for image denoising based on partial differential equations" Computer Engineering and Applications, Vol. 44, Iss. 20, 2008, pp. 198-201.

[14] A. N. Tikhonov, and V.Y. Arsenin, Solutions of ILL-Posed Problem, Washington, D.C., USA, Winston and Sons, 1977.

[15] S. Kalavathy, and R. M.Suresh, "Analysis of image denoising using wavelet coefficient and adaptive subband thresholding technique", International Journal of Computer Science Issues, Vol. 8, No. 1, 2011, pp. 166-172.

[16] A. Adeli, F. Tajeripoor, M. J. Zomorodian, and M. Neshat, "Comparison of the fuzzy-based wavelet shrinkage image denoising techniques", International Journal of Computer Science Issues, Vol. 9, No. 3, 2012, pp. 211-216.

[17] R. Volar, and A. Tayal, "Image restoration using thresholding techniques on wavelet coefficients", International Journal of Computer Science Issues, Vol. 8, No. 3, 2011, pp. 400-404.

[18] S. Karthik, V. K. Hemanth, K.P. Soman, V. Balaji, Sachin Kumar S, and M. S. Manikandan, "Directional total variation filtering based image denoising method", International Journal of Computer Science Issues, Vol. 9, No. 9, 2012, pp. 161-170.

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