Heterogeneous Vehicle Routing Problem with profits Dynamic solving by Clustering Genetic Algorithm

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Abstract

The transport problem is known as one of the most important combinatorial optimization problems that have drawn the interest of many researchers. Many variants of these problems have been studied in this decade especially the Vehicle Routing Problem. The transport problem has been associated with many variants such as the Heterogeneous Vehicle Routing Problem and dynamic problem.

We propose in this study dynamic performance measures added to HVRP that we call "Heterogeneous Vehicle Routing Problem with Dynamic profits" (HVRPD), and we solve this problem by proposing a new scheme based on a clustering genetic algorithm heuristics that we will specify later.

Computational experiments with the benchmark test instances confirm that our approach produces acceptable quality solutions compared with previous results in similar problems in terms of generated solutions and processing time. Experimental results prove that the method of clustering genetic algorithm heuristics is effective in solving the HVRPD problem and hence has a great potential.

Keywords:*The Heterogeneous Vehicle Routing Problem, dynamic problems, genetic algorithm heuristics, k-means clustering.*

1. Introduction

Within the wide scope of logistics management, transportation plays a central role and is a crucial activity in the delivery of goods and services. The transport problem is one of the mainly essential combinatorial optimization problems that have taken the interest of several researchers. Huge research efforts have been devoted to the study of logistic problems and thousands of papers have been written on many variants of this problem such as Traveling Salesman Problem (TSP), Vehicle Routing Problem (VRP), supply chain management (SCM) and so on [6].

The VRP is one of the most studied combinatorial optimization problems and it consists of the optimal design of routes to be used by a fleet of vehicles to satisfy the demands of customers. In general, the number of vehicles used in VRP is a variable decision.

A variant of VRP has a dynamic nature and can be modelled as Dynamic Combinatorial Optimization Problem (DCOP). This variant has been used in many researches on transportation problems. The results of these studies show that the complex real transportation problems are dynamic and change over time in terms of their objective functions, decision variables and constraints. This implies that the optimal solution might change at any time due to the changes in the transport environment.

Many other related problems are associated with VRP such as the Heterogeneous Fleet Vehicle Routing Problem (HVRP). The HVRP differs from the classical VRP in that it deals with a heterogeneous fleet of vehicles having various capacities and both fixed and variable costs. Therefore, the HVRP involves designing a set of vehicle routes, each starting and ending at the depot, for a heterogeneous fleet of vehicles which services a set of customers with specific demands. Each customer is visited only once, and the total demand of a route should not exceed the loading capacity of the vehicle assigned to it. The routing costs of a vehicle is the sum of its fixed costs and a variable costs incurred proportionately to the travel distance.

The objective is to minimize the total of such routing costs. The number of available vehicles of each type is assumed to be unlimited [5].

In the literature, three HVRP versions have been studied. The first one was introduced by Golden and al. [10], in which variable costs are uniformly given over all vehicle types with the number of available vehicles assumed to be unlimited for each type. This version is also called the Vehicle Fleet Mix (VFM) [19], the Fleet Size and Mix VRP

[11] or the Fleet Size and Composition VRP [8]. This version that we are consider in this paper.

The second version considers the variable costs, dependent on the vehicle type, which are ignored in the first version. This version is referred to as the HVRP [7], the VFM with variable unit running costs [18].

The third one, called the VRP with a heterogeneous fleet of vehicles [20] or the heterogeneous fixed fleet VRP [21], generalizes the second version by limiting the number of available vehicles of each type.

The remaining part of the paper is organized as follows: the next section introduces the notation used throughout the paper and describes the variants of heterogeneous VRPs studied in the literature then we introduce our problem: heterogeneous vehicle routing problem with dynamic profits associated to the customer. In section 3 we propose a hybrid algorithm that we call “Clustering Genetic Algorithm”. Finally, Section 4 reviews the experimental results.

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2. Heterogeneous Vehicle Routing Problem with Dynamic profits associated to the customer priority

In this part, we describe the formulation of HVRP then we explain our problem and we propose a new mathematic formulation for HVRPD.

2.1 The Heterogeneous Vehicle Routing Problem: formulation of the problem

The Heterogeneous Vehicle Routing Problem is defined as follows [2]. Let's $G=(V,A)$ is a directed graph where $V=\{v_0,v_1,...,v_n\}$ is the set of $n+1$ nodes and $A=\{(v_i,v_j):v_iv_j\in V, i\neq j\}$ is the arc set. Node v_0 represents a depot on which a fleet of vehicles is based, while the remaining node set $V'=V \setminus \{0\}$ vertices corresponds to n cities or customers.

Each customer $v_i\in V'$ has a non negative demand q_i. The vehicle fleet is composed of m different vehicle types, with $M=\{1,...,m\}$. For each type $k\in M$, m_k vehicles are available at the depot, which have a capacity equal to Q_k. Each vehicle type is also associated with a fixed cost, equal to F_k and a variable cost G_k per distance unit.

The number of vehicles of each type is assumed to be unlimited. With each arc (v_i,v_j) is associated a distance or cost C_{ij}. The HVRP aims at designing a set of vehicle routes, each starting and ending at the depot, visiting each customer only once, limiting the total demand of a route to the loading capacity of the vehicle assigned to it, and minimizing the total cost of the route [7].

All the HVRP variants are NP-hard as they include the classical VRP as a special case. All the presented studies have thus focused on developing heuristic algorithms as a substitute for exact algorithms. They can be generally grouped into two kinds: classical heuristics [11], [18], [16], frequently derived from the classical VRP heuristics, and meta-heuristics like the tabu search method [7], [21]. For more information, see [18] and [21] for a review of HVRP variants.

In our approach, we propose a new problem that we called HVRPD it is to introduce the priority of the customers with dynamic profits in the formulation of HVRP.

2.2 Proposed problem DHVRP

Practically, “dynamism” can be attributed to several factors, such as new customer order, cancellation of old demands… Demands of customers that have been studied by Taillard [20], Gendreau and al. [7], Ismail and Irhamah [12] have already used “dynamism” in their study. Other factors are stochastic like travel times between customers, customers to be visited, locations of customers, capacities of vehicles, and number of vehicles available have been dealt with by Prins [15], Renaud [16]…

In the static vehicle routing problem, information is assumed to be known, including all attributes of the customers such as the geographical location, the service time and all the details about the customer's demand.

However, in the dynamic applications, information on the problem is not completely known in advance.

The problem is said dynamic when not all information related to the planning of the routes is known by the planner when the routing process begins. Besides, the information is progressively reveled to the decision maker and it is likely to change according to the dynamic nature of the transport environment.
For the problem under study, we will combine HVRP and dynamic VRP including the priority of customers. We will present in this paper, a mathematical model for HVRPD.
Hence, the mathematical formulation is similar to that of general HVRP [9] with the introduction of the dynamism and priorities by clients.
Note $z_{ir} = 1$ if the client i is at the rank r and 0 otherwise.

The modeling of the problem is similar to standard HVRP [8] and is as follows:

$$Min \sum_{k\in M} F_k \sum_{j\in V'} x_{0j}^k + \sum_{k\in M}\sum_{i,j\in V} c_{ij}x_{ij}^k - \sum_{i,r\in V} q_i g_{ir} z_{ir}$$

Subject to

$$\sum_{k\in M}\sum_{i\in V} x_{ij}^k = 1 \ \forall j\in V' \quad (1)$$

$$\sum_{i\in V} x_{iv}^k - \sum_{i\in V} x_{vj}^k = 0 \ \forall v\in V', \forall\ k\in M \quad (2)$$

$$\sum_{j\in V'} x_{0j}^k \le p_k \ \forall k\in M \quad (3)$$

$$\sum_{i\in V} y_{ij} - \sum_{i\in V} y_{ji} = q_j \ \forall j\in V' \quad (4)$$

$$q_j x_{ij}^k \le y_{ij} \le (Q^k - q_i)x_{ij}^k \forall i,j\in V, i\neq j, \quad (5)$$

$$y_{ij} \ge 0, \ \forall i,j\in V, \ \forall i\neq j \quad (6)$$

$$x_{ij}^k \in \{0,1\}, \ \forall i,j\in V, i\neq j, \ \forall\ k\in M \quad (7)$$

With the addition of other constraints to introduce the principle of dynamic gains are:

$$\sum_{r=1}^{n} rz_{ir} + 1 - n\left(1 - x_{ij}^k\right) \le \sum_{r=1}^{n} rz_{jr} \quad \forall\ i,j\in V, i\neq j \quad (8)$$

$$\sum_{r=1}^{n} rz_{jr} \le \sum_{r=1}^{n} rz_{ir} + 1 + n\left(1 - x_{ij}^k\right) \quad \forall\ i,j\in V, i\neq j \quad (9)$$

And $$\sum_{r=1}^{n} z_{ir} = 1 \quad \forall\ i\in V \quad (10)$$

Given the random character of the needs q_i, changing continually, and availability of vehicles, we assume a probability distribution according to the history of the customer's request.
In the above formulation, constraint (1) ensures that the customer is visited only once and the constraint (2) ensures the continuity of a tour that means: if a vehicle visits a customer i, it marks the end for this customer and the start of the next one. The maximum number available for each type of vehicles is imposed by constraint (3). Constraint (4) states that the difference between the quantities of goods that a vehicle carries raised before and after visiting a customer is equal to the demand of this customer. Constraint (5) ensures that the vehicle capacity does never exceed the demand of customers.
The two constraints (6) and (7) show that the quantity transported must be positive and the value of x_{ij}^k can take only 0 or 1.
For the two new constraints (8) and (9), we introduce the profits: the lower the rank of the client is the higher his gains are.
The ranking of client can replace the principle of time window and it makes it easier to implement and reduce the computation time with the same efficiency. It will be beneficial to have a small place.
In this paper, and to solve this problem, we will develop hybrid approaches that incorporate the best features of the metaheuristic "genetic algorithms" and the method of classification "k-means" that we called "Clustering Genetic Algorithm". So, we develop five steps, that we will mention later, that begin by clustering method and finish with finding the optimal tours at the same time, they satisfy the demand of customers.

3. Clustering Genetic Algorithm

The growing importance of combinatorial optimization problems requires the search for efficient algorithms to find optimal and near optimal solutions. Clustering methods are important tool to analyze and manage data. They are used to classify data into homogeneous classes. Inspired by the high performance of Genetic Algorithms (GA); we propose to introduce a clustering process into a genetic algorithm in order to design an efficient tool to deal with complex optimization data. In this domain, some proposed works have appeared recently [4].
To solve the HVRPD, taking into account the new capacity constraints and priorities for customers, a hybrid algorithm will be initialized named Clustering Genetic Algorithm (CGA), which must make appropriate changes on the structural representation of the elements of the problem and reset, and that crossing operations with regard to the specifics of the problem in five steps.
First, client groups within the capacity of the vehicles are formed. They are based on clustering techniques of *k*-means. Then, a traveling salesman problem is solved in each group by the GA. In the third phase, the vehicles are assigned to tours obtained within the constraints of capacity. They should also aim at minimizing the costs. The fourth phase is to introduce two variables that show the

degree of certainty with whish all the requirements must be satisfied by a given vehicle k.
Finally, the fifth phase aims to identify the customers who are satisfied, taking into account the levels of priority, by which groups of customers are aggregated into super-priority nodes.

3.1 Clustering Method

Clustering can be considered the most important unsupervised learning problem; so, as every other problem of this kind, it deals with finding a structure in a collection of unlabeled data.
A loose definition of clustering could be "the process of organizing objects into groups whose members are similar in some way". A cluster is therefore a collection of objects which are "similar" between them and are "dissimilar" to the objects belonging to other clusters [22].
There are many clustering methods available, and each of them may give a different grouping of a dataset. The choice of a particular method will depend on the type of output desired, The known performance of method with particular types of data, the hardware and software facilities available and the size of the dataset.
In general, clustering methods may be divided into two categories based on the cluster structure which they produce. The non-hierarchical or unsupervised methods divide a dataset of N objects into M clusters, with or without overlap and the hierarchical clustering algorithm is based on the union between the two nearest clusters. The beginning condition is realized by setting every datum as a cluster. After a few iterations it reaches the final clusters wanted.

The specific method used in our approach is the k–means. It's one of the simplest unsupervised learning algorithms that solve the will-known clustering problem. k-means consists in partitioning the data into groups, or "clusters". These clusters are obtained by positioning "centroids" in regions of space that have the largest number of population.
Each observation is then assigned to the closest prototype. Each class therefore contains observations that are nearer to a prototype than any other.
Then, the centroids are positioned by an iterative procedure (the centroid of the classes is calculated: the weight of a center of gravity is equal to the sum of the weights of the class of individual forms) which leads progressively to their final stable position.
The weight of a center of gravity aims at minimizing an objective function, in this case a square error function. The objective function [22]:

$$J = \sum_{j=1}^{k} \sum_{i=1}^{n} \left\| x_i^{(j)} - c_j \right\|^2$$

where $\left\| x_i^{(j)} - c_j \right\|^2$ is a chosen distance measure between a data point $x_i^{(j)}$ and the centroid c_j, is an indicator of the distance of the n data points from their respective cluster center.
At this state, the final number of classes will be defining.
k-means is an algorithm that has been adapted to many problem domains. As we are going to see, it is a good candidate for extension to work with fuzzy feature vectors.

Once the classes are identified in the final partition of the data space, it is possible, to run the Genetic Algorithm cluster by cluster.

3.2 Genetic Algorithm

The Genetic Algorithm (GA) starts from an initial population of candidate solutions or individuals and proceeds for a certain number of iterations until one or more stopping criteria is /are satisfied. This evolution is directed by a fitness measure function that assigns to each solution (represented by a chromosome) a quality value.

Once the population is evaluated, the selection operator chooses which chromosomes in the population will be allowed to reproduce. The stronger an individual is, the greater chance of contributing to the production of new individuals it has.
The new individuals inherit the properties of their parents and they may be created by crossover (the probabilistic exchange of values between chromosomes) or mutation (the random replacement of values in a chromosome). Continuation of this process through a number of generations will result in a group of solutions with better fitness in which optimal or near-optimal solutions can be found [1].
When the size of the problem is becoming wider, genetic algorithms find a difficulty in identifying the optimal solution. Nevertheless, the question is why sacrifying an approach that can have a good opportunity to develop a new area of expertise and keep the problem in touch with reality?
The purpose of the implementation of genetic algorithms is to get a hamiltonian cycle for the clusters identified. This means the genetic algorithms are applied cluster by cluster and the outcome of this step is the tours $T_1, T_2, ..., T_n$ for the clusters $K_1, K_2, ..., K_n$.

3.3 Clustering Genetic Algorithm

In our approach CGA, we have first formed the clusters using *k*-means then we have built the route by solving TSP problems for each tour. So, we have divided our vehicle routing problem (search space) into areas to locate the task of GA. Therefore, we will use a two-phase method. It has shown in many applications [3], [16] of the vehicle routing problem and its generalizations that the set of routes is the most important phase of the algorithm.

The two-phase method uses the principle of "cluster first - route second" [5]. In the case of TSP, such a method comprises, first, splitting the set of vertices (customers) into sub-classes (sets) of customers, and then issuing a routing process on each class to have a solution.

The general outline of the two-phase method is as follows:

General diagram of the two-phase method

step 1 : $i = 0$, initialize *iMax*.

step 2 : Repeat steps 3 through 4 until $i = iMax$.

step 3 : Generate a *Phase1* partition into classes of all customers using a specific method of classification.

step 4 : *Phase2*
Apply a routing procedure on each class of peaks generated by step 3,
i++.

The diversification of class divisions allows the two phase method to provide several solutions to the problem by applying alternatively the classification procedures and routing.

After clustering and genetic algorithm, the next step is to find the optimal tour. Therefore, we must find the complete tour by eliminating some edges and joining clusters. We will take the adjacent centroids and choose the shorts edges that we have to eliminate and repeat the same operation until we have only one tour and ultimately find the minimal cost of the whole tour.
Once the tours have been completed, we will assign the vehicles in a way that makes the fixed costs associated with the trucks the lowest possible.
The next step of our algorithm is to introduce two variables that allow to identify the degree of certainty with which a given vehicle *k* will meet all requirements must be satisfied.

Finally, the fifth phase aims to identify the satisfied customers, taking into account the levels of priorities in which groups of customers are aggregated into super-priority nodes.

4. Experiment results

In our research, we include the k-means method to classify data into identical classes in order to facilitate the procedure of the genetic algorithm and it tackles the travelling salesman problem with many cities. The performances of the heuristics are tested, with a few exceptions, on two sets of benchmark instances: the first one consists of the 20 VFM instances [10], and the second contains 8 instances only with fixed costs [19].
The following table illustrates the results found by comparing the clustering genetic algorithm (Clu.GA) with a classical genetic algorithm (Cla.GA) (two-function point mutation by inversion), including the minimum length of the tour and finally the ratio = average / optimal for different instances of benchmarks from TSP library LIB see table 1.

Table 1: Examples of instances per cluster

Instances	***Best know solutions***	***Minimal tour***		***Ratio***	
		Cla. GA	***Clu. GA***	***Cla. GA***	***Clu. GA***
A 280	2579	4500 .447	3954.456	1.744	1.533
Brg 180	1950	3160.621	2267.540	1.620	1.162
Ch 130	6110	15818	8853.430	2.588	1.448
Ch 150	6528	19140	10436	2.931	1.598
Eil 101	629	1954.734	1409.157	3.106	2.240
Eil 76	538	2349.051	1172.204	4.39	2.178
Eil 51	426	1651.796	808.400	3.875	1.890
Gil 262	2378	4460.268	3789.650	1.875	1.563
Fl 417	11861	33140	29436	2.931	1.598
Fl 1400	20127	34500	25354	1.744	1.833

Results from the experiment in table 1 show the advantages of the proposed algorithm concerning the quality of the solution (the ratio is: Average/optimale) .
The proposed algorithm, Clu. GA, is compared with standard genetic algorithms considering a set of instances from the TSP LIB. Our experimental results prove the efficiency of the proposed algorithm in many instances when compared to the best-known solutions of Taillard [19].
We have also tested these instances with Lindo and CPLEX algorithm that gives optimal solutions but only for small instances. Both tests have taken much time to find these solutions.

Even though, our algorithm gives solutions that are not optimal but it has proved to be more efficient then Lindo and CPLEX ones in terms of processing time.

The resolution of the problem shows that our algorithm needs less time to solve big instances.

Let be the following instances with a number of different cities. We compare our algorithm with [12], [19] and [10].

The results are summarized in the following table:

Table 2. Comparable instances

Inst.	*n*	*Osman and Salhi*	*Taillard*	*Golden*	*Our Avreage Solution*	*Time*
3	20	965	961.03	961.03	965	2.65
4	20	6445	6437.33	6437.33	6536	5.98
5	20	1009	1008.59	1007.05	1016	3.95
6	20	6516	6516.47	6516.47	6596	6.97
13	50	2437	2413.78	2408.62	2785	6.95
14	50	9125	9119.03	9119.28	9150	6.74
15	50	2600	2586.37	2586.37	2960	7.16
16	50	2745	2741.5	2741.5	2830	4.29
17	75	1762	1747.24	1749.5	2086	6.14
18	75	2412	2373.63	2381.43	3360	5.12
19	100	8685	8661.81	8675.16	9642	7.95
20	100	4166	4047.55	4086.76	4213	6.96

The results presented in table 2 show that our algorithm produces solutions that are comparable quality to those of Osman and Salhi[12] and Taillard [19] and it improves the computational time (in secondes). Comparisons with other authors are the only way to evaluate our solutions because it is a new problem in the literature.

Our average results found by Clu.GA are close to those of Osman and Salhi [12], but they are bit lower than those of Taillard [19].

Table 3. Our average results

Ins	*n*	*Veh.*	*Vehicles nature*	*Vehicles capacity*	*Our average value*	*Fixed costs*	*Time*
3	21	5	5	20-30-40-70-120	965	20-35-50-120-225.	2.65
4	21	5	3	60-80-150	6536	1000-1500-3000.	5.98
5	21	5	5	20-30-40-70-120	1016	20-35-50-120-225.	3.95
6	21	5	3	60-80-150	6596	1000-1500-3000.	6.97
13	51	8	6	20-30-40-70-120-200	2785	20-35-50-120-225-400.	6.95
14	51	8	3	120-160-300	9150	1000-1500-3500.	6.74
15	51	5	3	50-100-160	2960	100-250-450.	7.16
16	51	5	3	40-80-140	2830	100-200-400.	4.29
20	101	8	3	60-140-200	4213	100-300-500.	6.96

Our algorithm has better performance on instances with fixed costs. Here, our average solution values are lower than those of Taillard in most cases, but they allow a good classification of vehicles and their fixed costs and also our algorithm allocate customers in a better way than that presented by Golden (See table 3).

5. Conclusions

In this paper, we suggest a new mathematic model for HVRPD and we solve this problem by proposing a hybrid metaheuristic based on GA and k-means clustering.

The various experiments give indicate on the sensitivity of the algorithms in different configurations. They can exhibit the advantage of using a definition of compromise proposed to be specifically incorporated into an evolutionary algorithm. In this study, we have investigated a class of dynamic optimization problems. These algorithms need to be evaluated on other classes of problems including other variants and constraints.

We have used the clustering approach as an additional means to control intensification in order to obtain robust algorithms able to produce good solutions.

The results are encouraging especially in relation to processing time if compared with similar problems and similar algorithms.

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