

# BER Analysis of 3x3 MIMO Spatial Multiplexing under AWGN & Rician Channels for Different Modulation Techniques

Anuj Vadhera<sup>1</sup>, Lavish Kansal<sup>2</sup>

<sup>1</sup> School of Electronics Engineering, Lovely Professional University  
Phagwara, Punjab, India

<sup>2</sup> Department of Electronics and Communication, Lovely Professional University  
Phagwara, Punjab, India

## Abstract

The multiple input multiple output (MIMO) antenna system provides very promising gain in capacity without increasing the use of spectrum, throughput, and power consumption. This is also less sensitive to fading, hence leading to a breakthrough in the data rate of wireless communication systems. In such systems, the antenna properties as well as the multipath channel characteristics play a key role in determining communication performance.

This paper proposes the analysis and performance of Spatial Multiplexing technique of MIMO system. Here different fading channels like AWGN and Rician are used for analysis purpose. Moreover we analyzed the technique using high level modulations (i.e. M-PSK for different values of M). Detection algorithms used are Zero- Forcing and Minimum mean square estimator.

**Keywords:** Multiple Input Multiple Output (MIMO), Spatial Multiplexing (SM), Additive White Gaussian Noise (AWGN), Zero-Forcing (ZF), Minimum Mean Square Estimator (MMSE).

## 1. Introduction

The idea of using multiple receive and multiple transmit antennas has emerged as one of the most significant technical breakthroughs in modern wireless communications. As a result, MIMO is considered a key technology for improving the throughput of future wireless broadband data systems. MIMO is the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. A core idea in MIMO system is space-time signal processing in which time (the natural dimension of digital communication data) is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. A key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user.

Two typical approaches in the MIMO systems are to provide diversity gain as in space-time coding (STC) or to allow spatial multiplexing (SM). While STC systems are capable of improving system reliability through coding across space and/or time, SM systems are capable of increasing data transmission rate through spatial multiplexing. In this paper, we focus on the SM technique.

Spatial multiplexing is a transmission technique in MIMO wireless communication to transmit independent and separately encoded data signals, so-called streams, from each of the multiple transmit antennas. Therefore, the space dimension is reused, or multiplexed, more than one time. If the transmitter is equipped with  $N_t$  antennas and the receiver has  $N_r$  antennas, the maximum spatial multiplexing order (the number of streams) is

$$N_s = \min(N_t, N_r) \quad (1)$$

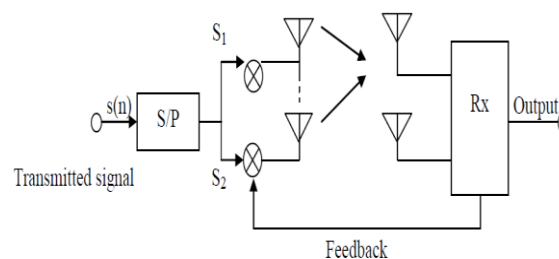


Fig. 1 Spatial Multiplexing Concept

MIMO spatial multiplexing achieves this by utilizing the multiple paths and effectively using them as additional channels to carry data such that receiver receives multiple data at the same time. The tenet in spatial multiplexing is to transmit different symbols from each antenna and the receiver discriminates these symbols by taking advantage of the fact that, due to spatial selectivity,

each transmit antenna has a different spatial signature at the receiver. This allows an increased number of information symbols per MIMO symbol. In any case for MIMO spatial multiplexing, the number of receiving antennas must be equal to or greater than the number of transmit antennas such that data can be transmitted over different antennas. Therefore the space dimension is reused or multiplexed more than one time. The data streams can be separated by equalizers if the fading processes of the spatial channels are nearly independent. Spatial multiplexing requires no bandwidth expansion and provides additional data bandwidth in multipath radio scenarios [2].

The general concept of spatial multiplexing can be understood using MIMO antenna configuration. In spatial multiplexing, a high data rate signal is split into multiple lower data rate streams and each stream is transmitted from a different transmitting antenna in the same frequency channel. If these signals arrive at the receiver antenna array with different spatial signatures, the receiver can separate these streams into parallel channels thus improving the capacity. Thus spatial multiplexing is a very powerful technique for increasing channel capacity at higher SNR values. The maximum number of spatial streams is limited by the lesser number of antennas at the transmitter or receiver side. Spatial multiplexing can be used with or without transmit channel knowledge.

## 2. Multiple Input Multiple Output (MIMO)

Multiple antennas can be used at the transmitter and receiver, an arrangement called a MIMO system. A MIMO system takes advantage of the spatial diversity that is obtained by spatially separated antennas in a dense multipath scattering environment. MIMO systems may be implemented in a number of different ways to obtain either a diversity gain to combat signal fading or to obtain a capacity gain. Generally, there are three categories of MIMO techniques. The first aims to improve the power efficiency by maximizing spatial diversity. Such techniques include delay diversity, STBC and STTC. The second class uses a layered approach to increase capacity. One popular example of such a system is V-BLAST suggested by Foschini et al. [2] where full spatial diversity is usually not achieved. Finally, the third type exploits the knowledge of channel at the transmitter. It decomposes the channel coefficient matrix using SVD and uses these decomposed unitary matrices as pre- and post-filters at the transmitter and the receiver to achieve near capacity [3].

### 2.1 Benefits of MIMO system

MIMO channels provide a number of advantages over conventional Single Input Single Output (SISO)

channels such as the array gain, the diversity gain, and the multiplexing gain. While the array and diversity gains are not exclusive of MIMO channels and also exist in single-input multiple-output (SIMO) and multiple-input single-output (MISO) channels, the multiplexing gain is a unique characteristic of MIMO channels. These gains are described in brief below

### Array Gain

Array gain denotes the improvement in receive signal-to-noise ratio (SNR) that results from a coherent combining effect of the information signals. The coherent combining may be realized through spatial processing at the receive antenna array and/or spatial pre-processing at the transmit antenna array. Formally, the array gain characterizes the horizontal shift of the error probability versus transmitted or received power curve (in a log-log scale), due to the gain in SNR

### Spatial Diversity Gain

Diversity gain is the improvement in link reliability obtained by receiving replicas of the information signal through (ideally independent) fading links. With an increasing number of independent copies, the probability that at least one of the signals is not experiencing a deep fade increases, thereby improving the quality and reliability of reception. A MIMO channel with  $n_T$  transmit and  $n_R$  receive antennas offers potentially  $n_T n_R$  independently fading links and, hence, a spatial diversity order of  $n_T n_R$ . Formally, the diversity gain characterizes the slope of the error probability versus transmitted or received power curve (in a log-log scale) in the high-SNR regime.

### Spatial Multiplexing Gain

MIMO systems offer a linear increase in data rate through spatial multiplexing, i.e., transmitting multiple, independent data streams within the bandwidth of operation. Under suitable channel conditions, such as rich scattering in the environment, the receiver can separate the data streams. Furthermore, each data stream experiences at least the same channel quality that would be experienced by a SISO system, effectively enhancing the capacity by a multiplicative factor equal to the number of substreams. In general, the number of data streams that can be reliably supported by a MIMO channel coincides with the minimum of the number of transmit antennas  $T$  and the number of receive antennas  $n_R$ , i.e.,  $\min\{n_T; n_R\}$ .

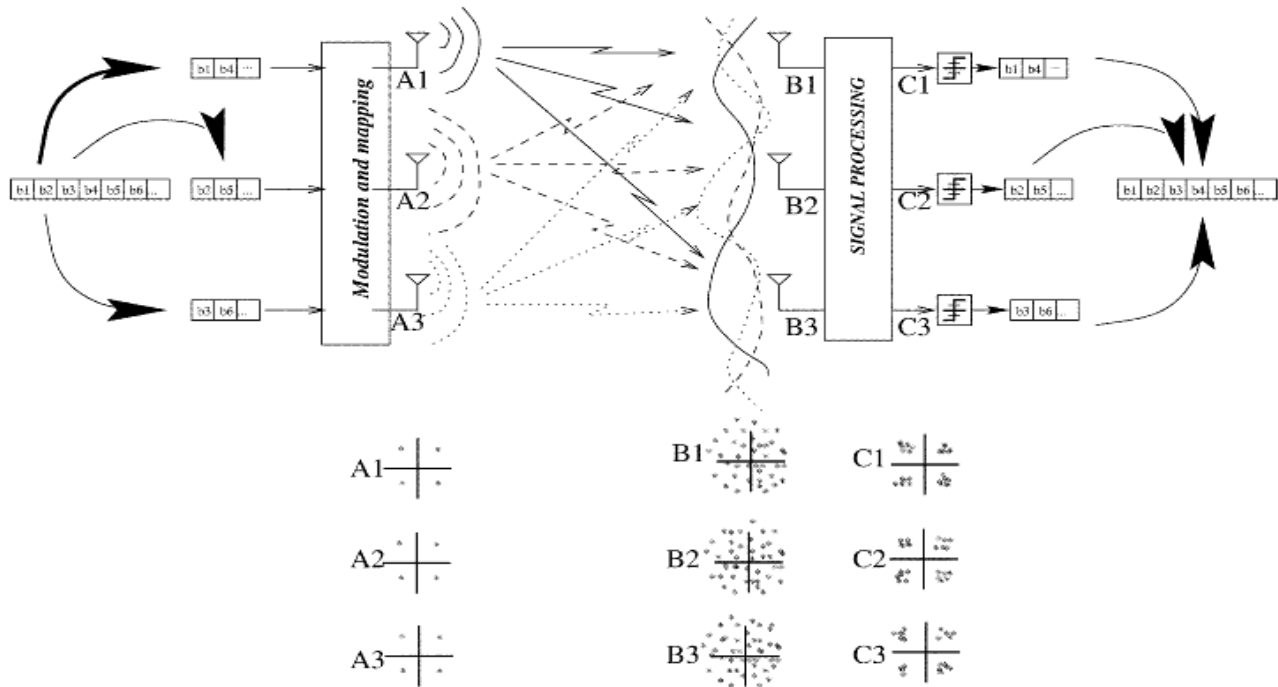


Fig. 2 Basic spatial multiplexing (SM) scheme with three Transmit and three Receive antennas  $A_i$ ,  $B_i$ , and  $C_i$  represent symbol constellations for the three inputs at the various stages of transmission and reception.

## 2.2 Channels

Wireless transmission uses air or space for its transmission medium. The radio propagation is not as smooth as in wire transmission since the received signal is not only coming directly from the transmitter, but the combination of reflected, diffracted, and scattered copies of the transmitted signal.

### 2.2.1 AWGN Channel

Additive white Gaussian noise (AWGN) channel is universal channel model for analyzing modulation schemes. In this model, the channel does nothing but add a white Gaussian noise to the signal passing through it. This implies that the channel's amplitude frequency response is flat (thus with unlimited or infinite bandwidth) and phase frequency response is linear for all frequencies so that modulated signals pass through it without any amplitude loss and phase distortion of frequency components. Fading does not exist. The only distortion is introduced by the AWGN. AWGN channel is a theoretical channel used for analysis purpose only. The received signal is simplified to:

$$r(t) = x(t) + n(t) \quad (2)$$

where  $x(t)$  is the transmitted signal and  $n(t)$  is additive white Gaussian noise.

### 2.2.1 Rician Channel

When there is line of sight, direct path is normally the strongest component goes into deeper fade compared to the multipath components. This kind of signal is approximated by Rician distribution. As the dominating component run into more fade the signal characteristic goes from Rician to Rayleigh distribution. The received signal can be simplified to:

$$r(t) = x(t) * h(t) + n(t) \quad (3)$$

where  $h(t)$  is the random channel matrix having Rician distribution and  $n(t)$  is the additive white Gaussian noise. The Rician distribution is given by:

$$P(r) = \frac{r^2}{\sigma^2} e^{-\frac{r^2 + A^2}{\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) \quad \text{for } (A \geq 0, r \geq 0) \quad (4)$$

where  $A$  denotes the peak amplitude of the dominant signal and  $I_0[\cdot]$  is the modified Bessel function of the first kind and zero-order.

## 2.3 Modulation

Modulation is the process of mapping the digital information to analog form so it can be transmitted over the channel. Consequently every digital communication

system has a modulator that performs this task. Closely related to modulation is the inverse process, called demodulation, done by the receiver to recover the transmitted digital information [4].

Modulation of a signal changes binary bits into an analog waveform. Modulation can be done by changing the amplitude, phase, and frequency of a sinusoidal carrier. There is several digital modulation techniques used for data transmission.

### 2.3.1 Phase Shift Keying

Phase Shift Keying is a digital modulation scheme that conveys data by changing or modulating, the phase of a reference signal (the carrier wave). In M-ary PSK modulation, the amplitude of the transmitted signals was constrained to remain constant, thereby yielding a circular constellation. Modulation equation of M-PSK signal is:

$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi i}{M}\right) \quad (5)$$

$i=0,1,\dots,M$

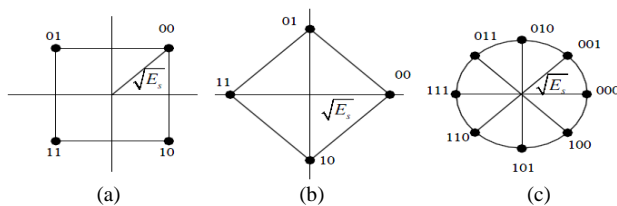


Fig. 3 Constellation Diagrams of M-PSK  
 a) QPSK (b) QPSK (c) 8-PSK

## 2.4 Detection Techniques

There are numerous detection techniques available with combination of linear and non-linear detectors. The most common detection techniques are ZF, MMSE and ML detection technique. The generalized block diagram of MIMO detection technique is shown in Figure 4.

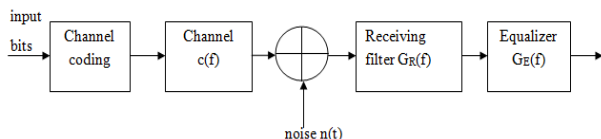


Fig. 4 Block Diagram of system with equalizer

### 2.4.1 Zero Forcing detection

The ZF is a linear estimation technique, which inverse the frequency response of received signal, the inverse is taken for the restoration of signal after the

channel. The estimation of strongest transmitted signal is obtained by nulling out the weaker transmit signal. The strongest signal has been subtracted from received signal and proceeds to decode strong signal from the remaining transmitted signal. ZF equalizer ignores the additive noise and may significantly amplify noise for channel.

The basic Zero force equalizer of 2x2 MIMO channel can be modeled by taking received signal  $y_1$  during first slot at receiver antenna as:

$$r_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (6)$$

The received signal  $y_2$  at the second slot receiver antenna is:

$$r_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (7)$$

Where  $i=1, 2$  in  $x_i$  is the transmitted symbol and  $i=1, 2$  in  $h_{i,j}$  is correlated matrix of fading channel, with  $j$  represented transmitted antenna and  $i$  represented receiver antenna, is the noise of first and second receiver antenna. The ZF equalizer is given by:

$$W_{ZF} = (H^H)^{-1}H^H \quad (8)$$

Where  $W_{ZF}$  is equalization matrix and  $H$  is a channel matrix. Assuming  $M_R \geq M_T$  and  $H$  has full rank, the result of ZF equalization before quantization is written as:

$$y_{ZF} = (H^H H)^{-1}H^H y \quad (9)$$

### 2.4.2. Minimum Mean Square Estimator (MMSE)

Minimum mean square error equalizer minimizes the mean –square error between the output of the equalizer and the transmitted symbol, which is a stochastic gradient algorithm with low complexity. Unlike a ZF equalizer, an MMSE equalizer maximizes the signal–to distortion ratio by penalizing both residual ISI and noise enhancement. Instead of removing ISI completely, an MMSE equalizer allows some residual ISI to minimize the overall distortion. Compared with a ZF equalizer, an MMSE equalizer is much more robust in presence of deepest channel nulls and noise. Most of the finite tap equalizers are designed to minimize the mean square error performance metric but MMSE directly minimizes the bit error rate. The channel model for MMSE is same as ZF [13],[14]. The MMSE equalization is

$$W_{MMSE} = \text{arg} \min_{x,n} E_{x,n} [\|x - x^{\wedge}\|^2] \quad (10)$$

Where is  $W_{MMSE}$  equalization matrix,  $H$  channel correlated matrix and  $n$  is channel noise

$$y_{MMSE} = H^H(HH^H + n_o I_n)^{-1}y \quad (11)$$

### 3. Results and Discussions

The system discussed above has been designed and results are shown in the form of SNR vs. BER plot for different modulations and different channels. Here antenna configuration 3x3 is analysed using ZF and MMSE detection techniques. Analyses have been done for two channels AWGN and Rician channel using MMSE and ZF detection algorithms.

#### 3.1 Using ZF detection algorithm:

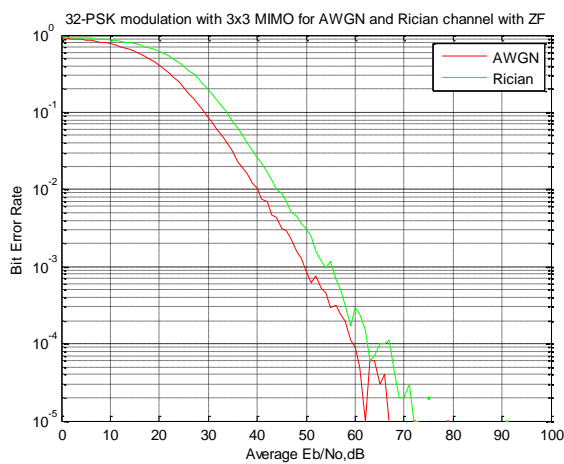


Fig. 5(a)

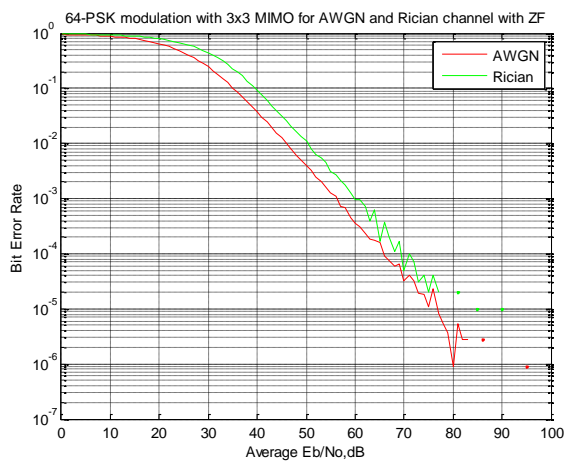


Fig 5(b)

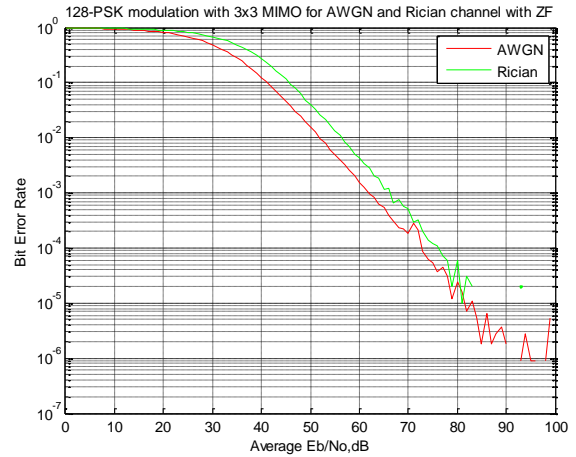


Fig. 5(c)

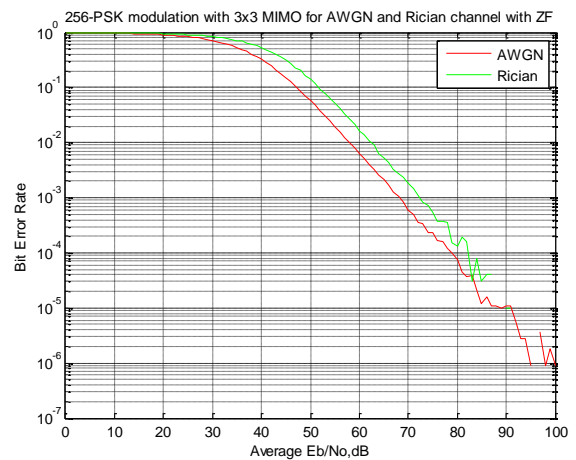


Fig. 5(d)

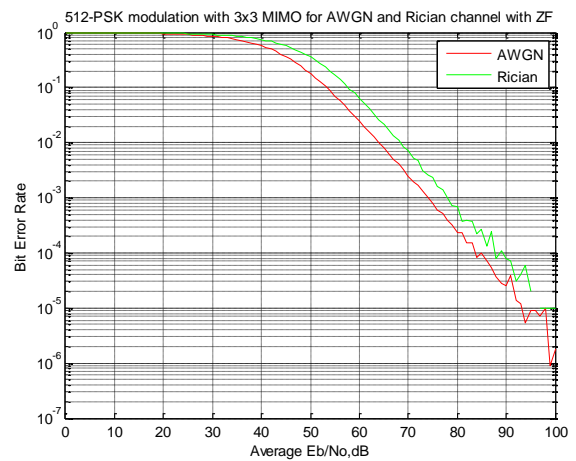


Fig. 5(e)



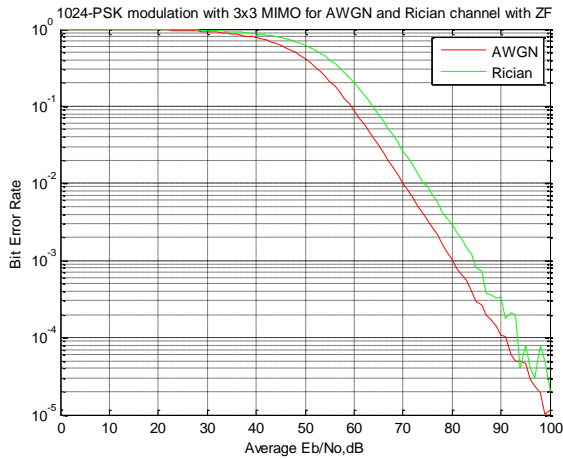


Fig. 5(f)

Fig. 5 BER vs. SNR plots over AWGN & Rician channel for SM technique using 3x3 MIMO using ZF Equalization  
 a) 32 PSK b) 64 PSK c) 128 PSK d) 256 PSK e) 512 PSK  
 f) 1024 PSK

Table 1. Comparison of different Modulation Techniques for AWGN & Rician Channel for 3x3 MIMO Spatial Multiplexing using ZF Equalization

Modulations	Rician Channel	AWGN Channel	Improvement
32-PSK	62dB	59 dB	3 dB
64-PSK	70dB	66 dB	4 dB
128-PSK	77dB	73 dB	4 dB
256-PSK	82 dB	79 dB	3 dB
512-PSK	88 dB	84 dB	4 dB
1024-PSK	93 dB	90 dB	3 dB

Table 1 presents that at 32-PSK, 256-PSK, 1024-PSK there is an improvement of 3dB and at 64-PSK, 128-PSK and 512-PSK there is an improvement of 4dB at BER  $10^{-4}$ . Hence table 1. shows the improvement in terms of decibels shown by proposed system employing SM technique for 3x3 MIMO system for different modulation schemes over different environments (channels).

### 3.2 Using MMSE detection algorithm:

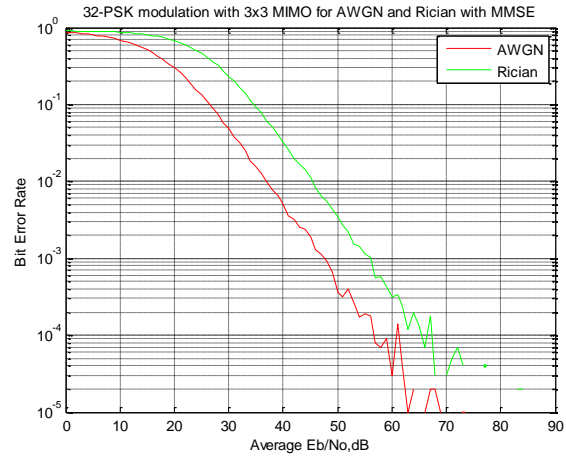


Fig. 6(a)

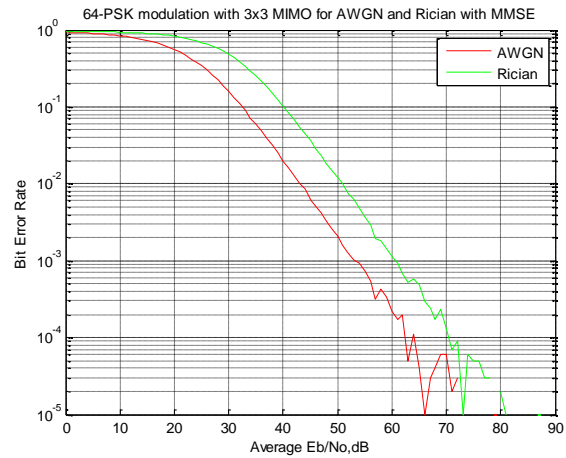


Fig. 6(b)

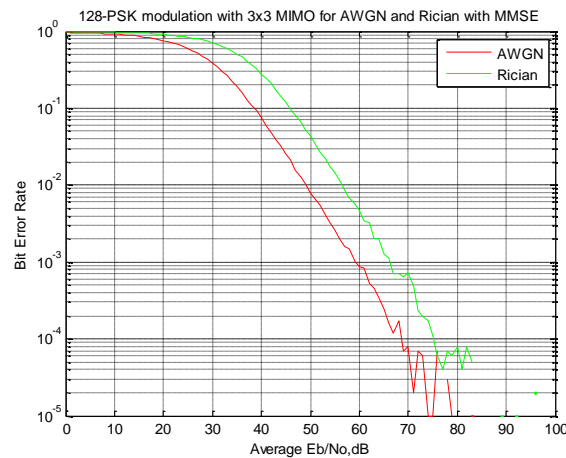


Fig. 6(c)

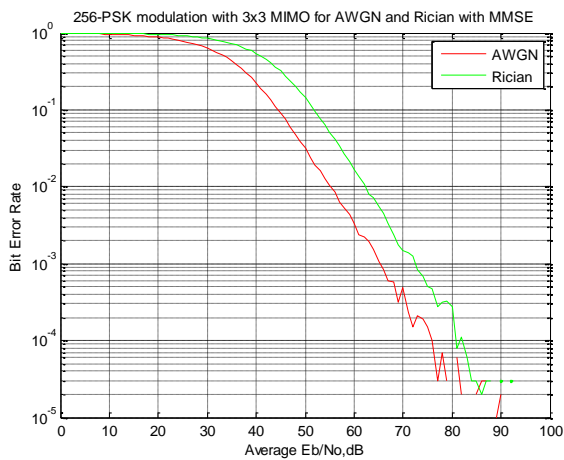


Fig. 6(d)

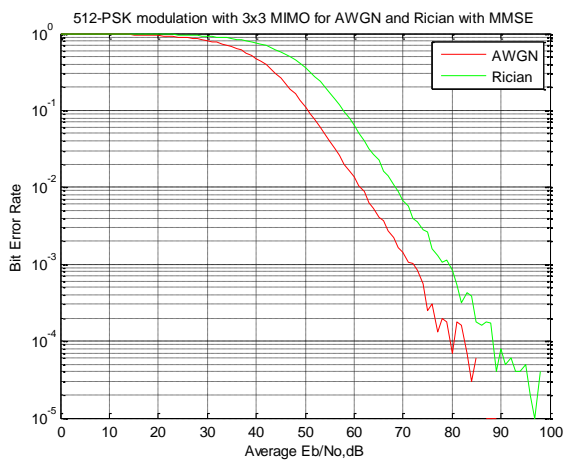


Fig. 6(e)

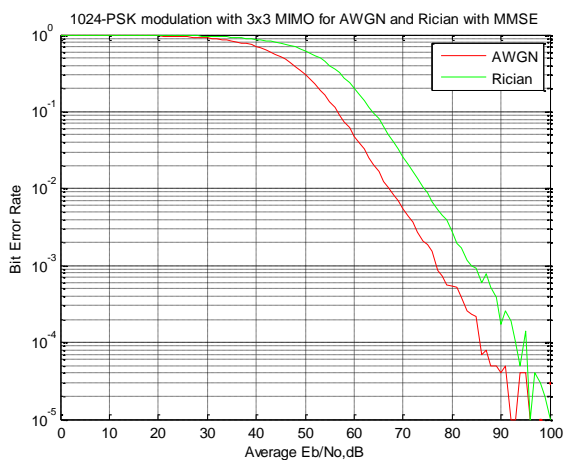


Fig. 6(f)

Fig. 6 BER vs. SNR plots over AWGN & Rician channel for SM technique using 3x3 MIMO using MMSE Equalization a) 32 PSK b) 64 PSK c) 128 PSK d) 256 PSK e) 512 PSK f) 1024 PSK

Table 2. Comparison of different Modulation Techniques for AWGN & Rician Channel for 3x3 MIMO Spatial Multiplexing using MMSE Equalization

Modulations	Rician Channel	AWGN Channel	Improvement
32-PSK	65dB	62 dB	3 dB
64-PSK	70 dB	64 dB	6 dB
128-PSK	75 dB	69 dB	6 dB
256-PSK	80 dB	71 dB	9 dB
512-PSK	85 dB	77 dB	8 dB
1024-PSK	95 dB	86 dB	9 dB

Table 2 depicts that at 64-PSK, 128-PSK there is an improvement of 6dB, at 256-PSK, 1024-PSK there is an improvement of 9dB and at 512-PSK there is an improvement of 8dB. Hence table 2 shows improvement in terms of decibels shown by proposed system employing SM technique for 3x3 MIMO system for different modulation schemes over different environments (channels) at BER  $10^{-4}$ .

#### 4. Conclusions

In this paper, an idea about the performance of the MIMO-SM technique at higher modulation levels and for 3x3 antenna configuration using different signal detection techniques is presented. MIMO-SM technique can be implemented using higher order modulations to achieve large data capacity. But there is a problem of BER (bit error rate) which increases as the order of the modulation increases. The solution to this problem is to increase the value of the SNR so, that the effect of the distortions introduced by the channel will also goes on decreasing, as a result of this, the BER will also decrease at higher values of the SNR for high order modulations.

Several different diversity modes are used to make radio communications more robust, even with varying channels. These include time diversity (different timeslots and channel coding), frequency diversity (different channels, spread spectrum, and OFDM), and also spatial diversity. Spatial diversity requires the use of multiple antennas at the transmitter or the receiver end. Multiple antenna systems are typically known as Multiple Input, Multiple Output systems (MIMO). Multiple antenna

technology can also be used to increase the data rate (spatial multiplexing) instead of improving robustness. In future, we can make a single integrated circuit that uses both methods combination.

### Acknowledgments

With deep sense of gratitude, I express my sincere thanks to my esteemed and worthy guide Mr. Lavish Kansal, Assistant Professor, Electronics and Communication Engineering Department, Lovely Professional University, Phagwara, for his valuable advice, motivation, guidance, encouragement, efforts, timely help and the attitude with which he solved all of my queries regarding thesis work.

### References

- [1] H. Jiang and P. A. Wilford, "A hierarchical modulation for upgrading digital broadcasting systems," IEEE Transaction on Broadcasting, vol. 51, pp. 222-229, June 2005.
- [2] P. W. Wolniansky, G. J. Foschini, G. D. Golden and R. A. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel," In Proceeding of International symposium on Signals, Systems Electronics, pp. 259-300, October 1998.
- [3] J. Ha, A. N. Mody, J. H. Sung, J. Barry, S. Mclaughlin and G. L. Stuber, "LDPC coded OFDM with Alamouti/SVD diversity technique," IEEE Journal on Wireless Personal Communication, Vol. 23, Issue 1, pp. 183-194, Oct. 2002.
- [4] P. S. Mundra, T. L. Singal and R. Kapur, "The Choice of A Digital Modulation Schemes in A Mobile Radio System", In proceedings of IEEE Vehicular Technology Conference, Issue 5, pp 1-4, (Secaucus, NJ)1993.
- [5] P. Liu & Il-Min Kim, "Exact and Closed-Form Error Performance Analysis for Hard MMSE-SIC Detection in MIMO Systems", IEEE Transactions on Communication, Vol. 59, no. 9, September 2011.
- [6] P. Sanghoi & L. Kansal, "Analysis of WiMAX Physical Layer Using Spatial Multiplexing Under Different Fading Channels", SPIJ, Vol.(6), Issue(3), 2012.
- [7] C. Wang & E. K. S. Au, R. D Murch, W. H. Mow & V. Lau, "On the Performance of the MIMO Zero-Forcing Receiver in the Presence of Channel Estimation Error", IEEE Transactions on Wireless Communication, Vol. 6, no. 3, 2007.
- [8] X. Zhang, Y. Su & G. Tao, "Signal Detection Technology Research of MIMO-OFDM System", 3rd International Congress on Image and Signal Processing, pp 3031-3034, 2010.
- [9] I. Ammu & R. Deepa, "Performance Analysis of Decoding Algorithms in multiple antenna systems", IEEE, pp 258-262, 2011.
- [10] H. B. Voelcker, "Phase-shift keying in fading channels", In IEEE Proceeding on Electronics and Communication Engineering, Vol. 107, Issue 31, pp 31-38, 1960.
- [11] D. S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems", IEEE Transaction on Communication, Vol. 48, pp. 502-513, 2000.
- [12] G. J. Foschini, K. Karakayali, and R. A. Valenzuela, "Coordinating multiple antenna cellular networks to achieve

enormous spectral efficiency," Communications, IEE Proceedings, Vol. 153, pp. 548-555, 2006.

[13] J. S. Thompson, B. Mulgrew and Peter M. Grant, "A comparison of the MMSE detector and its BLAST versions for MIMO channels", IET seminar on Communication System from Concept to Implementation, pp. 1911-1916, 2001.

[14] X. Zhang and Sun-Yuan Kung, "Capacity analysis for parallel and sequential MIMO equalizers", IEEE Transaction on Signal processing, Vol. 51, pp. 2989-3002, 2003.

**Anuj Vadhera** was born in Fazilka. She received her B.Tech degree in Electronics and Communication Engineering from Punjab Technical University, Jalandhar, in 2009, and pursuing M.Tech degree in Electronics and communication engineering from Lovely Professional University, Phagwara, India. Her research interests include MIMO systems, cognitive radios and wireless systems.

**Lavish Kansal** was born in Bathinda. He received his B.Tech degree in Electronics and Communication Engineering from PTU, Jalandhar in 2009 and M.E. degree in Electronics and Communication Engineering from Thapar University, Patiala in 2011. He is working as Assistant Professor in the department of Electronics and communication Engineering, Lovely Professional University, Phagwara, India. He has published 15 papers in International journals. His research area includes Digital Signal Processing, Digital Communication & Wireless Communication.