

Handwriting and Hand Drawing Velocity Modeling by Superposing Beta Impulses and Continuous Training Component

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Abstract

We present in this paper a new strategy of handwriting or hand drawing velocity modeling Based on the Beta theory. The introduced approach aims to improve the interpretability of the dynamic profile model, reduce the data redundancy, and ameliorate the features accuracy. Indeed, we showed that the curvilinear velocity of handwritten or hand drawn trajectory can be rebuilt by superposing two components; consecutive Beta impulses representing its amplitude alternation imposed by the trajectory curvature variation and a velocity gain part of persistent pen carrying called "continuous training component" interpreting the learning level of the hand drawing faculty and the control of neuromuscular pulses synchronization. The proposed strategy was validated by the reduction of the error of curvilinear velocity fitting and the improvement of the recognition rate of Arabic handwriting characters represented by its model features vector.

Keywords: Online hand drawing – Beta theory – velocity profile modeling – beta impulse – continuous training component.

1. Introduction

The velocity modeling is a useful stage in various on-line hand drawn trajectory analyses and recognition process as: handwriting and hand drawn symbols recognition, writer identification, signature authenticity verification, biomechanical system diagnostic, ... Different approaches are addressed for the hand drawing velocity modeling. From the oscillatory model of Hollerbach [8], to the Beta elliptic model of Bezine [5] and Kherallah et al [3], preceded by the delta-lognormal model of Plamondon et al [6, 7] and the Beta model of Alimi et al [1, 2], the hand movement drawing speed was always approximated by an association of bell shaped function. We propose in this paper a new strategy of handwriting and hand drawing curvilinear speed modeling based on the Beta approach. It suggests that for a planed hand movement trajectory, the effect of the overlapped neuromuscular subsystems actions on the velocity profile appear as an optimal arrangement to ensure a continuous component to the movement which is

superposed with another impulsive component to meet the trajectory curvature variation.

The optimization level result as a compromise between increasing the average level of the continuous training component and the constraint of drawing precision.

Thus, to improve the interpretability of the dynamic profile model, the proposed approach decomposes the velocity profile in two superposed components: beta impulses to represent the trajectory curvature variation and continuous training component to describe the control and training level of the hand drawing faculty.

In order to validate the proposed strategy, we tested its pertinence for the velocity profile modeling of on-line handwriting character and signature by considering the error of velocity profile rebuilding and their rate of recognition.

In the second section of this article, we study the dynamics characteristics of the hand drawing movement. The third section presents the classic overlapped Beta impulse approach for hand movement velocity modeling. Then we introduce the strategy of dynamic profile modeling by Beta function and continuous training component. Finally, we conclude with the result of the model pertinence tests and perspectives.

2. Velocity Profile Modeling by Beta Impulse Overlapping

A handwriting or hand drawing movement, which is generated by neuromuscular excitations, is characterized by its velocity and trajectory profiles. Based on kinematics studies, the Beta model is proposed as modeling tool for the dynamic data of on-line hand drawing movements [1, 2, 5, 3, 7, 9, 14, 15].

3.1 Segmentation of the handwritten trajectory

The hand drawing movement, as any other driving process, is programmed partially in advance. The movements are represented and organized in the velocity

fields [6, 7, 4]. In this context, trajectory model is the result of the activation of N neuromuscular subsystems which are characterized by a standardized impulse response. According to the works of Alimi et al. [1, 2], the response of global impulse converges with a Beta curve. Curvilinear velocity $V_{\sigma}(t)$, calculated by the equation (2), represents the resulting response to the finished impulses. It is smoothed by a second order derivative filters [5, 3] :

$$V_{\sigma}(t) = \sqrt{\left(\frac{dx(t)}{dt}\right)^2 + \left(\frac{dy(t)}{dt}\right)^2} \quad (2)$$

The trajectory of the handwriting is segmented in simple movements that called strokes. The number of strokes of one script is determined by an inspection of extremums [1, 7, 5, 3] to know, the local extremas of the horizontal or vertical direction [4], the local extremas of the curvilinear velocity signal of handwriting, and its inflexion points [3] (see figure 2).

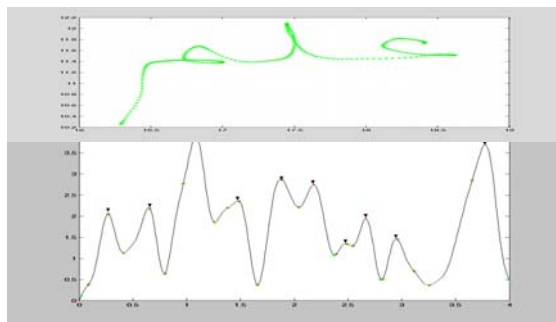


Fig. 2 Detection of the extremum points.

The inflexion points given by acceleration signal are considered and allotted to forms of Beta functions rides in the profile velocity.

3.2 Modeling of the velocity profile

The curvilinear velocity of each stroke obeys to "Beta" approach. Thus, the generation of a model for the trajectory is the algebraic result of the addition of the velocity profiles of the successive strokes (see eq 3).

$$V_{\sigma}(t) = \sum_{i=1}^n V_i(t - t_{0i}) \quad (3)$$

Consequently, the complete velocity profile, which is generated by the neuromuscular system, is described by the following Beta model :

$$V_r(t) = \sum_{i=1}^n K_i \cdot \beta(t, q, p, t_0, t_1) \quad (4)$$

With :

$$(5)$$

$$\beta(t, q, p, t_0, t_1) = \begin{cases} \left(\frac{t-t_0}{t_c-t_0}\right)^p \cdot \left(\frac{t_1-t}{t_1-t_c}\right)^q & \text{if } t \in [t_0, t_1] \\ 0 & \text{elsewhere} \end{cases}$$

Where t_0 is the starting time of Beta function, t_c is the instant when the curvilinear velocity reaches the amplitude of the inflexion point, t_1 is the ending time of Beta function checking $t_0 < t_1 \in \mathbb{R}$, and p, q are intermediate parameters, which have an influence on the symmetry of Beta shape and verifying :

$$\frac{t_c - t_0}{t_1 - t_c} = \frac{p}{q} \quad (6)$$

The shape of a symmetrical Beta signal is given by figure 3. The parameter K is the amplitude of the beta signal.

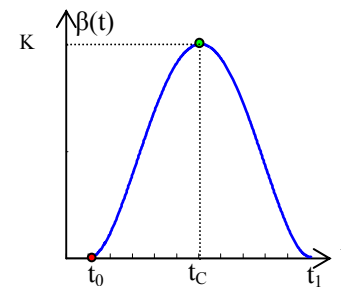


Fig. 3 Shape of a symmetrical Beta impulse function for $p = q = 2.5$

The curvilinear velocity results as the superposition of the neuromuscular finished actions with the impulsive character which is modeled by overlapping Beta impulses in the course of time (see Figures 4a and 4b).

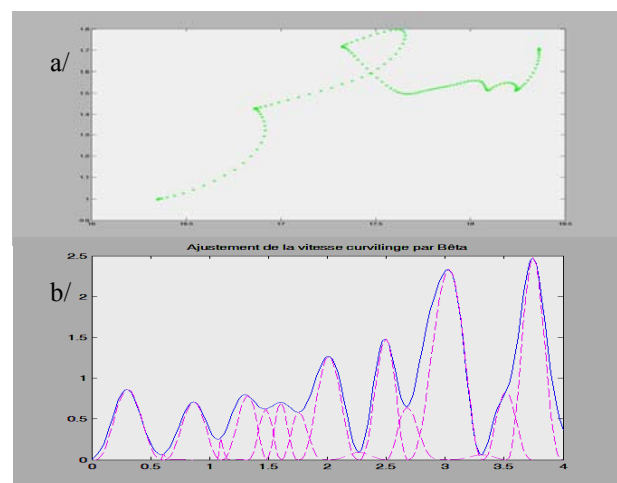


Fig. 4 Velocity signal modeling by overlapping Beta functions

4. News strategy of velocity profile Modeling

To enhance the interpretability of the Beta model, we introduce in this section a new strategy for velocity profile modeling by superposing successive beta impulses to a persistent component of pen carrying called continuous training component.

4.1 Principle

Observing the evolution of handwriting or hand drawing executed by young children (4 to 8 years), we note that they generally actuate their pencils in an impulsive and discontinuous mode. Their drawing or script displays acute forms, discontinuities and distortions [11, 12]. Their pencils velocity is cancelled at several times during a continuous line drawing. Later learned and trained to perform hand drawing (more then 9 years), their drawn trajectory becomes more continuous, cursive, faster and less acute (see Figures 5). Indeed, their control of the acceleration and braking actions becomes precise what enables them to avoid the cancellation of the trajectory velocity during a continuous line drawing by maintaining a not null component of drag [11, 12].

However, the hand drawing velocity variation keeps always a relative impulsive character which interprets the intrinsic curvature radius variation of the executed trajectory. These successive impulses are superposed to the component of continuous drag developed by training.

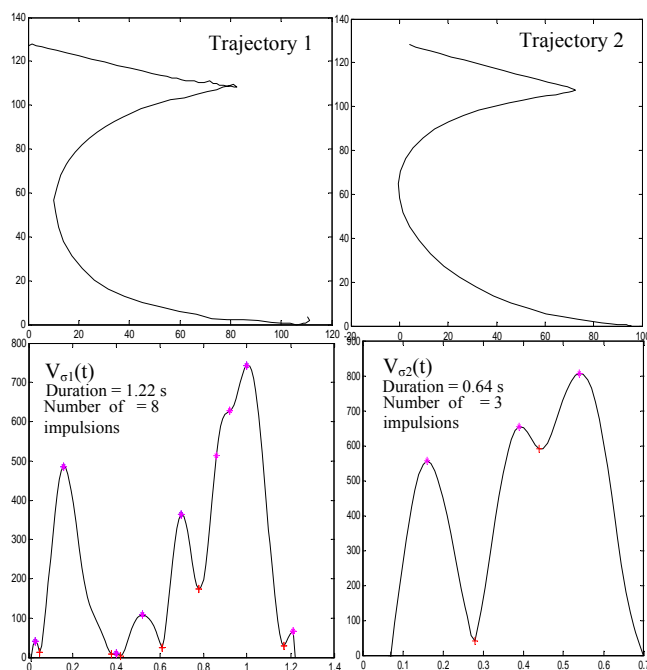


Fig. 5 samples of handwriting trajectory and corresponding velocity profile executed by two schoolchildren of respectively 8 and 13 years

4.2 Velocity profile modeling by superposing Beta impulses and continuous training component

We decompose the time axis of the profile velocity into intervals which represent cycles of acceleration, deceleration and braking. Each time interval $T=[t_0, t_1]$ is limited by a successive local minimums or double inflexion points of velocity: $V_i=V_{\sigma}(t_0)$ and $V_f = V_{\sigma}(t_1)$. During each interval, the curvilinear velocity can be fictitiously divided into two components:

- An **impulsive** component: $V_{Imp}(t)$

It is a velocity impulsion during the interval T with finished energy, engendered by a cycle of acceleration, deceleration and braking. It can be modeled by a Beta function :

$$V_{Imp}(t) = K \cdot \left(\frac{t - t_0}{t_C - t_0} \right)^p \cdot \left(\frac{t_1 - t}{t_1 - t_C} \right)^q$$

- A continuous **training** component: $V_{Tra}(t)$

It engenders the energy which allows the continuous passage (with a not null velocity) from a trajectory segment to another separated by a local minimum of curvature radius. Its variation must have the most monotonous and softest character in order to reserve the velocity impulsive character to $V_{Imp}(t)$ component.

Thus, $V_{Tra}(t)$ represents the initial velocity gain V_i to which we add the algebraic effect of a supplementary energy (of acceleration or braking) added to the finished impulse $V_{Imp}(t)$ in order to ensure the assymetry of the curvature radius variation from $R_i = R(t_0)$ to $R_f = R(t_1)$ (see Figure 6).

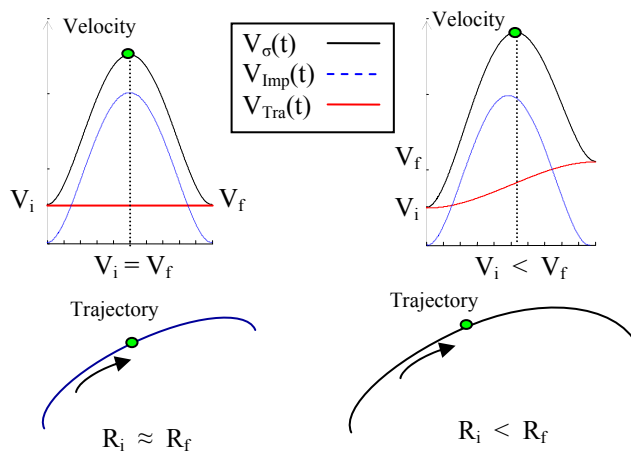


Fig. 6 Correspondence between asymmetry of curvature radius and velocity variations

The variation according to the time of the continuous training component $V_{Tra}(t)$ is given by a monotonous polynomial function of third degree :

$$V_{Tra}(t) = a \cdot \left[\frac{(t-t_0)^3}{3} - \frac{(t_1-t_0) \cdot (t-t_0)^2}{2} \right] + V_i \quad (7)$$

where $a = -6 \cdot \frac{V_f - V_i}{(t_1 - t_0)^3}$

The reconstituted curvilinear speed of tracing is obtained by the sum of its impulsive component with the component of continues drag :

$$V_R(t) = V_{Imp}(t) + V_{Tra}(t) \quad (8)$$

(see Figure 7)

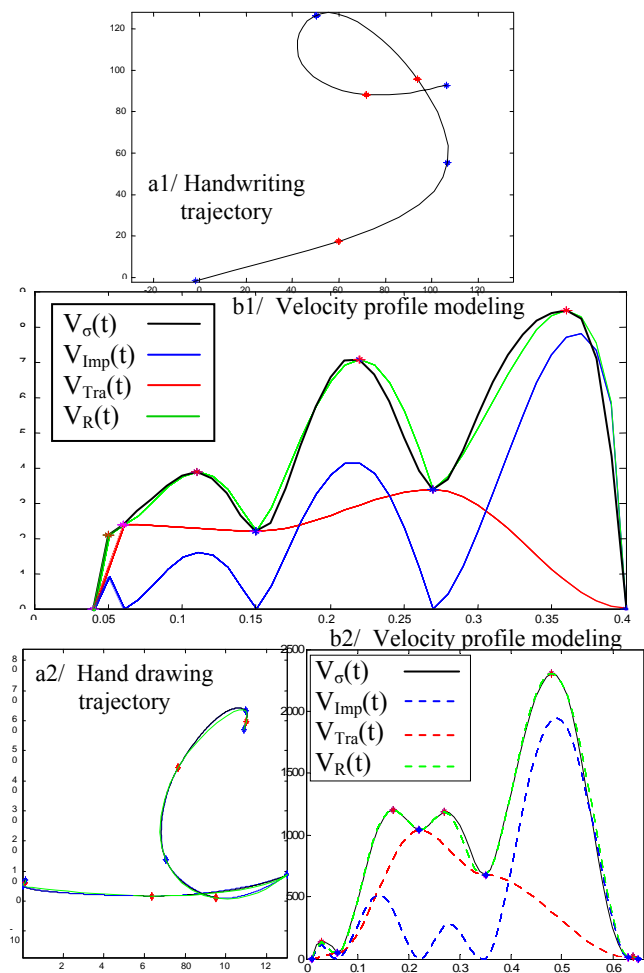


Fig. 7 Examples of velocity profile modeling by superposing successive Beta impulses and continuous training component

The parameters K_c and t_c of a given beta impulse component $V_{Imp}(t)$ are given from the original velocity profile. The t_c moment corresponds to the local maximum velocity: $\frac{dV_R(t_c)}{dt} = 0$. This leads to the following relation between p , q and t_c :

$$\frac{p}{t_c - t_0} - \frac{K \cdot q}{t_1 - t_c} = \left[\frac{a}{2} \cdot (t_1 - t_c) \cdot (t_c - t_0) \right] \quad (9)$$

4.3 Simplified variant of the Beta modeling approach

This new strategy adopts a simplified representation of the real effect of neuromuscular impulses overlapping in its various modes of implementation (see figure 8). Indeed, the resulting effect of this overlap is modeled by the superposition of consecutive and finite speed impulses modeled by Beta functions on a continuous training component representing the inferior limit of the curvilinear speed variation envelope.

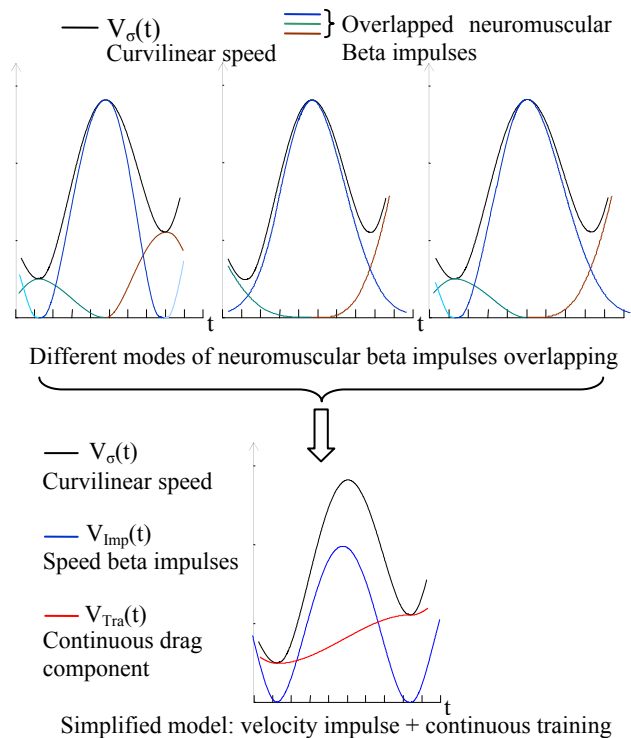


Fig. 8 Strategy of simplification of the velocity Beta modeling approach

5. Evaluation of the Modeling Approach Pertinence

5.1 Reduction of the data load

The new modeling strategy reduces the data redundancy by adopting a simple segmentation approach of not – overlapped velocity Beta strokes. In fact, a hand drawn trajectory with $n = (2 \cdot m) + 1$ successive speed extremums; $m + 1$ minimums alternated by m maximum speed, is segmented into m strokes by the new strategy when it is segmented into $(2 \cdot m) - 1$ strokes using the overlapped Beta approach. The decrease in the number of segmented strokes reduces the total number of the considered parameters despite the increment of the features vector size from 4 parameters $[K, \Delta t_1 = (t_1 - t_0), \Delta t_c = (t_c - t_0), p]$ for the overlapped Beta approach to 6 parameters $[K, \Delta t_1 = (t_1 - t_0), \Delta t_c = (t_c - t_0), p, a, V_i]$ with the new strategy where the added couple of parameters $[a, V_i]$ model the continuous training (drive) component.

Thus, to model the velocity profile of the drawing of a wavy line of 7 peaks or the handwriting of the word "mini" succeeding each one 15 minimums alternated with 14 local maximums of speed, Beta overlapped approach would use: $((2 \times 14) - 1) \text{ strokes} \times 4 \text{ parameters} = 108 \text{ parameters}$ while the strategy of Beta impulse and continuous training component would need : $14 \text{ strokes} \times 6 \text{ parameters} = 84 \text{ parameters}$

However this parameters load easing in no way affects the modeling accuracy. In fact, the proposed strategy makes it possible to simplify the problem of calculating the shape parameters p and q of the velocity Beta pulses avoiding the interdependence of this parameters for all the generated impulses due to their overlap that will be modeled locally by the continuous training component $V_{Tra}(t)$. Thus, determining the parameters of the speed Beta impulse $V_{Imp}(t)$ correspondent to a specified time interval depends only on the variation of the curvilinear speed $V_\sigma(t)$ during this interval and that of the continuous training (drive) component $V_{Tra}(t)$.

5.2 Evaluation of the Accuracy of velocity profile fitting

The reduction in the number of unknowns allows more stability for the regression system computing the values of the parameters p and q and then better precision on their estimation which leads to reduces the velocity profile reconstruction error. In fact, tests are conducted on a set composed of 5000 samples of isolated Arabic characters from the LMCA database [4], 500 signatures (see in Figure 9 an example of signature dynamic profile modeling) and 1000 handwritten symbols, to study the accuracy of the new strategy of hand movement velocity profile modeling. For each sample we calculate the rate in

percent of the fitting error of curvilinear velocity profile at each point of the trajectory:

$$Err_V(t) = \frac{|V_\sigma(t) - V_R(t)|}{V_\sigma(t)} \times 100 \quad (10)$$

$$\text{where } V_R(t) = (V_{Imp}(t) + V_{Tra}(t)) \quad (11)$$

Then the average error over the whole trajectory of the sample:

$$Ave_Err_V = \frac{\int_{t_1}^{t_2} Err_V(t) \cdot dt}{|t_2 - t_1|} \quad (12)$$

The following table compares the results of the average error of dynamic profile reconstruction obtained for different types of hand - drawn graphics respectively by the simplified strategy (superposition of Beta impulses and continuous training component) and the overlapped Beta impulses model taking the parameter q modeling the shape of the decay phase as a constant $q = 2$:

Table 1: Dynamic profile reconstruction error rates obtained by the overlapped and the simplified Beta strategies

Strategy of velocity profile modeling	Ave_Err_V		
	Isolated characters	signatures	Symbols
Overlapped Beta impulses with approximation $q = \text{constant}$	22.8 %	23.4 %	20.6 %
Superposition of Beta impulses and continuous training component	9.5 %	10.3 %	8.7 %

The results show an improvement of the accuracy of velocity profile fitting by adopting the simplified strategy of the Beta modeling.

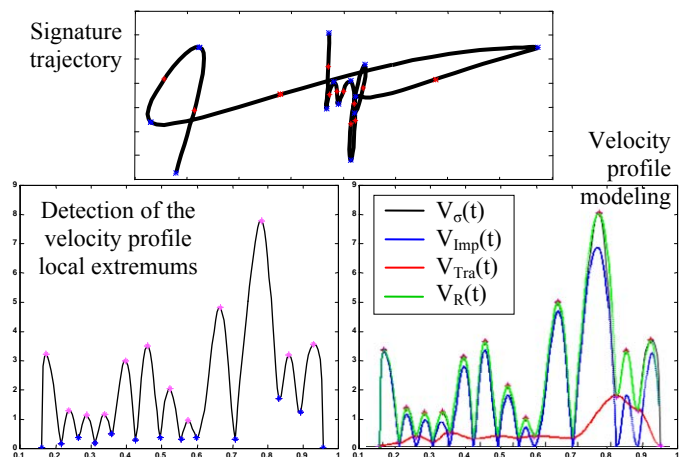


Fig. 9 Examples of Signature velocity profile modeling by superposing Beta impulses and continuous training component

5.3 Evaluation of the model discrimination power

Both strategies of dynamic profile modeling were also tested for recognition of isolated Arabic characters. These latter are grouped into categories defined by the number of strokes of the characters samples that include. We have listed eight categories for the overlapped Beta strategy from 5 to 12 strokes characters and only five categories for the simplified strategy going from 3 to 7 strokes characters.

The overall recognition system consists of a number of neural network subsystems, each of which is associated with a category of stroke number. Each subsystem is composed by neural networks type OCON (One Class One Network). The number of OCON in each subsystem is equal to the number of existing classes in the category of features. For example, the subsystem of 3 strokes is composed by four OCON, one for each following character label : letter 'ا' 'alif', letter 'ل' 'lam', letter 'ن' 'noun' and letter 'ر' 'ra'.

The recognition system learning is performed on two-thirds of the 5000 samples composing the set of on-line Arabic handwriting characters of the LMCA database [4]. The remainder third of the database is used as a test set [10, 13].

After its trajectory dynamic profile modeling, each sample is first assigned to the subsystem of category that corresponds to the number of Beta strokes composing it. Then, the activated recognition subsystem presents the features vector of the tested sample to the different OCON associated to its category. Finally, the tested sample is assigned to the character label corresponding to the OCON that maximizes its recognition rate. The following tables show the results of recognition tests for the character class: 'ا' 'Alif', 'ن' 'Noun', 'هـ' 'Haa', 'ص' 'Sad', 'و' 'Waw', 'ف' 'Fa', and 'س' 'sin', obtained using respectively the simplified strategy superposing Beta impulse and training component and the strategy of Beta impulses overlapping when fixing the parameter q as a constant equal to 2:

Table 2: Results of recognition tests obtained using the simplified strategy superposing Beta impulse and continuous training component

Category	Handwritten character	Recognition rate in (%)
3 Strokes	Alif	99.90
	Noun	82.13
4 Strokes	Haa	100
	Noun	86.45
5 Strokes	Sad	88.41
	Haa	87.50
	Waw	89.57
6 Strokes	Fa	88.93
	Fa	84.38
7 Strokes	Sin	84.79

Table 3: Results of recognition tests obtained using the strategy of Beta impulses overlapping ($q=2$)

Category	Handwritten character	Recognition rate in (%)
5 Strokes	Alif	97.78
	Noun	86.13
6 Strokes	Haa	89.12
	Noun	82.22
	Alif	83.85
	Waw	83.80
7 Strokes	Haa	87.09
	Noun	88.22
	Waw	74.62
8 Strokes	Haa	88.93
	Fa	78.04
9 Strokes	Sad	82.87
	Fa	80.60
10 Strokes	Sad	81.67
	Fa	75.34
11 Strokes	Sad	84.21
12 Strokes	Sin	82.85

The results show an improvement in the average recognition rate obtained with the implementation of the simplified strategy of the Beta model compared to that obtained by the overlapped Beta impulses approach.

6. Conclusions

We have presented in this work a new strategy of handwriting and hand drawing dynamic profile modeling based on the Beta model theory. The introduced strategy decomposes the curvilinear velocity into two components: a component of pen continuous training (drag) that simplifies the representation of the dynamic effect of neuromuscular impulses overlapping, and a second component of sequenced and not overlapped Beta impulses that represent the alternate variation of the curvilinear velocity amplitude. The proposed strategy is a simplified variant of the Beta modeling approach that reduce the data redundancy and improve the model interpretability. Its validation is proved by the reduction of the error of curvilinear velocity fitting and the amelioration of the recognition rate of Arabic handwriting characters.

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