A NEW METHOD TO SOLVE MULTI-OBJECTIVE NON-LINEAR FRACTIONAL PROGRAMMING PROBLEMS

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Abstract

The multi-objective optimization method proposed in this paper does not convert multi-objective problem into a single objective problem. The given objective functions are treated as they are. A new type of transformation known as sum of ratios of objective functions is adopted. The contribution of decision variables to all the objective functions is determined in each iteration of the proposed algorithms. Solution that is extreme to all the objectives is determined.

1.0 Introduction

Existing methods to solve separable non-linear multi-objective optimization[11] problems transform the given objective functions in one way or other and convert them into single objective optimization problem and a compromise solution is obtained. When all the objectives are important, a compromise solution may not be optimal.

In this paper, a new method to solve large scale multi objective separable non-linear programming problems is presented. The proposed method solves the multi objective optimization problem by performing a new type of transformation on objective functions.

2.0 Linear Multi objective Optimization Problem

A separable non-linear multi objective optimization problem has a number of functions which are to be extremized. A seperable non-linear multi objective optimization problem with **t** number of objective functions, m number of constraints and n number of decision variables in its general form:

$$\text{Extremize} \quad Z(X) = \begin{Bmatrix} z_1(X) \\ z_2(X) \\ \dots \\ z_t(X) \end{Bmatrix}$$

$$where \quad z_i = \phi_i(x_1, x_2 \dots x_n) + \phi_{i0} \qquad i = 1 \dots t$$

Subject to the constraints,

$$g_1(x_1, x_2 \dots x_n) \le b_1$$
$$g_2(x_1, x_2 \dots x_n) \le b_2$$
$$g_3(x_1, x_2 \dots x_n) \le b_3$$
$$\vdots$$
$$g_m(x_1, x_2 \dots x_n) \le b_m$$
$$and \quad x_j \ge 0 \qquad j = 1, 2, \dots n$$

Solution is a vector of n decision variables X=(x1, x2, ….xn) which extremizes multiple objectives.

2.1 Proposed Method

Proposed method finds solution to multi objective optimization problem by employing the idea of optimizing a sum of separable non-linear fractional functions[13]. Different orderings (permutation) of the given objective functions are formed. Problem is solved by taking different orderings of objective functions. Each ordering is used to form a sum of ratios objective function. Promising variables are identified, arranged and allowed to enter into basis for each ordering of objective functions. Promising variables are allowed to enter into basis only if they improve the objective fraction value compared to the value in previous iteration. Solution is found in each iteration. Finally, the solution corresponding to the ordering that gives the best ratio value is chosen as the optimal solution.

Let z_1, z_2, z_3 …. z_t be the objective functions. All the possible ordering of objective functions are formed. There will be ${}^tP_2$ number of different orderings. For example, with four objectives (t=4), there will be 24 unique orderings to form sum of objective fractions. Permutation set will contain 12 unique orderings after the removal of duplicate entries as shown below:

Let **tc** represent total number of permutations, i.e.

$$tc = 4p_2$$

PN_set[tc][t]={(1,2,3,4),(1,2,4,3),(1,3,2,4),(1,3,4,2),

(1,4,2,3),(1,4,3,2),(2,1,4,3),(2,4,3,1),
(3,1,4,2),(3,4,2,1),(4,1,3,2),(2,3,4,1)}

A sum of ratio of 't/2' separable non-linear fractional functions[1,5,6,] with 't' number of objective functions. The sum of ratios[7] of objective fractions are formed for each ordering with objective functions as numerator and denominator part of the fractions as follows:

$$\frac{z_1}{z_2}+\frac{z_3}{z_4},\quad \frac{z_1}{z_2}+\frac{z_4}{z_3},\quad \frac{z_1}{z_3}+\frac{z_2}{z_4},\quad \frac{z_1}{z_3}+\frac{z_4}{z_2},$$

$$\frac{z_1}{z_4}+\frac{z_2}{z_3},\quad \frac{z_1}{z_4}+\frac{z_3}{z_2},\quad \frac{z_2}{z_1}+\frac{z_4}{z_3},\quad \frac{z_2}{z_4}+\frac{z_3}{z_1},$$

$$\frac{z_3}{z_1}+\frac{z_4}{z_2},\quad \frac{z_3}{z_4}+\frac{z_2}{z_1},\quad \frac{z_4}{z_1}+\frac{z_3}{z_2},\quad \frac{z_2}{z_3}+\frac{z_4}{z_1}$$

In general, the numerator and denominator parts of the objective fractions are formed using the recurrence relation:
For every **q** varying from **1** to **tc**

$$z_{ni} = z_{PN\_SET[q][2i-1]} \qquad z_{di} = z_{PN\_SET[q][2i]}$$

$$i = 1, 2 \ldots t/2$$

2.2 Multi Objective separable non-linear Optimization Algorithm

The steps in multi objective separable non-linear optimization algorithm are as follows:

Step 1 Convert the separable non-linear function in to a linear function using the steps involved in piecewise linear approximation[12].

Step 2: Slack variables are added and the initial basic solution XB is determined. B matrix, CB and DB matrices are formed.

2a: Create the Permutation set PN_set with tc number of orderings where $tc = 4P_2$ and t' is the number of objective functions.

2b: Let q=1

2c: Let Z_q = PN_set[q][o] where o = 1,2, … t.

2d: Exchange the objective rows in M^{-1} matrix as per the permutation PN_set[q].

2e: The optimality conditions of the solution are checked and promising variables are identified and arranged using the procedures 3.7.1 and 3.7.2. The set J is constructed. Let *l* be the number of elements in set J. Let p=1 and go to step 3a.

2f: Under non optimal conditions increment p, goto step 8 without updating the value of Z.

2g: Under optimal conditions, the feasibility is verified. The process is terminated if the solution is feasible.

Step 3a: Take the variable corresponding to the p^{th} element of the set J. Let the subscript of the selected variable be j.

For each of the decision variables x_j, the values of the net evaluations of the numerator and denominator are computed using the formula:

$$\begin{bmatrix} z_{n1}-c_1 \\ z_{d1}-d_1 \\ . \\ . \\ . \\ z_{nt}-c_t \\ z_{dt}-d_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \ldots\ldots & 0 & 0 & C_{B1}B^{-1} \\ 0 & 1 & 0 & \ldots\ldots & 0 & 0 & D_{B1}B^{-1} \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ 0 & 0 & 0 & \ldots\ldots & 1 & 0 & C_{B_t}B^{-1} \\ 0 & 0 & 0 & \ldots\ldots & 0 & 1 & D_{B_t}B^{-1} \end{bmatrix} \begin{bmatrix} -c_1 \\ -d_1 \\ . \\ . \\ . \\ -c_t \\ -d_t \\ P_j \end{bmatrix}$$

3b: Compute the ratios

$$R_i = \frac{(z_{ni}-c_i)}{(z_{di}-d_i)} \; \{i = 1,2,3,..t\}$$

If all the ratios do not satisfy optimality conditions in, then go to step 8. If at least one ratio satisfies the optimality conditions, then x_j may still be promising.

3c: A new column α is constructed using the formula $\alpha = (B^{-1}P_j)$.

The solution vector $(B^{-1}P_0)$ is also obtained.

Step 4a: Procedure to find leaving vector.

If the i^{th} constraint is an equality constraint and the i^{th} basic variable is not a decision variable, find

$$r_{1j} = \min_{i=1...m} \left\{ \frac{(B^{-1}P_0)_i}{\alpha_i} \ ; \alpha_i \ > \ 0 \right\}$$

If r_{1j} exists then the ith basic variable corresponding to r_{1j} is the leaving variable, and r_{1j} is the value of the leaving variable.

i.e., $x_j = r_{1j}$, Goto step 5, else goto step 4b.

Step 4b: If the ith constraint is a lower bound constraint and the ith variable is not a decision variable, find

$$r_{2j} = \max_{i=1...m} \left\{ \frac{(B^{-1}P_0)_i}{\alpha_i} ; \alpha_i \ > \ 0 \right\}$$

If r_{2j} exists then the ith variable corresponding to r_{2j} is the leaving variable and $x_j = r_{2j}$ Go to Step 5 else go to Step 2c.

Step 4c: If the ith basic variable is a decision variable or ith constraint is an upper bound constraint, then find

$$\theta_i = \left\{ \frac{(B^{-1}P_0)_i}{\alpha_i} ; \alpha_i > 0 \right\}$$

If the ith basic variable is the feasible slack variable, then find

$$\theta_i = \left\{ \frac{(B^{-1}P_0)_i}{\alpha_i} ; \text{if } \alpha_i < 0, (B^{-1}P_0)_i < 0 \right\}$$

Compute $r_{3j} = \min_{i=1...m} \{\theta_i\}$

If r_{3j} exists, then ith variable corresponding to r_{3j} is the leaving variable and $x_j = r_{3j}$

Step 5: Compute improvement formula

$$u_j = \sum_{i=1}^{t} \frac{-(z_{ni} - c_i) * x_j + z_{ni}}{-(z_{di} - d_i) * x_j + z_{di}}$$

If value of u_j is greater than previous value of Z then go to step 6 else goto step 8.

Step 6: Computation of E matrix

The transformation matrix corresponding to the new entering and leaving variables can be obtained by using the product form of inverse. In the first step η vector is computed. Let j be the subscript of the entering variable and r be the column vector in which jth variable enters. The column vector corresponding to the jth vector is used to calculate η vector by using the relation.

$$\eta_1 = z_{n1} - c_1$$
$$\eta_2 = z_{d1} - d_1$$
$$\eta_3 = z_{n2} - c_2$$
$$\eta_4 = z_{d2} - d_2$$
$$\eta_n = z_{nt} - c_t$$
$$\eta_d = z_{dt} - d_t$$
$$\eta_{t+2} = \left[B^{-1}P_j \right]_i \quad i = 1 .. m$$

Computation of η_{new}

Since the variable corresponding to the rth row is the leaving variable (r+2t)th element in the η vector is the pivotal element. η_{new} can be obtained using the following relation:

$$\eta_{i\ new} = \frac{-\eta_{i\ old}}{\left(B^{-1}P_j\right)_r} \quad when \ \ i \neq r$$

$$\eta_{r\ new} = \frac{-\eta_{i\ old}}{\left(B^{-1}P_j\right)_r}$$

Replace the rth column of (m+2t) x (m+2t) unit matrix by the η_{new} vector. The resulting matrix is the transformation matrix E.

Step 7: Computation of M^{-1}

Compute the inverse matrix for the next iteration as

$M^{-1}_{next} = E * M^{-1}_{current}$

Step 8: Let p = p + 1. If $p \leq l$ go to step 3a else go to step 1f.

Step 9: Compute the optimal solution using the relation

$$\begin{bmatrix} z_1 \\ z_2 \\ .. \\ z_t \\ x_B \end{bmatrix} = M^{-1} \text{x} \begin{bmatrix} c_{01} \\ d_{01} \\ .. \\ c_{0t} \\ d_{0t} \\ P_0 \end{bmatrix}$$

Step 10: Compute value of Z for the current ordering of objective functions and save optimal solution and value of Z.

Step 11: If q < tc, Let q = q + 1 and go to step 2d.

Step 12: Arrange the Z value in ascending order and output the best solution set with maximum Z value.

2.3 Illustration of Multi-objective non-linear programming problem

Maximize

$$Z_1 = 5x_1^2 + 3x_2^2 + 1$$
$$Z_2 = 5x_1^2 + 2x_2^2 + 1$$
$$Z_3 = 4x_1^2 + 9x_2^2 + 2$$
$$Z_4 = 7x_1^2 + 5x_2^2 + 2$$

Subject to

$$3x_1^2 + 5x_2 \le 15$$
$$5x_1 + 2x_2^2 \le 10$$
$$\text{where } x_1, x_2 \ge 0$$

Phase I

Step 1: Conversion of Multi-objective Non-linear programming problem into sum of ratios linear fractional form. Find the maximum value t_j of the variable x_j which satisfies all the constraints.

$t_1 = The\ \max imum\ value\ of$

$$x_1 = \min\left\{\sqrt{\frac{15}{3}}, \frac{10}{5}\right\} = 2$$

$t_2 = The\ \max imum\ value\ of$

$$x_2 = \min\left\{\frac{15}{5}, \sqrt{\frac{10}{2}}\right\} = \sqrt{5}$$

Step 2: Let m_1=8 and m_2=8 where m1, m2 are the number of mesh points for the variables x_1 and x_2 respectively.

Then

$$\Delta\ for\ x_1 = \frac{t_1}{m_1} = \frac{2}{8} = \frac{1}{4}$$

$$and\ \Delta\ for\ x_2 = \frac{t_2}{m_2} = \frac{\sqrt{5}}{8}$$

The modified objective functions are

$$Z_1 = 0y_1 + \frac{5}{16}y_2 + \frac{5}{4}y_3 + \frac{45}{16}y_4 + 5y_5 + \frac{125}{16}y_6 + \frac{45}{4}y_7 + \frac{245}{16}y_8 + 20y_9 +$$
$$0y_{10} + \frac{15}{64}y_{11} + \frac{15}{16}y_{12} + \frac{135}{64}y_{13} + \frac{15}{4}y_{14} + \frac{375}{64}y_{15} + \frac{135}{16}y_{16} + \frac{735}{64}y_{17} + 15y_{18}$$

$$Z_2 = 0y_1 + \frac{5}{16}y_2 + \frac{5}{4}y_3 + \frac{45}{16}y_4 + 5y_5 + \frac{125}{16}y_6 + \frac{45}{4}y_7 + \frac{245}{16}y_8 + 20y_9 +$$
$$0y_{10} + \frac{5}{32}y_{11} + \frac{5}{8}y_{12} + \frac{45}{32}y_{13} + \frac{5}{2}y_{14} + \frac{125}{32}y_{15} + \frac{45}{8}y_{16} + \frac{245}{32}y_{17} + 10y_{18}$$

$$Z_3 = 0y_1 + \frac{1}{4}y_2 + y_3 + \frac{9}{4}y_4 + 4y_5 + \frac{25}{4}y_6 + 9y_7 + \frac{49}{4}y_8 + 16y_9 + 0y_{10} +$$
$$\frac{45}{64}y_{11} + \frac{45}{16}y_{12} + \frac{405}{64}y_{13} + \frac{45}{4}y_{14} + \frac{1125}{64}y_{15} + \frac{405}{16}y_{16} + \frac{2205}{64}y_{17} + 45y_{18}$$

$$Z_4 = 0y_1 + \frac{7}{16}y_2 + \frac{7}{4}y_3 + \frac{63}{16}y_4 + 7y_5 + \frac{175}{16}y_6 + \frac{63}{4}y_7 + \frac{343}{16}y_8 + 28y_9 +$$
$$0y_{10} + \frac{25}{64}y_{11} + \frac{25}{16}y_{12} + \frac{225}{64}y_{13} + \frac{25}{4}y_{14} + \frac{625}{64}y_{15} + \frac{225}{16}y_{16} + \frac{1225}{64}y_{17} + 25y_{18}$$

The Constraints are

$$0y_1 + \frac{3}{16}y_2 + \frac{3}{4}y_3 + \frac{27}{16}y_4 + 3y_5 + \frac{75}{16}y_6 + \frac{27}{4}y_7 +$$
$$\frac{147}{16}y_8 + 12y_9 + 0y_{10} + \frac{5\sqrt{5}}{8}y_{11} + \frac{5\sqrt{5}}{4}y_{12} + \frac{15\sqrt{5}}{8}y_{13} +$$
$$\frac{5\sqrt{5}}{2}y_{14} + \frac{25\sqrt{5}}{8}y_{15} + \frac{15\sqrt{5}}{4}y_{16} + \frac{35\sqrt{5}}{8}y_{17} + 5\sqrt{5}y_{18} = 15$$

$$0y_1 + \frac{5}{4}y_2 + \frac{5}{2}y_3 + \frac{15}{4}y_4 + 5y_5 + \frac{25}{4}y_6 + \frac{15}{2}y_7 +$$
$$\frac{35}{4}y_8 + 10y_9 + 0y_{10} + \frac{5}{32}y_{11} + \frac{5}{8}y_{12} + \frac{45}{32}y_{13} + \frac{5}{2}y_{14} +$$
$$\frac{125}{32}y_{15} + \frac{45}{8}y_{16} + \frac{245}{32}y_{17} + 10y_{18} = 10$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 = 1$$

$$y_{10} + y_{11} + y_{12} + y_{13} + y_{14} + y_{15} + y_{16} + y_{17} + y_{18} = 1$$

In a multi-objective fractional problem various sum of ratios are formed taking any two objective function to form a ratio.

Problem 2.3 $\frac{Z_2}{Z_1}+\frac{Z_3}{Z_4}$ Initial Solution

	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10	y11	y12	y13	y14	y15	y16	y17	y18	RHS
Z_2	0	$-\frac{5}{16}$	$-\frac{5}{4}$	$-\frac{45}{16}$	–5	$-\frac{125}{16}$	$-\frac{45}{4}$	$-\frac{245}{16}$	–20	0	$-\frac{5}{32}$	$-\frac{5}{8}$	$-\frac{45}{32}$	$-\frac{5}{2}$	$-\frac{125}{32}$	$-\frac{45}{8}$	$-\frac{245}{32}$	–10	1
Z_1	0	$-\frac{5}{16}$	$-\frac{5}{4}$	$-\frac{45}{16}$	–5	$-\frac{125}{16}$	$-\frac{45}{4}$	$-\frac{245}{16}$	–20	0	$-\frac{15}{64}$	$-\frac{15}{16}$	$-\frac{135}{64}$	$-\frac{15}{4}$	$-\frac{375}{64}$	$-\frac{135}{16}$	$-\frac{735}{64}$	–15	1
$\frac{Z_2}{Z_1}$	0	1	1	1	1	1	1	1	1	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	1
Z_3	0	$-\frac{1}{4}$	–1	$-\frac{9}{4}$	–4	$-\frac{25}{4}$	–9	$-\frac{49}{4}$	–16	0	$-\frac{45}{64}$	$-\frac{45}{16}$	$-\frac{405}{64}$	$-\frac{45}{4}$	$-\frac{1125}{64}$	$-\frac{405}{16}$	$-\frac{2205}{64}$	–45	2
Z_4	0	$-\frac{7}{16}$	$-\frac{7}{4}$	$-\frac{63}{16}$	–7	$-\frac{175}{16}$	$-\frac{63}{4}$	$-\frac{343}{16}$	–28	0	$-\frac{25}{64}$	$-\frac{25}{16}$	$-\frac{225}{64}$	$-\frac{25}{4}$	$-\frac{625}{64}$	$-\frac{225}{16}$	$-\frac{1225}{64}$	–25	2
$\frac{Z_3}{Z_4}$	0	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	0	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{9}{5}$	1
$\frac{Z_2}{Z_1}+\frac{Z_3}{Z_4}$	0	$\frac{11}{7}$	$\frac{11}{7}$	$\frac{11}{7}$	$\frac{11}{7}$	$\frac{11}{7}$	$\frac{11}{7}$	$\frac{11}{7}$	$\frac{11}{7}$	0	$\frac{37}{15}$	$\frac{37}{15}$	$\frac{37}{15}$	$\frac{37}{15}$	$\frac{37}{15}$	$\frac{37}{15}$	$\frac{37}{15}$	$\frac{37}{15}$	2
$C1$	0	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{27}{16}$	3	$\frac{75}{16}$	$\frac{27}{4}$	$\frac{147}{16}$	12	0	$\frac{5\sqrt{5}}{8}$	$\frac{5\sqrt{5}}{4}$	$\frac{15\sqrt{5}}{8}$	$\frac{5\sqrt{5}}{2}$	$\frac{25\sqrt{5}}{8}$	$\frac{15\sqrt{5}}{4}$	$\frac{35\sqrt{5}}{8}$	$5\sqrt{5}$	15
$C2$	0	$\frac{5}{4}$	$\frac{5}{2}$	$\frac{15}{4}$	5	$\frac{25}{4}$	$\frac{15}{2}$	$\frac{35}{4}$	10	0	$\frac{5}{32}$	$\frac{5}{8}$	$\frac{45}{32}$	$\frac{5}{2}$	$\frac{125}{32}$	$\frac{45}{8}$	$\frac{245}{32}$	10	10
$C3$	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1
$C4$	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

SOLUTION TABLE

Sl.No	Objective	Solution		Objective Function Value
		X_1	X_2	
1.	$\frac{Z_1}{Z_2}+\frac{Z_3}{Z_4}$	0	$\sqrt{5}$	$\frac{930}{319}$
2.	$\frac{Z_2}{Z_1}+\frac{Z_3}{Z_4}$	0	$\sqrt{5}$	$\frac{1049}{432}$
3.	$\frac{Z_1}{Z_2}+\frac{Z_4}{Z_3}$	0	$\sqrt{5}$	$\frac{1049}{517}$
4.	$\frac{Z_2}{Z_1}+\frac{Z_4}{Z_3}$	0	$\sqrt{5}$	$\frac{949}{297}$
5.	$\frac{Z_1}{Z_3}+\frac{Z_2}{Z_4}$	2	0	$\frac{28}{15}$
6.	$\frac{Z_1}{Z_3}+\frac{Z_4}{Z_2}$	0	$\sqrt{5}$	$\frac{1445}{517}$
7.	$\frac{Z_3}{Z_1}+\frac{Z_2}{Z_4}$	0	$\sqrt{5}$	$\frac{1445}{432}$
8.	$\frac{Z_3}{Z_1}+\frac{Z_4}{Z_2}$	0	$\sqrt{5}$	$\frac{949}{176}$
9.	$\frac{Z_1}{Z_4}+\frac{Z_2}{Z_3}$	2	0	$\frac{1008}{540}$
10.	$\frac{Z_1}{Z_4}+\frac{Z_3}{Z_2}$	2	0	$\frac{109}{42}$
11.	$\frac{Z_4}{Z_1}+\frac{Z_2}{Z_3}$	2	0	$\frac{981}{378}$
12.	$\frac{Z_4}{Z_1}+\frac{Z_3}{Z_2}$	0	$\sqrt{5}$	$\frac{1049}{176}$

The Value of the ratio $\frac{Z_4}{Z_1}+\frac{Z_3}{Z_2}$ is 5.960227 which is maximum, and the best solution obtained.

TABLE I

	Sign of z_{nj}	Sign of z_{dj}	Sign of $z_{nj}-c_j$	Sign of $z_{dj}-d_j$	Sign of R_j	Condition for Improvement
1	Positive	Positive	+	+	+	Positive $R_j < Z_{nj} / Z_{dj}$
2	Positive	Positive	-	-	+	Positive $R_j > Z_{nj} / Z_{dj}$
3	Positive	Positive	+	-	-	No Improvement
4	Positive	Positive	-	+	-	**1.1 Negative $R_j < Z_{nj} / Z_{dj}$**
5	Negative	Positive	+	+	+	No Improvement (i) Provided there is no change of sign for Z_{dj} (ii) Check for condition Number 16 if there is change in sign of Z_{dj}
6	Negative	Positive	-	-	+	Positive $R_j > Z_{nj} / Z_{dj}$
7	Negative	Positive	+	-	-	Negative $R_j > Z_{nj} / Z_{dj}$
8	Negative	Positive	-	+	-	Negative $R_j < Z_{nj} / Z_{dj}$
9	Positive	Negative	-	-	+	No improvement (i) Provided there is no change of sign for Z_{dj} (ii) Check for condition number 16 if there is change in sign of Z_{dj}.
10	Positive	Negative	-	+	-	Negative $R_j > Z_{nj} / Z_{dj}$
11	Positive	Negative	+	-	-	Negative $R_j < Z_{nj} / Z_{dj}$
12	Positive	Negative	+	+	+	Positive $R_j > Z_{nj} / Z_{dj}$
13	Negative	Negative	-	-	+	Positive $R_j < Z_{nj} / Z_{dj}$
14	Negative	Negative	+	-	-	Negative $R_j < Z_{nj} / Z_{dj}$
15	Negative	Negative	-	+	-	**1.2 No Improvement**
16	Negative	Negative	+	+	+	Positive $R_j > Z_{nj} / Z_{dj}$

Conclusion:

A new algorithm for solving a multi objective non-linear optimization problems with 't' number of objectives is presented. Optimal solution is determined by exploring different orderings of objective functions playing the roles of numerators and denominators of objective fractions. In this way the multi objective optimization problem is approached in a new pattern.

Variables are allowed to enter only if they improve the value of Z (sum of ratios of objectives). Promisibility is checked by calculating the Z value every time a variable enters into basis. Variables are not merely allowed to enter the basis even after ordering them, because previously entered variable may change the value of Z and affects the next promising variable.

The algorithm finds the best optimal solution among the solutions found for each ordering of objectives. The solution is best if it gives the best value for the sum of ratios. Algorithm is implemented using C language program. Efficiency of the algorithm is computationally tested by solving large scale multi objective optimization problems. The results are tabulated.

The proposed algorithm is computationally tested and it is found that the execution time and number of iterations are saved by this algorithm.

References:

[1] Frenk, J.B.G., Scheible, S.: Fractional programming. In: Floudas, C.A., Pardalos, P.M. (eds.) Encyclopedia of Optimization, vol. II, pp. 162–172. Kluwer, Dordrecht (2001)

[2] Rao, M.R.: Cluster analysis and mathematical programming. J. Am. Stat. Assoc. **66**, 622–626 (1971)

[3] Almogy, Y., Levin, O.: Parametric analysis of a multi-stage stochastic shipping problem. In: Proceedings of the 5th IFORS Conference (IFORS), pp. 359–370 (1964)

[4] Colantoni, C.S., Manes, R.P., Whinston, A.: Programming, profit rates, and pricing decisions. Account. Rev. 44, 467–481 (1969)

[5] Konno, H., Inori, M.: Bond portfolio optimization by bilinear fractional programming. J. Oper. Res. Soc. Jpn. 32, 143–158 (1989)

[6] Schaible, S.: Fractional programming. In: Horst, R., Pardalos, P. (eds.) Handbook of Global Optimization, pp. 495–608. Kluwer, Dordrecht (1995)

[7] Schaible, S.: Fractional programming with sums of ratios. In: Castagnoli, E., Giorgi, G. (eds.) Scalar and Vector Optimization in Economic and Financial Problems, Proceedings of the Workshop in Milan, 1995, pp. 163–175. Elioprint, Modena (1995)

[8] Naganathan E R (1997). A Ratio Multiplex Algorithm to solve large scale linear programming problems, Ph.D. thesis, Department of Computer Science and Engineering, Alagappa University, Karaikudi

[9] Nagarajan S, "A Complex Algorithm to Solve Large Scale Linear Programming Problems", Ph.D. thesis, Department of Computer Science and Engineering, Alagappa University, Karaikudi

[10] Sakthivel,S., "A Multiplex algorithm to solve large scale linear programming problems", Ph.D. thesis, Anna University, India, 1984.

[11] Meyyappan. T "A new Computer Oriented Technique To Solve Large Scale Multi Objective Optimization Problems", ", Ph.D. thesis, Department of Computer Science and Engineering, Alagappa University, Karaikudi

[12] Palanisamy.V "Computer oriented algorithms to solve special cases of non-linear fractional programming problems", ", Ph.D. thesis, Department of Computer Science and Engineering, Alagappa University, Karaikudi

[13] Ramesh babu.R Divyaa devi.S Palanisamy.V "A New Computer Oriented Technique to Solve Sum Of Ratios Non-Linear Fractional Programming Problems" International Journal of Computing Algorithm Volume: 01 Issue: 01 January-June 2012.