# Image Denoising with Linear and Non-Linear Filters: A REVIEW

**Mrs. Bhumika Gupta[1], Mr. Shailendra Singh Negi[2]**

**[1] Assistant professor, G.B.Pant Engineering College
Pauri Garhwal, Uttarakhand, 246194, India**

**[2] M.Tech (CSE) Scholar, G.B.Pant Engineering College
Pauri Garhwal, Uttarakhand, 246194, India**

## Abstract

*Image denoising is the manipulation of the image data to produce a visually high quality image. The existing or current denoising algorithms or approaches are filtering approach, multifractal approach and wavelet based approach. Different noise models include noise as additive and multiplicative type. They include Gaussian noise, salt and pepper noise (impulsive noise), Brownian noise and speckle noise. Noise arises due to various factors like bit error rate, speed, dead pixels. denoising algorithm is application dependent i.e. the application of a specific filter is beneficial against a specific kind of noise. The filtering approach has been proved to be the best when the image is corrupted with salt and pepper noise. In the filtering approach Median filter provides best result against impulsive noise i.e. salt and pepper noise. The wavelet based approach finds applications in denoising images corrupted with Gaussian noise. If the noise characteristics are complex, then multifractal approach can be used.*

*Keywords:* Image denoising, mean filter, LMS (least mean square) adaptive filter, median filter, Noises, Filter Mask.

## 1. Introduction

A very large portion of digital image processing is devoted to image restoration. This includes research in algorithm development and routine goal oriented image processing. Image restoration is the removal or reduction of degradations that are incurred while the image is being obtained. Degradation [1] comes from blurring [1] as well as noise due to photometric and electronic sources. Blurring is a form of bandwidth contraction of the image caused by the imperfect image formation process such as relative motion between the camera and the original scene or by an optical system that is out of focus. A noise [2] is introduced in the transmission medium due to noisy channel, errors during the measurement process and during sampling [2] and quantization [2] of the data for digital storage (in the form of arrays).

### 1.1. Representation of digital image.

A 2-dimensional digital image can be represented as a 2-dimensional array of data s(x, y), where (x, y) represent the pixel [2] position. The pixel value corresponds to the brightness of the image at position (x, y). Some of the most frequently used image types are binary, gray-scale and color images. Binary images [14] are the simplest type of images and can take only two discrete values, black and white. Black is represented with the value '0' while white with '1'. They are also referred to as 1 bit/pixel images. Gray-scale images [14] are known as monochrome or one-color images. They represent no color information but represent the brightness or intensity of the image. This image contains 8 bits per pixel data, which means it can have up to 256 (0 to 255) different brightness levels. A '0' represents black and '255' denotes white. As they contain the intensity information, they are also referred to as intensity images. Color images [14] are called as three band monochrome images, in which each band is of a different color. Each band provides the brightness or intensity information of the corresponding spectral band. Normal color images are red, green and blue images and are also referred to as RGB images. This is 24 bits per pixel image.

### 1.2. Denoising Concept

The image s(x, y) is blurred by a linear operation and noise n(x, y) is added to make the degraded image w(x, y). w(x, y) is then convolved with the restoration procedure g(x, y) to generate the restored image z(x, y).
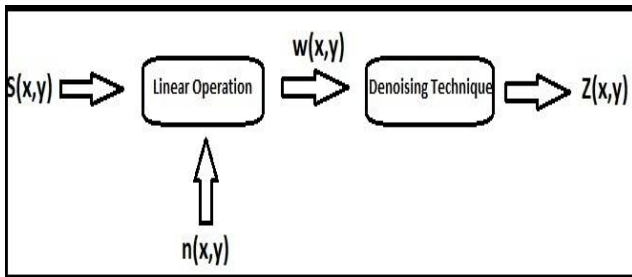
Fig. 1 Denoising Concept

The "Linear operation" shown in figure 1 is the addition or multiplication of the Noise n(x, y) to the signal or image s(x, y). Once the corrupted or noised image w(x, y) is obtained, it is subjected to the denoising technique i.e. algorithm to get the denoised image z(x, y). Noise removal or noise reduction can be done on an image by filtering, by wavelet analysis, or by multi fractal analysis. Each technique has its advantages and disadvantages. Wavelet techniques consider thresholding while multifractal analysis is based on improving the Holder regularity of the corrupted image.

## 2. Additive and Multiplicative Noises

An additive noise [3] follows the rule: - **W(x, y) = s(x, y) + n(x, y)**, examples of additive noise includes Gaussian, Uniform, or Salt and Pepper distribution. The multiplicative noise [3] satisfies the rule: - **W(x, y) = s(x, y) × n(x, y)**, example of multiplicative noise includes Speckle noise. In the above equations s(x, y) is the original signal or image, n(x, y) denotes the noise introduced into the image to produce the corrupted image w(x, y), and (x, y) represents the pixel location. Image addition also finds applications in image morphing. By image multiplication, we mean the brightness of the image is varied. The digital image acquisition process converts an optical image into a continuous electrical signal that is, then, sampled. At every step in the process there are fluctuations caused by natural phenomena, adding a random generated value to the exact brightness value for a given pixel.

### 2.1. Gaussian Noise

Gaussian noise [4] is uniformly distributed over the signal. It means that each pixel in the noisy image is the sum of the true pixel value and a random value of Gaussian distributed noise. As the name shows, this type of noise has a Gaussian distribution, that has a bell shaped probability distribution function (PDF)given by:-

$$F(g) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(g-m)^2/2\sigma^2} \quad \text{.............. (1)}$$

Where g = gray level,
    m = mean or average of the function,
    $\sigma^2$ = variance of the noise.
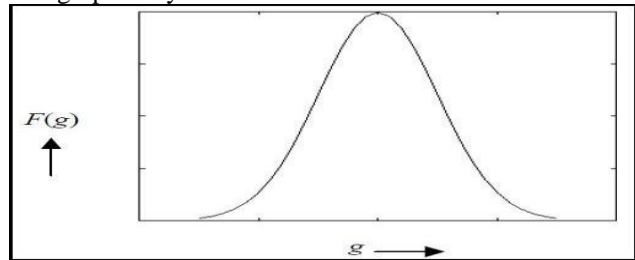
It is graphically shown as:-



Fig. 2 Graphical Representation of Gaussian Noise

### 2.2. Salt and Pepper Noise

Salt and pepper noise [4] is an impulse type of noise, which is also called as intensity spikes. This is generally caused due to errors in transmission of data through the channel. It has only two possible values, 'a' and 'b'. The probability of each is less than 0.1. The corrupted or noised pixels are set alternatively to the minimum or to the maximum value, giving the image a "salt and pepper" like appearance. For an 8-bit image, the typical value for pepper noise is '0' and for salt noise are '255'. The salt and pepper noise (impulsive noise) is generally caused by malfunctioning of pixel elements in the camera sensors, timing errors, or faulty memory location in the digitization process. The probability density function (PDF) for impulsive noise is shown below:-

$$F(g) = \begin{cases} P_a & g = a \\ P_b & g = b \\ O & otherwise \end{cases} \quad \text{........ (2)}$$
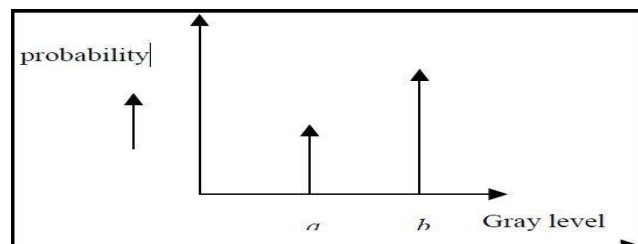
It is graphically shown as:-



Fig. 3 Graphical Representation of Impulsive Noise

## 2.3. Speckle Noise

Speckle noise [4] is a kind of multiplicative noise. This type of noise generally occurs in almost all coherent imaging systems such as laser, acoustics and SAR (Synthetic Aperture Radar) imagery. Speckle noise follows a gamma distribution and is given as: -

$$F(g) = \frac{g^{\alpha - 1}}{(\alpha - 1)! a^{\alpha}} e^{-g/a} \quad \dots\dots\dots (3)$$

Where $\mathbf{a^2\,\alpha}$ = variance
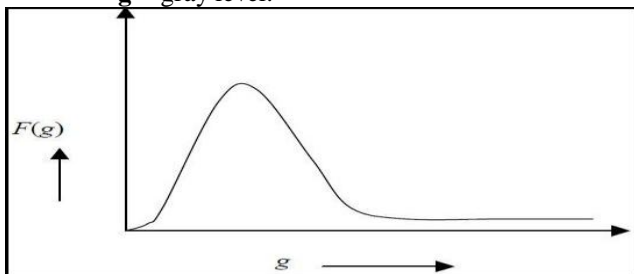$\mathbf{g}$ = gray level.


Fig. 4 Graphical Representation of Speckle Noise

## 2.4. Brownian Noise

Brownian noise [4] comes under the class of 1/f or fractal noises. The mathematical model for 1/f noise is fractional Brownian motion. Brownian noise is a special case of 1/f noise or fractal noises. It is obtained by mixing white noise with the image. It can be graphically represented as shown below:-
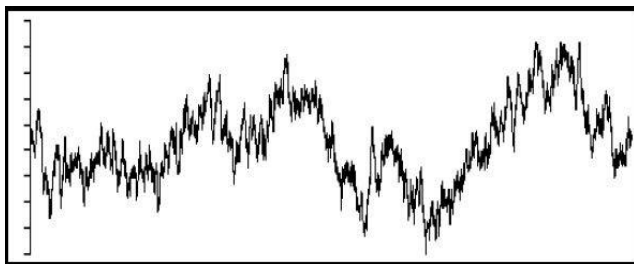

Fig. 5 Graphical Representation of Brownian Noise

## 3.  Linear and Non-Linear Filtering Approach

Linear filtering [5] is of two types: - **mean filter** and **Least Mean Square (LMS) adaptive filter** and nonlinear filtering is based on **median filter**. These filtering approaches are discussed below:-

## 3.1. Introduction

Filters [5] play a major role in the Image Restoration process. The basic Concept behind Image Restoration is digital Convolution using linear filters and moving window principle. Let w(x) is the input signal subjected to filtering, and z(x) is the filtered output. In 1D the output filter can be expressed mathematically in simple form as z(x) = ∫ w(t)h(x − t)dt, where h(t) = point spread function or impulse response. The integral represents a convolution integral and, can be expressed as z = w* h. For the 2-dimensional case, h (t) is h (t, u), and above Equation becomes:-

$$Z(i, j) = \sum_{t=i-k}^{i+k} \sum_{u=j-l}^{j+l} w(t,u)h(i-t, j-u) \quad \dots\dots\dots (4)$$

Values of h (t, u) are referred to as the filter Weights, the filter kernel, or filter mask. The total output is created by a series of shift- multiply-sum operations, and this forms a discrete convolution.

## 3.2. Linear Filtering
### 3.2.1.   Mean Filtering

A mean filter [15] acts on an image by smoothing it, i.e., it reduces the intensity variation between adjacent pixels. The mean filter is a simple sliding window spatial filter that replaces the center value in the window with the average of all the neighboring pixel values including itself. It is implemented on the basis of digital convolution using linear filters, which provides a result that is a weighted sum of the values of a pixel and its neighbors. It is also called as a linear filter. The mask or kernel is a square. If the sum of the coefficients of the mask is one, then the average brightness of the image is not altered. If the sum of coefficients is zero, then the average brightness is lost, and it returns an image which is dark. The average or mean filter works on the principle of shift-multiply-sum. This principle in the two-dimensional image can be illustrated as shown below.
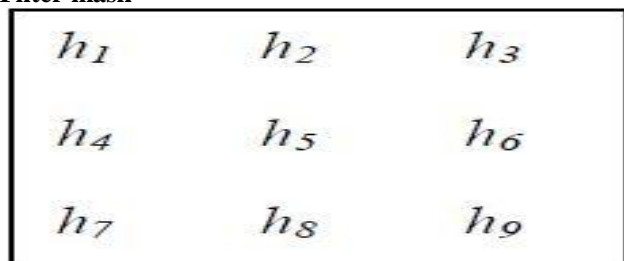
**Filter mask**


Fig. 6 A Filter mask of 3×3

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Multiply and sum for the pixel at (4, 3) = $h_1w_{32} + h_2w_{33} + h_3w_{34} + h_4w_{42} + h_5w_{43} + h_6w_{44} + h_7w_{52} + h_8w_{53} + h_9w_{54}$. Since the sum of coefficients of the mask or filter shown below is one, so the image brightness is not lost, and the coefficients are all positive, hence it will tend to blur the image.

| | | |
|---|---|---|
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |

Fig. 7 A general filter mask of 3×3

Computing the straightforward convolution of an image with this kernel carries out the mean filtering process. It is effective and beneficial when the noise in the image is of salt and pepper (impulsive) type. The mean or averaging filter works like a low pass filter, and it does not allow the high frequency components present in the noise to pass through. Larger kernels of size $5 \times 5$ or $7 \times 7$ produces more denoising but make the image more blurred. A trade off is to be made between the kernel size and the amount of denoising. The filter discussed above is also known as a constant coefficient filter because the weight matrix does not change during the whole process. The pixel values of an image are read into the program by using the function **imread().** Salt and pepper noise is added to this image by using the function **imnoise().** The modified pixel matrix is now converted to the image format with the help of the function **imwrite().** The mean filter is used in applications where the noise is present in certain regions of the image and it needs to be removed. Hence the mean filter is useful when only a part of the image needs to be processed.



Fig. 8 Image corrupted with salt and pepper (Impulsive) noise.

The above Image of fig 8 is input to all the filters discussed below.



Fig. 9 Output of mean filter.

### 3.2.2. LMS Adaptive Filter

The difference between the mean filter and the adaptive filter [15] is that the weight matrix changes after each iteration in the adaptive filter while it remains constant throughout the iterations in the mean filter. Adaptive filters are capable of denoising non-stationary images, i.e., images that have abrupt changes in intensity. An adaptive filter iteratively adjusts its parameters during scanning the image to match the image generating mechanism. The basic model of LMS Adaptive filter is a linear combination of a stationary low-pass image and a non-stationary high-pass component through a weighting function. The LMS adaptive filter incorporating a local mean estimator works on the following concept. A window 'W' of size 'm × n' is scanned over the image. The mean of this window is 'μ' which is subtracted from the elements in the window to get the residual matrix '$W^r$'.

$W^r = W - \mu$. A weighted sum $\tilde{z}$, is computed in a way similar to the mean filter using:-

$$\tilde{z} = \sum_{i,j \in W} h(i, j)W^r \qquad \text{......... (5)}$$

Where h (i, j) = elements of the weight matrix. A sum of the weighted sum, $\tilde{z}$, and the mean, μ, of the window under filter replaces the center element of the window. Thus, the resultant modified pixel value is given as:-

$z = \tilde{z} + \mu$.

For the next iteration, the window is shifted over one pixel in row major order and the weight matrix is modified. The deviation '**e**' is computed by taking the difference between the center value of the residual matrix '$W^r$' and the weighted sum as Equation: - $e = W^r - \tilde{z}$.

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The largest eigen value $\lambda$ of the original window is calculated from the autocorrelation matrix of the window considered. The use of the largest eigen value in computing the modified weight matrix for the next iteration reduces the minimum mean squared error. A value $\eta$ is selected such that it lies in the range **(0, 1/λ)**. The new weight matrix $h_{k+1}$ is:-

$h_{k+1} = h_k + \eta \times e \times W^r$ . Where $h_k$ = weight matrix from the previous iteration. The weight matrix obtained this way is used in the next iteration. This process continues until the window covers the entire image. Similar to the mean filter, the LMS adaptive filter works well for images corrupted with salt and pepper type of noise. But this filter does a better denoising job compared to the mean filter.



Fig. 10 Output from LMS Adaptive Filter

### 3.3. Non Linear Filter
### 3.3.1. Median Filter

The median filter [15][2] also follows the moving window principle similar to the mean filter. A 3×3, 5×5, or 7×7 kernel or filter mask of pixels is scanned over pixel matrix of the entire image. The median of the pixel values in the window is calculated, and the center pixel of the window is replaced with the calculated median value. Median filtering is done by, first sorting all the pixel values from the surrounding neighborhood into numerical order and then replacing the pixel being considered with the middle pixel value.

$$g(p) = median\{f(p), where \ p \in N_8(p)\} \quad \text{....... (6)}$$

The mathematical formula for the median filter is shown above:-
Where g(p) = median pixel value.
     f(p) = all pixel values under mask.
     $N_8(p)$ = 8-neighbourhood of pixel 'p'.

Concept of median filtering:-



Fig. 11 Concept of Median Filter

Neighborhood values for figure 11:
115,119,120,123,124,125,126,127,150
Median value: 124

The median filter is popular because of its ability to reduce random impulsive noise without blurring edges. It often fails to perform well as linear filters in providing sufficient smoothing of non impulsive noise components such as additive Gaussian noise. The main drawback of median filter is that it is not location variant in nature, and thus also tends to alter the pixels not disturbed by noise.



Fig. 12 Output from Median Filter

## 4. Conclusion

In this paper, we have focused on the denoising of images using linear and nonlinear filtering techniques. Linear filtering is done using the mean filter and LMS adaptive filter while the nonlinear filtering is performed using a median filter. These filters are beneficial for removing noise that is impulsive in nature i.e. salt and pepper noise. The mean filters find applications where the noise is concentrated in the small portion of the image. Besides, implementation of such filters is easy, cost effective and fast. It can be observed from the output Images of Mean and LMS Adaptive filter that the filtered images are

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blurred [6][10]. The median filter overcomes this problem by providing a solution to this, in which the sharpness of the image is retained after denoising. The result below supports our approach towards median filter. Thus median filter is best among all the filters in filtering approach

| Filter Name | De-noising result for Speckle noise (%) | De-noising result for Gaussian noise (%) | De-noising result for Salt & Pepper noise (%) |
|---|---|---|---|
| Median Filter | 70% | 60% | 95% |

Fig. 13 Result of median filter to support conclusion

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**Mrs. Bhumika Gupta** presently persuing Ph.D in Iterated Function System in CHAOS and FRACTALS and completed M.Tech(Gold Medalist) from G.G.S.I.P.U. Delhi in 2010. Completed B.Tech. in 2005. Presently working as Asst. Professor in Gobind Ballabh Pant Engineering College, Pauri Garhwal, Uttarakhand, India-246194.

**Mr. Shailendra Singh Negi** is persuing M.Tech (CSE) from Gobind Ballabh Pant Engineering College, Pauri Garhwal, Uttarakhand, India-246194. Graduated in B.Tech (CSE) in 2012 with 79.30%. Qualified GATE (2012) with 97 percetile and a GATE score of 489. Presently working in the area of '**Image Denoising**'.