New LMI robust stability criteria of uncertain bidirectional associative memory neural networks

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***Abstract*—This paper deals with the uniqueness and stability problem for uncertain bidirectional associative memory (BAM) neural networks with time-varying delays. Based on the Lyapunov-Krasovskii functional approach and free-weighting matrices method, new delay-dependent stability criteria with two classes of system uncertainties are presented in terms of linear matrix inequalities (LMIs). By using the Jensen integral inequality, the obtained results are less conservative than some previous ones. Four examples are given to illustrate the effectiveness of our proposed conditions.**

***Index Terms*—Global robust exponential stability; globally exponential stability; linear matrix inequality(LMI); bidirectional associative memory (BAM) neural networks; Jensen integral inequality**

I. INTRODUCTION

In the most recent two decades, neural networks have been extensively studied in many aspects and successfully applied to many fields such as pattern identifying, voice recognizing, system controlling, signal processing systems, static image treatment, and solving nonlinear algebraic equations, etc. As one of the most important neural networks, bidirectional associative memory (BAM) neural networks generalized the single-layer autoassociative Hebbian correlator to a two-layer pattern-matched heteroassociative circuits(see [9], [10]). In hardware implementation of neural networks, time delays are inevitably present due to the finite switching speeds of the amplifiers. It is well known that time delays not only deteriorate dynamical performance such as the boundary of the basin of attraction of the stable equilibria but also affect the stability of a network creating oscillatory and unstable characteristics. Hence, it is of primary importance to investigate the stability of delayed neural networks. There exist some results of stability for delayed neural networks, see, for example, [2]–[5], [7], [11], [14], [15], [17], [18].

On the other hand, in the design and hardware implementation of neural networks, however, a common problem is that parameters acquired in neural networks are inaccurate due to the tolerances of electronic components employed in the design. This fact implies that a good neural network should have certain robustness which paves the way for introducing the theory of interval matrices and interval dynamics to investigate the global stability of interval neural networks. Therefore the robust stability of the neural networks have been extensively investigated, e.g., [11], [14]–[17], [23]–[26]. By using the well known mean value inequality, Cao et al [3] derived exponential stability conditions of BAM neural networks with constant delays; Based on the Lyapunov-Krasovskii functional in combination with linear matrix inequality approach, Huang et al. [7] proposed exponential stability results of BAM neural networks with constant and varying delays; By means of linear matrix inequality technique, Wang and Zhang et al. [17], [18] presented asymptotic and exponential stability results of BAM neural networks with constant delays; From the Jensen integral inequality, Gau et al. [5] obtained the uniqueness and robust exponential stability conditions of delayed BAM neural networks with uncertainties also in terms of LMIs; By constructing a novel Lyapunov functional, Sheng [15] established robust exponential stability criteria of delayed BAM neural networks with uncertainties also by means of LMIs; Based on an inequality and free-weighting matrix method, Park et al. [12] achieved a robust stability criterion for delayed BAM systems; By using the Jensen integral inequality and introducing some free-weighting matrices, Park et al. [13] recently brought out an exponential stability condition for BAM systems with time varying delays. But in order to become sufficient conditions, the Theorems in [12], [13] need to be revised.

Motivated by the preceding discussions, the aim of this paper is to study the global robust exponential stability for delayed BAM neural networks with the norm-bounded uncertainty. By constructing a new type of Lyapunov functional and adopting the idea of introducing additional free-weighting matrices [8], [27], we derive several new sufficient conditions for the global exponential stability of delayed BAM neural networks with two classes of system uncertainties. The derived conditions are expressed in terms of linear matrix inequalities, which can be checked numerically very efficiently via the LMI toolbox. Some comparisons between the obtained results in this paper and previous results are made and four examples are used to illustrate the effectiveness of the obtained results.

The rest of this paper is organized as follows. In Section 2, problem formulation and preliminaries are given. In Section 3, the new delay-dependent exponential stability conditions are established. In Section 4, the new stability conditions are extended to neural networks with norm-bounded uncertainties. Section 5 points out the mistakes of [12], [13] and provides four illustrative examples. Finally, some conclusions are drawn in Section 6.

II. Problem description

Considering the following BAM neural networks with time-varying delays:

$$\begin{cases} \dot{x}(t) = -\bar{A}_1 x(t) + \bar{B}_1 \tilde{f}(y(t)) + \bar{C}_1 \tilde{f}(y(t-\tau(t))) + J_1, \\ \dot{y}(t) = -\bar{A}_2 y(t) + \bar{B}_2 \tilde{g}(x(t)) + \bar{C}_2 \tilde{g}(x(t-\sigma(t))) + J_2, \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$ and $y(t) = (y_1(t), y_2(t), ..., y_m(t))^T$ are the neural state vectors, J_1, J_2 are the constant external input vectors. $\bar{A}_1 = A_1 + \Delta A_1(t)$, $\bar{B}_1 = B_1 + \Delta B_1(t)$, $\bar{C}_1 = C_1 + \Delta C_1(t)$, $\bar{A}_2 = A_2 + \Delta A_2(t)$, $\bar{B}_2 = B_2 + \Delta B_2(t)$, $\bar{C}_2 = C_2 + \Delta C_2(t)$. $A_1 = \mathrm{diag}\{a_{11}, a_{12}, ..., a_{1n}\}$ and $A_2 = \mathrm{diag}\{a_{21}, a_{22}, ..., a_{2m}\}$ are positive diagonal matrices, $B_1 = (b_{1ij})_{n\times m}$, $C_1 = (c_{1ij})_{n\times m}$, $B_2 = (b_{2ij})_{m\times n}$, $C_2 = (c_{2ij})_{m\times n}$ are known constant matrices, $\Delta A_1(t), \Delta B_1(t), \Delta C_1(t), \Delta A_2(t), \Delta B_2(t), \Delta C_2(t)$ are parametric uncertainties, $0 \le \tau(t) \le \bar{\tau}$, $0 \le \sigma(t) \le \bar{\sigma}$ are the time-varying delays, where $\bar{\tau}, \bar{\sigma}$ are positive constants. $\tilde{f}(x(t)) = (\tilde{f}_1(y_1(t)), \tilde{f}_2(y_2(t)), ..., \tilde{f}_m(y_m(t)))^T$, $\tilde{g}(x(t)) = (\tilde{g}_1(x_1(t)), \tilde{g}_2(x_2(t)), ..., \tilde{g}_n(x_n(t)))^T$ denote the neural activation functions. It is assumed that $\tilde{f}_i(y_i(t))$, $\tilde{g}_j(x_j(t))$ are bounded and there exist constants $l_{1i}, l_{2i}, l_{3j}, l_{4j}$ such that

$$l_{1i} \le \frac{\tilde{f}_i(s_1) - \tilde{f}_i(s_2)}{s_1 - s_2} \le l_{2i}, \tag{2}$$

$$l_{3j} \le \frac{\tilde{g}_j(s_1) - \tilde{g}_j(s_2)}{s_1 - s_2} \le l_{4j}, \tag{3}$$

for any $s_1, s_2 \in \mathcal{R}, s_1 \ne s_2$, $i = 1, 2, ..., m; j = 1, 2, ..., n$.

Moreover, we assume that the initial condition of system (1) has the form

$$x_j(t) = \phi_j(t),\ y_i(t) = \varphi_i(t), \quad t \in [-\max\{\bar{\sigma}, \bar{\tau}\}, 0]$$

where $\phi_j(t), \varphi_i(t) (i = 1, 2, ..., m; j = 1, 2, ..., n)$ are continuous functions.

From the well-known Brouwer's fixed point theorem, system (1) always has an equilibrium point (x^*, y^*).

Throughout this paper, let $||y||$ denotes the Euclidean norm of a vector $y \in \mathcal{R}^n$, $W^T, W^{-1}, \lambda_M(W), \lambda_m(W)$ and $||W|| = \sqrt{\lambda_M(W^T W)}$ denote the transpose, the inverse, the largest eigenvalue, the smallest eigenvalue, and the spectral norm of a square matrix W, respectively. Let $W > 0 (< 0)$ denote a positive (negative) definite symmetric matrix, I denote an identity matrix with compatible dimension.

In order to prove the robust stability of the equilibrium point (x^*, y^*) of system (1), we will first simplify system (1) as follows. Let $u(\cdot) = x(\cdot) - x^*$, $v(\cdot) = y(\cdot) - y^*$, then we have

$$\begin{cases} \dot{u}(t) = -\bar{A}_1 u(t) + \bar{B}_1 f(v(t)) + \bar{C}_1 f(v(t-\tau(t))), \\ \dot{v}(t) = -\bar{A}_2 v(t) + \bar{B}_2 g(u(t)) + \bar{C}_2 g(u(t-\sigma(t))), \end{cases} \tag{4}$$

where $f_i(v_i(t)) = \tilde{f}_i(v_i(t) + y_i^*) - \tilde{f}_i(y_i^*)$, $g_j(u_j(t)) = \tilde{g}_j(u_j(t) + x_j^*) - \tilde{g}_j(x_j^*)$ with $f_i(0) = g_j(0) = 0$, $i = 1, 2, ..., m;$ $j = 1, 2, ..., n$. By assumptions (2) and (3), we can see that

$$l_{1i} \le \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le l_{2i}, \tag{5}$$

$$l_{3j} \le \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \le l_{4j}. \tag{6}$$

The definition of exponential stability is now given.

Definition 1([19]) The system (4) is said to be globally exponentially stable if there exist constants $k \ge 0$ and $M > 1$ such that

$$||u(t)|| + ||v(t)|| \le M \sup_{-\max\{\bar{\sigma},\bar{\tau}\} \le \theta \le 0} \{||u(\theta)||, ||v(\theta)||\} e^{-kt},$$

where k is called the exponential convergence rate.

Clearly, the equilibrium point of system (1) is robust stable if and only if the zero solution of system (4) is robust stable.

Consider the following two classes of time-varying uncertain matrices defined by:

Assumption 1

$$\begin{aligned} [\Delta A_1(t), \Delta B_1(t), \Delta C_1(t)] &= H_0 E_0(t) [G_1, G_2, G_3], \\ [\Delta A_2(t), \Delta B_2(t), \Delta C_2(t)] &= H_1 E_1(t) [G_4, G_5, G_6], \end{aligned} \tag{7}$$

where $H_0, H_1, G_i (i = 1, ..., 6)$ are known real constant matrices with appropriate dimensions. $E_0(t), E_1(t)$ are unknown time-varying matrices satisfying

$$E_0^T(t) E_0(t) \le I,\ E_1^T(t) E_1(t) \le I. \tag{8}$$

Assumption 2

$$\begin{aligned} &||\Delta A_1(t)|| \le \rho_1,\ ||\Delta B_1(t)|| \le \rho_2,\ ||\Delta C_1(t)|| \le \rho_3, \\ &||\Delta A_2(t)|| \le \rho_4,\ ||\Delta B_2(t)|| \le \rho_5,\ ||\Delta C_2(t)|| \le \rho_6. \end{aligned} \tag{9}$$

In order to obtain the results, we need the following lemmas.

Lemma 1 (see [6]) *For any positive symmetric constant matrix $M \in \mathcal{R}^{n\times n}$, scalars $r_1 < r_2$ and vector function $\omega : [r_1, r_2] \to \mathcal{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_{r_1}^{r_2} \omega(s) \mathrm{d}s\right)^T M \left(\int_{r_1}^{r_2} \omega(s) \mathrm{d}s\right) \le (r_2 - r_1) \int_{r_1}^{r_2} \omega^T(s) M \omega(s) \mathrm{d}s.$$

Lemma 2 (see [1], [22]) *Let X, Y and P be real matrices of appropriate dimensions with $P > 0$. Then for any positive scalar ε the following matrix inequality holds:*

$$X^T Y + Y^T X \le \varepsilon^{-1} X^T P^{-1} X + \varepsilon Y^T P Y.$$

III. Uniqueness and globally exponential stability result of neural networks

First, we will present the exponential stability results for system (4) without uncertainties, that is

$$\begin{cases} \dot{u}(t) = -A_1 u(t) + B_1 f(v(t)) + C_1 f(v(t-\tau(t))), \\ \dot{v}(t) = -A_2 v(t) + B_2 g(u(t)) + C_2 g(u(t-\sigma(t))). \end{cases} \tag{10}$$

Theorem 1. *Under the assumptions* (2),(3) *and* $0 \le \tau(t) \le \bar{\tau}$, $0 \le \sigma(t) \le \bar{\sigma}$, $0 \le \dot{\tau}(t) \le \eta_1 < 1$, $0 \le \dot{\sigma}(t) \le \eta_2 < 1$, *given a constant* $k \ge 0$, *suppose that there exist positive definite symmetric matrices* $P = [P_{ij}]_{2\times 2}$, $Q = [Q_{ij}]_{2\times 2}$, *nonnegative definite symmetric matrices* $R = [R_{ij}]_{3\times 3}$, $S = [S_{ij}]_{3\times 3}$, $Z = [Z_{ij}]_{3\times 3}$, $W = [W_{ij}]_{3\times 3}$, $U_i (i = 1, ..., 6)$, *positive diagonal matrices* $T_i(i = 1, 2, 3, 4)$, *real matrices* $X = [\ X_1\ \ X_2\ \ 0\ ], Y = [\ 0\ \ Y_1\ \ Y_2\ ], M = [\ M_1\ \ M_2\ \ 0\ ]$, $N = [\ 0\ \ N_1\ \ N_2\ ]$ *with compatible dimensions such that the following LMIs hold:*

$$\Xi_1 = \begin{bmatrix} Z & X^T \\ X & e^{-2k\bar{\sigma}}U_2 \end{bmatrix} \geq 0, \quad (11)$$

$$\Xi_2 = \begin{bmatrix} Z & Y^T \\ Y & e^{-2k\bar{\sigma}}U_2 \end{bmatrix} \geq 0, \quad (12)$$

$$\Xi_3 = \begin{bmatrix} W & M^T \\ M & e^{-2k\bar{\tau}}U_5 \end{bmatrix} \geq 0, \quad (13)$$

$$\Xi_4 = \begin{bmatrix} W & N^T \\ N & e^{-2k\bar{\tau}}U_5 \end{bmatrix} \geq 0, \quad (14)$$

$$\begin{bmatrix} \Omega & \mathcal{A}_1^T F_1 & \mathcal{A}_2^T F_2 \\ F_1\mathcal{A}_1 & -F_1 & 0 \\ F_2\mathcal{A}_2 & 0 & -F_2 \end{bmatrix} < 0, \quad (15)$$

where

$$\begin{aligned}
&\Omega = [\ \Omega_{ij}\ ]_{12\times 12}, \\
&\Omega_{11} = 2kP_{11} - (P_{11} + R_{13})A_1 - A_1(P_{11} + R_{13}^T) + R_{11} \\
&\qquad + U_1 - e^{-2k\bar{\sigma}}U_3 + X_1 + X_1^T + \bar{\sigma}Z_{11} - 2L_3T_3L_4, \\
&\Omega_{12} = (2kI - A_1)P_{12} + e^{-2k\bar{\sigma}}U_3 - X_1^T + X_2 + \bar{\sigma}Z_{12}, \\
&\Omega_{13} = \bar{\sigma}Z_{13}, \quad \Omega_{14} = P_{12}, \\
&\Omega_{15} = (P_{11} + R_{13})B_1, \quad \Omega_{16} = (P_{11} + R_{13})C_1, \\
&\Omega_{1,11} = R_{12} - A_1R_{23}^T + (L_3 + L_4)T_3, \\
&\Omega_{22} = 2kP_{22} - (1 - \eta_2)e^{-2k\bar{\sigma}}R_{11} - 2e^{-2k\bar{\sigma}}U_3 - X_2 \\
&\qquad - X_2^T + Y_1 + Y_1^T + \bar{\sigma}Z_{22} - 2L_3T_4L_4, \\
&\Omega_{23} = e^{-2k\bar{\sigma}}U_3 - Y_1^T + Y_2 + \bar{\sigma}Z_{23}, \\
&\Omega_{24} = P_{22} - e^{-2k\bar{\sigma}}R_{13}, \\
&\Omega_{25} = P_{12}^TB_1, \quad \Omega_{26} = P_{12}^TC_1, \\
&\Omega_{2,12} = -(1 - \eta_2)e^{-2k\bar{\sigma}}R_{12} + (L_3 + L_4)T_4, \\
&\Omega_{33} = -e^{-2k\bar{\sigma}}U_1 - e^{-2k\bar{\sigma}}U_3 - Y_2 - Y_2^T + \bar{\sigma}Z_{33}, \\
&\Omega_{44} = -e^{-2k\bar{\sigma}}R_{33}, \quad \Omega_{4,12} = -e^{-2k\bar{\sigma}}R_{23}^T, \\
&\Omega_{55} = S_{22} - 2T_1, \quad \Omega_{57} = S_{12}^T - S_{23}A_2 + (L_1 + L_2)T_1, \\
&\Omega_{5,10} = C_2^TS_{23}^T, \quad \Omega_{5,11} = B_1^TR_{23}^T + S_{23}B_2, \\
&\Omega_{5,12} = S_{23}C_2, \quad \Omega_{66} = -(1 - \eta_1)e^{-2k\bar{\tau}}S_{22} - 2T_2, \\
&\Omega_{68} = -(1 - \eta_1)e^{-2k\bar{\tau}}S_{12}^T + (L_1 + L_2)T_2, \\
&\Omega_{6,10} = -e^{-2k\bar{\tau}}S_{23}^T, \quad \Omega_{6,11} = C_1^TR_{23}^T, \\
&\Omega_{77} = 2kQ_{11} - (Q_{11} + S_{13})A_2 - A_2(Q_{11} + S_{13}^T) + S_{11} \\
&\qquad + U_4 - e^{-2k\bar{\tau}}U_6 + M_1 + M_1^T + \bar{\tau}W_{11} - 2L_1T_1L_2, \\
&\Omega_{78} = (2kI - A_2)Q_{12} + e^{-2k\bar{\tau}}U_6 - M_1^T + M_2 + \bar{\tau}W_{12}, \\
&\Omega_{79} = \bar{\tau}W_{13}, \quad \Omega_{7,10} = Q_{12}, \\
&\Omega_{7,11} = (Q_{11} + S_{13})B_2, \quad \Omega_{7,12} = (Q_{11} + S_{13})C_2, \\
&\Omega_{88} = 2kQ_{22} - (1 - \eta_1)e^{-2k\bar{\tau}}S_{11} - 2e^{-2k\bar{\tau}}U_6 - M_2 \\
&\qquad - M_2^T + N_1 + N_1^T + \bar{\tau}W_{22} - 2L_1T_2L_2, \\
&\Omega_{89} = e^{-2k\bar{\tau}}U_6 - N_1^T + N_2 + \bar{\tau}W_{23}, \\
&\Omega_{8,10} = Q_{22} - e^{-2k\bar{\tau}}S_{13}, \\
&\Omega_{8,11} = Q_{12}^TB_2, \quad \Omega_{8,12} = Q_{12}^TC_2, \\
&\Omega_{99} = -e^{-2k\bar{\tau}}U_4 - e^{-2k\bar{\tau}}U_6 - N_2 - N_2^T + \bar{\tau}W_{33}, \\
&\Omega_{10,10} = -e^{-2k\bar{\tau}}S_{33}^T, \quad \Omega_{11,11} = R_{22} - 2T_3, \\
&\Omega_{12,12} = -(1 - \eta_2)e^{-2k\bar{\sigma}}R_{22} - 2T_4, \\
&F_1 = R_{33} + \bar{\sigma}U_2 + \bar{\sigma}^2U_3, \quad F_2 = S_{33} + \bar{\tau}U_5 + \bar{\tau}^2U_6, \\
&\mathcal{A}_1 = [-A_1\ \ 0\ \ 0\ \ 0\ \ B_1\ \ C_1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ ], \\
&\mathcal{A}_2 = [\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ -A_2\ \ 0\ \ 0\ \ 0\ \ B_2\ \ C_2\ ], \\
&L_i = \mathrm{diag}\{l_{i1}, l_{i2}, ..., l_{in}\},\ i = 1, 2, 3, 4,
\end{aligned}$$

and other parameters $\Omega_{ij}(i < j)$ *are all equal to zero's, then the equilibrium point of system* (10) *is unique and exponential stable.*

Proof. Firstly, we show the uniqueness of the equilibrium point by contradiction. To end this, let $(\hat{u}, \hat{v})$ be the equilibrium point of the delayed BAM neural network (10), then we have

$$\begin{aligned}
&- A_1\hat{u} + (B_1 + C_1)f(\hat{v}) = 0, \\
&- A_2\hat{v} + (B_2 + C_2)g(\hat{u}) = 0.
\end{aligned}$$

Now suppose $(\hat{u}, \hat{v}) \neq 0$. It is easy to see that

$$\begin{aligned}
&2[\hat{u}^T(P_{11} + P_{12}^T + R_{13}) + g^T(\hat{u})R_{23}] \\
&\qquad \times \{-A_1\hat{u} + (B_1 + C_1)f(\hat{v})\} = 0, \\
&2[\hat{v}^T(Q_{11} + Q_{12}^T + S_{13}) + f^T(\hat{v})S_{23}] \\
&\qquad \times \{-A_2\hat{v} + (B_2 + C_2)g(\hat{u})\} = 0.
\end{aligned}$$

By inequalities (5) and (6), we get

$$\begin{aligned}
&-2\hat{v}^TL_1(T_1 + T_2)L_2\hat{v} + 2\hat{v}^T(T_1 + T_2)(L_1 + L_2)f(\hat{v}) \\
&\qquad -2f^T(\hat{v})(T_1 + T_2)f(\hat{v}) \geq 0, \\
&-2\hat{u}^TL_3(T_3 + T_4)L_4\hat{u} + 2\hat{u}^T(T_3 + T_4)(L_3 + L_4)g(\hat{u}) \\
&\qquad -2g^T(\hat{u})(T_3 + T_4)g(\hat{u}) \geq 0.
\end{aligned}$$

Note that $k \geq 0, P > 0, Q > 0, R \geq 0, S \geq 0, Z \geq 0, W \geq 0, 0 \leq \eta_i < 1(i = 1, 2), \bar{\tau} \geq 0, \bar{\sigma} \geq 0$, thus the following inequalities hold.

$$2k \begin{bmatrix} \hat{u} \\ \hat{u} \end{bmatrix}^T P \begin{bmatrix} \hat{u} \\ \hat{u} \end{bmatrix} \geq 0, \quad 2k \begin{bmatrix} \hat{v} \\ \hat{v} \end{bmatrix}^T Q \begin{bmatrix} \hat{v} \\ \hat{v} \end{bmatrix} \geq 0,$$

$$[1 - (1 - \eta_2)e^{-2k\bar{\sigma}}] \begin{bmatrix} \hat{u} \\ g(\hat{u}) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} \begin{bmatrix} \hat{u} \\ g(\hat{u}) \end{bmatrix} \geq 0,$$

$$[1 - (1 - \eta_1)e^{-2k\bar{\tau}}] \begin{bmatrix} \hat{v} \\ f(\hat{v}) \end{bmatrix}^T \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} \begin{bmatrix} \hat{v} \\ f(\hat{v}) \end{bmatrix} \geq 0,$$

$$\big(1-e^{-2k\bar{\sigma}}\big)\hat{u}^TU_1\hat{u}\geq 0,\quad \big(1-e^{-2k\bar{\tau}}\big)\hat{v}^TU_4\hat{v}\geq 0,$$
$$\bar{\sigma}\left[\begin{array}{ccc}\hat{u}^T & \hat{u}^T & \hat{u}^T\end{array}\right]Z\left[\begin{array}{ccc}\hat{u}^T & \hat{u}^T & \hat{u}^T\end{array}\right]^T\geq 0,$$
$$\bar{\tau}\left[\begin{array}{ccc}\hat{v}^T & \hat{v}^T & \hat{v}^T\end{array}\right]W\left[\begin{array}{ccc}\hat{v}^T & \hat{v}^T & \hat{v}^T\end{array}\right]^T\geq 0.$$

Furthermore, it is easy to see that the following equations hold.

$$\left[\begin{array}{ccc}\hat{u}^T & \hat{u}^T & \hat{u}^T\end{array}\right]\Phi\left[\begin{array}{ccc}\hat{u}^T & \hat{u}^T & \hat{u}^T\end{array}\right]^T=0,$$
$$\left[\begin{array}{ccc}\hat{v}^T & \hat{v}^T & \hat{v}^T\end{array}\right]\Psi\left[\begin{array}{ccc}\hat{v}^T & \hat{v}^T & \hat{v}^T\end{array}\right]^T=0,$$

where

$$\begin{aligned}
&\Phi=[\ \Phi_{ij}\ ]_{3\times 3},\quad \Psi=[\ \Psi_{ij}\ ]_{3\times 3},\\
&\Phi_{11}=X_1+X_1^T,\quad \Phi_{12}=-X_1^T+X_2,\quad \Phi_{13}=0,\\
&\Phi_{22}=-X_2-X_2^T+Y_1+Y_1^T,\\
&\Phi_{23}=-Y_1^T+Y_2,\quad \Phi_{33}=-Y_2-Y_2^T,\\
&\Psi_{11}=M_1+M_1^T,\quad \Psi_{12}=-M_1^T+M_2,\quad \Psi_{13}=0,\\
&\Psi_{22}=-M_2-M_2^T+N_1+N_1^T,\\
&\Psi_{23}=-N_1^T+N_2,\quad \Psi_{33}=-N_2-N_2^T,
\end{aligned}$$

All these equations and inequalities together gives

$$\vartheta\left[\ \Omega_{ij}\ \right]_{12\times 12}\vartheta^T\geq 0, \tag{16}$$

where

$$\vartheta=[\hat{u}^T\ \ \hat{u}^T\ \ \hat{u}^T\ \ 0\ \ f^T(\hat{v})\ \ f^T(\hat{v})\ \ \hat{v}^T\ \ \hat{v}^T\ \ \hat{v}^T\ \ 0\ \ g^T(\hat{u})\ \ g^T(\hat{u})].$$

On the other hand, one can infer from inequality (15) that $\Omega<0$. Obviously, this contradicts with (16). The contradiction implies that $(\hat{u},\hat{v})=0$. That is, the origin of the delayed recurrent neural networks (10) is the unique equilibrium point.

Next, we show the unique equilibrium point of (4) is exponential stable. Consider the following Lyapunov-Krasovskii functional:

$$V(u(t))=\sum_{i=1}^{4}V_i(u(t)) \tag{17}$$

with

$$\begin{aligned}
V_1(u(t))=&e^{2kt}\alpha^T(t)P\alpha(t)+e^{2kt}\beta^T(t)Q\beta(t),\\
V_2(u(t))=&\int_{t-\sigma(t)}^{t}e^{2ks}\gamma^T(s)R\gamma(s)\mathrm{d}s\\
&+\int_{t-\tau(t)}^{t}e^{2ks}\delta^T(s)S\delta(s)\mathrm{d}s,\\
V_3(u(t))=&\int_{t-\bar{\sigma}}^{t}e^{2ks}u^T(s)U_1u(s)\mathrm{d}s\mathrm{d}\theta\\
&+\int_{-\bar{\sigma}}^{0}\int_{t+\theta}^{t}e^{2ks}\dot{u}^T(s)(U_2+\bar{\sigma}U_3)\dot{u}(s)\mathrm{d}s\mathrm{d}\theta,\\
V_4(u(t))=&\int_{t-\bar{\tau}}^{t}e^{2ks}v^T(s)U_4v(s)\mathrm{d}s\mathrm{d}\theta\\
&+\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}e^{2ks}\dot{v}^T(s)(U_5+\bar{\tau}U_6)\dot{v}(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

where $\alpha^T(t)=[u^T(t),u^T(t-\sigma(t))]$, $\beta^T(t)=[v^T(t),v^T(t-\tau(t))]$, $\gamma^T(s)=[u^T(s),g^T(u(s)),\dot{u}^T(s)]$, $\delta^T(s)=[v^T(s),f^T(v(s)),\dot{v}^T(s)]$.

For convenience, we denote $u_\sigma=u(t-\sigma(t))$, $v_\tau=v(t-\tau(t))$. The time derivative of functional (17) along the trajectories of system (10) is obtained as follows:

$$\begin{aligned}
\dot{V}_1(u(t))=&e^{2kt}\big\{2k\alpha^T(t)P\alpha(t)+2\alpha^T(t)P\dot{\alpha}(t)\\
&+2k\beta^T(t)Q\beta(t)+2\beta^T(t)Q\dot{\beta}(t)\big\},\\
\dot{V}_2(u(t))=&e^{2kt}\big\{\gamma^T(t)R\gamma(t)+\delta^T(t)S\delta(t)\\
&-(1-\dot{\sigma}(t))e^{-2k\sigma(t)}\gamma^T(t-\sigma(t))R\gamma(t-\sigma(t))\\
&-(1-\dot{\tau}(t))e^{-2k\tau(t)}\delta^T(t-\tau(t))S\delta(t-\tau(t))\big\},\\
\dot{V}_3(u(t))=&e^{2kt}\Big\{u^T(t)U_1u(t)-e^{-2k\bar{\sigma}}u^T(t-\bar{\sigma})U_1u(t-\bar{\sigma})\\
&+\bar{\sigma}\dot{u}^T(t)(U_2+\bar{\sigma}U_3)\dot{u}(t)\\
&-\int_{t-\bar{\sigma}}^{t}e^{2k(s-t)}\dot{u}^T(s)(U_2+\bar{\sigma}U_3)\dot{u}(s)\mathrm{d}s\Big\},\\
\dot{V}_4(u(t))=&e^{2kt}\Big\{v^T(t)U_4v(t)-e^{-2k\bar{\tau}}v^T(t-\bar{\tau})U_4v(t-\bar{\tau})\\
&+\bar{\tau}\dot{v}^T(t)(U_5+\bar{\tau}U_6)\dot{v}(t)\\
&-\int_{t-\bar{\tau}}^{t}e^{2k(s-t)}\dot{v}^T(s)(U_5+\bar{\tau}U_6)\dot{v}(s)\mathrm{d}s\Big\}.
\end{aligned}$$

It is clear that the following equations are true:

$$\begin{aligned}
&\int_{t-\bar{\sigma}}^{t}\dot{u}^T(s)(U_2+\bar{\sigma}U_3)\dot{u}(s)\mathrm{d}s\\
&=\int_{t-\sigma(t)}^{t}\dot{u}^T(s)(U_2+\bar{\sigma}U_3)\dot{u}(s)\mathrm{d}s\\
&\quad+\int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{u}^T(s)(U_2+\bar{\sigma}U_3)\dot{u}(s)\mathrm{d}s,
\end{aligned}\tag{18}$$

$$\begin{aligned}
&\int_{t-\bar{\tau}}^{t}\dot{v}^T(s)(U_5+\bar{\tau}U_6)\dot{v}(s)\mathrm{d}s\\
&=\int_{t-\tau(t)}^{t}\dot{v}^T(s)(U_5+\bar{\tau}U_6)\dot{v}(s)\mathrm{d}s\\
&\quad+\int_{t-\bar{\tau}}^{t-\tau(t)}\dot{v}^T(s)(U_5+\bar{\tau}U_6)\dot{v}(s)\mathrm{d}s.
\end{aligned}\tag{19}$$

By using the Jensen integral inequality (Lemma 1), we obtain

$$\begin{aligned}
&-\int_{t-\sigma(t)}^{t}\dot{u}^T(s)U_3\dot{u}(s)\mathrm{d}s\\
&\leq-\frac{1}{\sigma(t)}\Big(\int_{t-\sigma(t)}^{t}\dot{u}(s)\mathrm{d}s\Big)^TU_3\int_{t-\sigma(t)}^{t}\dot{u}(s)\mathrm{d}s\\
&\leq-\frac{1}{\bar{\sigma}}[u(t)-u_\sigma]^TU_3[u(t)-u_\sigma],
\end{aligned}\tag{20}$$

$$\begin{aligned}
&-\int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{u}^T(s)U_3\dot{u}(s)\mathrm{d}s\\
&\leq-\frac{1}{\bar{\sigma}-\sigma(t)}\Big(\int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{u}(s)\mathrm{d}s\Big)^TU_3\int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{u}(s)\mathrm{d}s\\
&\leq-\frac{1}{\bar{\sigma}}[u_\sigma-u(t-\bar{\sigma})]^TU_3[u_\sigma-u(t-\bar{\sigma})],
\end{aligned}\tag{21}$$

$$\begin{aligned}
&-\int_{t-\tau(t)}^{t}\dot{v}^T(s)U_6\dot{v}(s)\mathrm{d}s\\
&\leq-\frac{1}{\tau(t)}\Big(\int_{t-\tau(t)}^{t}\dot{v}(s)\mathrm{d}s\Big)^TU_6\int_{t-\tau(t)}^{t}\dot{v}(s)\mathrm{d}s
\end{aligned}$$

$$\leq -\frac{1}{\bar{\tau}}[v(t)-v_\tau]^T U_6[v(t)-v_\tau], \quad (22)$$

$$-\int_{t-\bar{\tau}}^{t-\tau(t)} \dot{v}^T(s)U_6\dot{v}(s)\mathrm{d}s$$
$$\leq -\frac{1}{\bar{\tau}-\tau(t)}\Big(\int_{t-\bar{\tau}}^{t-\tau(t)} \dot{v}(s)\mathrm{d}s\Big)^T U_6 \int_{t-\bar{\tau}}^{t-\tau(t)} \dot{v}(s)\mathrm{d}s$$
$$\leq -\frac{1}{\bar{\tau}}[v_\tau - v(t-\bar{\tau})]^T U_6[v_\tau - v(t-\bar{\tau})]. \quad (23)$$

On the other hand, based on Leibniz-Newton formula, for any real matrix $X_i, Y_i, M_i, N_i (i = 1, 2)$ with compatible dimensions, we get

$$0 = 2e^{2kt}\{u^T(t)X_1^T + u_\sigma^T X_2^T\} \times \Big\{u(t) - u_\sigma - \int_{t-\sigma(t)}^{t} \dot{u}(s)\mathrm{d}s\Big\}, \quad (24)$$
$$0 = 2e^{2kt}\{u_\sigma^T Y_1^T + u^T(t-\bar{\sigma})Y_2^T\} \times \Big\{u_\sigma - u(t-\bar{\sigma}) - \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{u}(s)\mathrm{d}s\Big\}, \quad (25)$$
$$0 = 2e^{2kt}\{v^T(t)M_1^T + v_\tau^T M_2^T\} \times \Big\{v(t) - v_\tau - \int_{t-\tau(t)}^{t} \dot{v}(s)\mathrm{d}s\Big\}, \quad (26)$$
$$0 = 2e^{2kt}\{v_\tau^T N_1^T + v^T(t-\bar{\tau})N_2^T\} \times \Big\{v_\tau - v(t-\bar{\tau}) - \int_{t-\bar{\tau}}^{t-\tau(t)} \dot{v}(s)\mathrm{d}s\Big\}. \quad (27)$$

Further, one can infer from inequalities (5),(6) that the following matrix inequalities hold for any positive diagonal matrices $T_i (i = 1, 2, 3, 4)$ with compatible dimensions

$$0 \leq -2e^{2kt}\{v^T(t)L_2T_1L_1v(t) + f^T(v(t))T_1f(v(t)) - v^T(t)T_1(L_1+L_2)f(v(t))\}, \quad (28)$$
$$0 \leq -2e^{2kt}\{v_\tau^T L_2T_2L_1v_\tau + f^T(v_\tau)T_2f(v_\tau) - v_\tau^T T_2(L_1+L_2)f(v_\tau)\}, \quad (29)$$
$$0 \leq -2e^{2kt}\{u^T(t)L_4T_3L_3u(t) + g^T(u(t))T_3g(u(t)) - u^T(t)T_3(L_3+L_4)g(u(t))\}, \quad (30)$$
$$0 \leq -2e^{2kt}\{u_\sigma^T L_4T_4L_3u_\sigma + g^T(u_\sigma)T_4g(u_\sigma) - u_\sigma^T T_4(L_3+L_4)g(u_\sigma)\}. \quad (31)$$

Moreover, for any real symmetric matrices Z, W with compatible dimensions, we have

$$0 = e^{2kt}\Big\{\bar{\sigma}\kappa^T(t)Z\kappa(t) - \int_{t-\sigma(t)}^{t} \kappa^T(t)Z\kappa(t)\mathrm{d}s - \int_{t-\bar{\sigma}}^{t-\sigma(t)} \kappa^T(t)Z\kappa(t)\mathrm{d}s\Big\}, \quad (32)$$
$$0 = e^{2kt}\Big\{\bar{\tau}\omega^T(t)W\omega(t) - \int_{t-\tau(t)}^{t} \omega^T(t)W\omega(t)\mathrm{d}s - \int_{t-\bar{\tau}}^{t-\tau(t)} \omega^T(t)W\omega(t)\mathrm{d}s\Big\}, \quad (33)$$

where $\kappa^T(t) = (u^T(t), u_\sigma^T, u^T(t-\bar{\sigma}))$, $\omega^T(t) = (v^T(t), v_\tau^T, v^T(t-\bar{\tau}))$.

From (17)-(33), we obtain

$$\dot{V}(u(t)) \leq e^{2kt}\Big\{\zeta^T(t)\big(\Omega + \mathcal{A}_1^T F_1\mathcal{A}_1 + \mathcal{A}_2^T F_2\mathcal{A}_2\big)\zeta(t) - \int_{t-\sigma(t)}^{t} \rho^T(t,s)\Xi_1\rho(t,s)\mathrm{d}s - \int_{t-\bar{\sigma}}^{t-\sigma(t)} \rho^T(t,s)\Xi_2\rho(t,s)\mathrm{d}s - \int_{t-\tau(t)}^{t} \mu^T(t,s)\Xi_3\mu(t,s)\mathrm{d}s - \int_{t-\bar{\tau}}^{t-\tau(t)} \mu^T(t,s)\Xi_4\mu(t,s)\mathrm{d}s\Big\},$$

where

$$\zeta^T(t) = [\ u^T(t),\ u_\sigma^T,\ u^T(t-\bar{\sigma}),\ (1-\dot{\sigma}(t))\dot{u}^T(t-\sigma(t)), f^T(v(t)),\ f^T(v_\tau),\ v^T(t),\ v_\tau^T,\ v^T(t-\bar{\tau}), (1-\dot{\tau}(t))\dot{u}^T(t-\tau(t)),\ g^T(u(t)),\ g^T(u_\sigma)\ ],$$
$$\rho^T(t,s) = [\kappa^T(t), \dot{u}^T(s)],\ \mu^T(t,s) = [\omega^T(t), \dot{v}^T(s))].$$

From the well known Schur complement, inequality

$$\Omega + \mathcal{A}_1^T F_1\mathcal{A}_1 + \mathcal{A}_2^T F_2 < 0,$$

is equivalent to (15), thus $\dot{V}(u(t)) < 0$ holds if (11-15) are true.

Furthermore, following the similar line in [26], [27], from Lemma 2 we have

$$V(u(0)) \leq M_1||\phi(t) - x^*||^2 + M_2||\varphi(t) - y^*||^2,$$

where

$$M_1 = 4\lambda_M(P) + 3\bar{\sigma}\lambda_M(R)\big(1 + \sigma_1^2 + 3\lambda_M(A_1^TA_1)\big) + \frac{3}{2}\bar{\sigma}^2\big((\lambda_M(U_2) + \bar{\sigma}\lambda_M(U_3))\big)\lambda_M(A_1^TA_1) + \bar{\sigma}\lambda_M(U_1) + \frac{3}{2}\sigma_1^2\bar{\tau}\big(\lambda_M(B_2^TB_2) + \lambda_M(C_2^TC_2)\big) \times \big(6\lambda_M(S) + \bar{\tau}\lambda_M(U_5) + \bar{\tau}^2\lambda_M(U_6))\big),$$
$$M_2 = 4\lambda_M(Q) + 3\bar{\tau}\lambda_M(S)\big(1 + \sigma_2^2 + 3\lambda_M(A_2^TA_2)\big) + \frac{3}{2}\bar{\tau}^2\big((\lambda_M(U_5) + \bar{\tau}\lambda_M(U_6))\big)\lambda_M(A_2^TA_2) + \bar{\tau}\lambda_M(U_4) + \frac{3}{2}\sigma_2^2\bar{\sigma}\big(\lambda_M(B_1^TB_1) + \lambda_M(C_1^TC_1)\big) \times \big(6\lambda_M(R) + \bar{\sigma}\lambda_M(U_2) + \bar{\sigma}^2\lambda_M(U_3))\big),$$

and

$$\sigma_1 = \max_{1\leq i\leq n}\{|l_{3i}|, |l_{4i}|\},\ \sigma_2 = \max_{1\leq i\leq n}\{|l_{1i}|, |l_{2i}|\}.$$

Meanwhile

$$V(u(t)) \geq e^{2kt}\big(\lambda_m(P_{11})||\phi(t) - x^*||^2 + \lambda_m(Q_{11}) \times ||\varphi(t) - y^*||^2\big) \geq \frac{1}{2}e^{2kt}\min\{\lambda_m(P_{11}), \lambda_m(Q_{11})\} \times \big(||\phi(t) - x^*|| + ||\varphi(t) - y^*||\big)^2,$$

by Lyapunov stability theory, the proof of Theorem 1 is completed.

Remark 1. One can notice that the augmented Lyapunov functional approach of this paper is quite different from previous ones. New terms $e^{2kt}u^T(t-\sigma(t))P_{22}u(t-\sigma(t))$ and $e^{2kt}v^T(t-\tau(t))Q_{22}v(t-\tau(t))$ are used to augment the Lyapunov functional, whose derivatives are directly coupled with the retarded systems. Therefore, the augmented Lyapunov functional can lead to an reduce in the conservativeness of the results, which will be illustrated by four examples.

Remark 2. It should be pointed out that the conditions of the Theorems in [12], [13] need to be revised. In the proofs of Theorems in [12], [13], one same key step is the following proposition (see inequality (13) in Page 440 of [12] and inequality (15) in Page 1088 of [13]):

Proposition. *Let $L_3 = -L_4$. The following inequality holds for any given $L_4 > 0$, positive definite matrices Z:*

$$g^T(y(t))Zg(y(t)) \leq y^T(t)L_4ZL_4y(t). \tag{34}$$

Unfortunately, the above proposition is not valid in general, this fact can be illustrated by the following example:

Example 3.1. Obviously $g_1(s) = g_2(s) = \frac{1}{2}(|s+1|-|s-1|)$ satisfy conditions (3) with $L_4 = I$. Set $y^T(t) = [1 \quad -2]$, then $g^T(y(t)) = [1 \quad -1]$. Further set $Z = \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix}$, we have

$$g^T(y(t))Zg(y(t)) = 3 > 2 = y^T(t)L_4ZL_4y(t),$$

i.e. inequality (34) is false.

Based on above analysis, the LMIs of the Theorems in [12], [13] may not be sufficient conditions assumed that Z is a positive definite matrix. In fact, if Z is revised to be a positive scalar matrix, then the above proposition and the Theorems in [12], [13] are still valid.

Remark 3. If any of $\sigma(t), \tau(t)$ are not differentiable or any of $\dot{\sigma}(t), \dot{\tau}(t)$ are unknown, by setting $P_{i2} = Q_{i2} = 0 (i = 1, 2), R = S = 0$ in functional (17), following the similar line in Theorem 1 we can obtain a stability criterion similar to LMIs (11-15).

Remark 4. In terms of LMIs, Theorem 1 and Remark 3 provide a sufficient condition for the global exponential stability of the delayed BAM neural network in (10). One of the advantages of the LMI approach is that the LMI condition can be checked numerically very efficiently by using the interior-point algorithms, which have been developed recently in solving LMIs [1].

IV. Robust exponential stability results of uncertain delayed neural network

Now, based on Lemma 2 we investigate the robust exponential stability problem for system (4) with uncertainties satisfying Assumption 1. Firstly, by using functional (17) in Theorem 1, we can easily obtain the following result.

Theorem 2. *Under the assumptions* (2),(3),(7),(8) *and* $0 \leq \tau(t) \leq \bar{\tau},\ 0 \leq \sigma(t) \leq \bar{\sigma},\ 0 \leq \dot{\tau}(t) \leq \eta_1 < 1,\ 0 \leq \dot{\sigma}(t) \leq \eta_2 < 1$, *given a constant* $k \geq 0$, *suppose that there exist positive scalars* $\varepsilon_0, \varepsilon_1$, *positive definite symmetric matrices* P, Q, *nonnegative definite symmetric matrices* $R, S, Z, W, U_i(i = 1, ..., 6)$, *positive diagonal matrices* $T_i(i = 1, 2, 3, 4)$, *real matrices* $X_i, Y_i, M_i, N_i(i = 1, 2)$ *with compatible dimensions such that LMIs (11-14) and the following LMI hold:*

$$\begin{bmatrix} \Omega+\Theta & \mathcal{A}_1^TF_1 & \mathcal{A}_2^TF_2 & \Upsilon_0^TH_0 & \Upsilon_1^TH_1 \\ F_1\mathcal{A}_1 & -F_1 & 0 & F_1H_0 & 0 \\ F_2\mathcal{A}_2 & 0 & -F_2 & 0 & F_2H_1 \\ H_0^T\Upsilon_0 & H_0^TF_1 & 0 & -\varepsilon_0 I & 0 \\ H_1^T\Upsilon_1 & 0 & H_1^TF_2 & 0 & -\varepsilon_1 I \end{bmatrix} < 0, \tag{35}$$

where

$$\begin{aligned} \Theta =& \varepsilon_0[\,-G_1 \ \ 0 \ \ 0 \ \ 0 \ \ G_2 \ \ G_3 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0\,]^T \\ & [\,-G_1 \ \ 0 \ \ 0 \ \ 0 \ \ G_2 \ \ G_3 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0\,] \\ & +\varepsilon_1[\,0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ -G_4 \ \ 0 \ \ 0 \ \ 0 \ \ G_5 \ \ G_6\,]^T \\ & [\,0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ -G_4 \ \ 0 \ \ 0 \ \ 0 \ \ G_5 \ \ G_6\,], \\ \Upsilon_0 =& [\,P_{11}+R_{13}^T \ \ P_{12} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ R_{23}^T \ \ 0\,], \\ \Upsilon_1 =& [\,0 \ \ 0 \ \ 0 \ \ 0 \ \ S_{23}^T \ \ 0 \ \ Q_{11}+S_{13}^T \ \ Q_{12} \ \ 0 \ \ 0 \ \ 0 \ \ 0\,], \end{aligned}$$

and other parameters are all defined in Theorem 1, then the equilibrium point of system (4) *is robust exponential stable.*

Remark 5. Similar to Remark 3, if any of $\sigma(t), \tau(t)$ are not differentiable or any of $\dot{\sigma}(t), \dot{\tau}(t)$ are unknown, by setting $P_{i2} = Q_{i2} = 0(i = 1, 2), R = S = 0$ in functional (17), following the similar line in Theorem 2 we can obtain a stability criterion similar to LMIs (35).

Next, we deal with the robust stability for system (4) with uncertainties satisfying Assumption 2.

Theorem 3. *Under the assumptions* (2),(3),(9) *and* $0 \leq \tau(t) \leq \bar{\tau},\ 0 \leq \sigma(t) \leq \bar{\sigma},\ 0 \leq \dot{\tau}(t) \leq \eta_1 < 1,\ 0 \leq \dot{\sigma}(t) \leq \eta_2 < 1$, *given a constant* $k \geq 0$, *suppose that there exist positive scalars* $\epsilon_i(i = 1, ..., 6)$, *positive definite symmetric matrices* P, Q, *nonnegative definite symmetric matrices* $R, S, Z, W, U_i(i = 1, ..., 6)$, *positive diagonal matrices* $T_i(i = 1, 2, 3, 4)$, *real matrices* $X_i, Y_i, M_i, N_i(i = 1, 2)$ *with compatible dimensions such that LMIs (11-14) and the following LMI hold:*

$$\begin{bmatrix} \Omega+\Gamma & \mathcal{A}_1^TF_1 & \mathcal{A}_2^TF_2 & \Pi \\ F_1\mathcal{A}_1 & -F_1 & 0 & \mathcal{F}_1 \\ F_2\mathcal{A}_2 & 0 & -F_2 & \mathcal{F}_2 \\ \Pi^T & \mathcal{F}_1^T & \mathcal{F}_2^T & -\Lambda \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned} &\Gamma = [\,\Gamma_{ij}\,]_{12\times12},\ \Pi = [\,\Pi_{ij}\,]_{12\times6}, \\ &\Gamma_{11} = \epsilon_1\rho_1^2, \quad \Gamma_{55} = \epsilon_2\rho_2^2, \quad \Gamma_{66} = \epsilon_3\rho_3^2, \\ &\Gamma_{77} = \epsilon_4\rho_4^2, \quad \Gamma_{11,11} = \epsilon_5\rho_5^2, \quad \Gamma_{12,12} = \epsilon_6\rho_6^2, \\ &\Pi_{11} = -P_{11} - R_{13}, \quad \Pi_{12} = \Pi_{13} = P_{11} + R_{13}, \\ &\Pi_{21} = -P_{12}^T, \quad \Pi_{22} = \Pi_{23} = P_{12}^T, \\ &\Pi_{54} = -S_{23}, \quad \Pi_{55} = \Pi_{56} = S_{23}, \\ &\Pi_{74} = -Q_{11} - S_{13}, \quad \Pi_{75} = \Pi_{76} = Q_{11} + S_{13}, \\ &\Pi_{84} = -Q_{12}^T, \quad \Pi_{85} = \Pi_{86} = Q_{12}^T, \\ &\Pi_{11,1} = -R_{23}, \quad \Pi_{11,2} = \Pi_{11,3} = R_{23}, \\ &\mathcal{F}_1 = [\,-F_1 \ \ F_1 \ \ F_1 \ \ 0 \ \ 0 \ \ 0\,], \\ &\mathcal{F}_2 = [\,0 \ \ 0 \ \ 0 \ \ -F_2 \ \ F_2 \ \ F_2\,], \\ &\Lambda = \mathrm{diag}\{-\epsilon_1 I,\ -\epsilon_2 I,\ -\epsilon_3 I,\ -\epsilon_4 I,\ -\epsilon_5 I,\ -\epsilon_6 I\}, \end{aligned}$$

other parameters $\Pi_{ij}(i = 1, ..., 12; j = 1, ..., 6)$ are all equal to zero's and other parameters are all defined in Theorem 1, then the equilibrium point of system (4) *is robust exponential stable.*

Proof. From (9), we have

$$\lambda_M(\Delta A_1^T(t)\Delta A_1(t)) \leq \rho_1,\ \lambda_M(\Delta B_1^T(t)\Delta B_1(t)) \leq \rho_2,$$
$$\lambda_M(\Delta C_1^T(t)\Delta C_1(t)) \leq \rho_3,\ \lambda_M(\Delta A_2^T(t)\Delta A_2(t)) \leq \rho_4,$$
$$\lambda_M(\Delta B_2^T(t)\Delta B_2(t)) \leq \rho_5,\ \lambda_M(\Delta C_2^T(t)\Delta C_2(t)) \leq \rho_6.$$

Therefore, the following inequalities hold for any positive scalars $\epsilon_i(i = 1, ..., 6)$.

$$\begin{aligned}
&0 \leq \epsilon_1\big\{\rho_1^2 u^T(t)u(t) - u^T(t)\Delta A_1^T(t)\Delta A_1(t)u(t)\big\},\\
&0 \leq \epsilon_2\big\{\rho_2^2 f^T(v(t))f(v(t)) - f^T(v(t))\Delta B_1^T(t)\\
&\qquad \times \Delta B_1(t)f(v(t))\big\},\\
&0 \leq \epsilon_3\big\{\rho_3^2 f^T(v_\tau)f(v_\tau) - f^T(v_\tau)\Delta C_1^T(t)\Delta C_1(t)f(v_\tau)\big\},\\
&0 \leq \epsilon_4\big\{\rho_4^2 v^T(t)v(t) - v^T(t)\Delta A_2^T(t)\Delta A_2(t)v(t)\big\},\\
&0 \leq \epsilon_5\big\{\rho_5^2 g^T(u(t))g(u(t)) - g^T(u(t))\Delta B_2^T(t)\\
&\qquad \times \Delta B_2(t)g(u(t))\big\},\\
&0 \leq \epsilon_6\big\{\rho_6^2 g^T(u_\sigma)g(u_\sigma) - g^T(u_\sigma)\Delta C_2^T(t)\Delta C_2(t)g(u_\sigma)\big\}.
\end{aligned}$$

By using functional (17) and following the same lines as in Theorem 1,we have

$$\begin{aligned}
\dot{V}(u(t)) \leq& e^{2kt}\Big\{\xi^T(t)\big(\Omega + \Gamma + \mathcal{A}_1^T F_1\mathcal{A}_1 + \mathcal{A}_2^T F_2\mathcal{A}_2\big)\xi(t)\\
&- \int_{t-\sigma(t)}^{t} \rho^T(t,s)\Xi_1\rho(t,s)\mathrm{d}s\\
&- \int_{t-\bar{\sigma}}^{t-\sigma(t)} \rho^T(t,s)\Xi_2\rho(t,s)\mathrm{d}s\\
&- \int_{t-\tau(t)}^{t} \mu^T(t,s)\Xi_3\mu(t,s)\mathrm{d}s\\
&- \int_{t-\bar{\tau}}^{t-\tau(t)} \mu^T(t,s)\Xi_4\mu(t,s)\mathrm{d}s\Big\},
\end{aligned}$$

where

$$\begin{aligned}
\xi^T(t) =& \big[\ \zeta^T(t),\ \big(\Delta A_1(t)u(t)\big)^T,\ \big(\Delta B_1(t)f(v(t))\big)^T,\\
&\big(\Delta C_1(t)f(v_\tau)\big)^T,\ \big(\Delta A_2(t)v(t)\big)^T,\\
&\big(\Delta B_2(t)g(u(t))\big)^T,\ \big(\Delta C_2(t)g(u_\sigma)\big)^T\ \big].
\end{aligned}$$

From the well known Schur complement, we can easily obtain Theorem 3.

Remark 6. Similar to Remark 3, if any of $\sigma(t), \tau(t)$ are not differentiable or any of $\dot{\sigma}(t), \dot{\tau}(t)$ are unknown, by setting $P_{i2} = Q_{i2} = 0 (i = 1, 2), R = S = 0$ in functional (17), following the similar line in Theorem 3 we can obtain a stability criterion similar to LMIs (36).

V. Comparison and Illustrative Examples

Next, we provide four numerical examples to demonstrate the effectiveness and less conservativeness of our delay-dependent stability criteria over some recent results in the literature.

Table I Calculated maximal upper bounds of time delays $\bar{\sigma}, \bar{\tau}$ for various η_1, η_2 of Example 5.1 with $k = 0.3$

$\eta_1 = \eta_2$	0.5	0.7	0.9	unknown η_1, η_2
[7], [15]	0.7755	–	–	–
[5]	1.2071	1.0378	0.9200	0.9162
Theorem 1	1.6789	1.6763	1.6761	–
Remark 3	–	–	–	1.6742

Example 5.1. Consider system (10) with

$$A_1 = I, A_2 = 2I, B_1 = B_2 = 0,$$
$$C_1 = \begin{bmatrix} 0.05 & 0.25 & 0.05 \\ 0.1 & 0.05 & 0.15 \\ 0.15 & 0.15 & 0.05 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.75 & 0.75 & 0.95 \\ 0 & 0.5 & 0.15 \\ 0.15 & 0.15 & 0.05 \end{bmatrix},$$
$$L_1 = L_3 = 0, L_2 = L_4 = I.$$

This model was studied in [13]. For this system, it is verified that the results given in [2], [3] fail to ascertain the stability for any time delay.

Furthermore, if we set exponential convergence rate k be fixed as 0.3, none of the criteria of [4], [17] can guarantee the stability for any time delay with $\dot{\tau}(t) \neq 0$ or $\dot{\sigma}(t) \neq 0$. Set $\eta_1 = \eta_2 \geq 1$, all of the criteria given in [7], [15] fail to verify the stability for any time delay, the allowable time delay upper bound obtained by Gau et al. [5] is 0.9162, while our method shows that the system is exponentially stable for any time delay with $\tau(t) \leq 1.6742, \sigma(t) \leq 1.6742$. This is much larger than the one of [5], which shows the less conservativeness of our developed method. The maximal upper bounds of time delays $\bar{\sigma}, \bar{\tau}$ for various η_1, η_2 from Theorem 1 and Remark 3 in this paper and those in [5], [7], [15] are listed in Table I, where "–" means that the result is not applicable to the corresponding case, and "unknown η_1, η_2" means that η_1, η_2 can be arbitrary value or $\tau(t), \sigma(t)$ can be not differentiable. It is clear that the results in this paper are markedly better than those in [2]–[5], [7], [15], [17].

Example 5.2. Consider system (10) with

$$A_1 = I, A_2 = 4I, B_1 = B_2 = 0,$$
$$C_1 = \begin{bmatrix} 0.05 & 0.10 & 0.15 \\ 0.25 & 0.05 & 0.15 \\ 0.05 & 0.15 & 0.05 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.75 & 0 & 0.15 \\ 0.75 & 0.50 & 0.95 \\ 0.95 & 0.75 & 0.95 \end{bmatrix},$$
$$L_1 = L_3 = 0, L_2 = L_4 = I.$$

This model was studied in [5], [7]. For this system, it is verified in [7] that the results given in [2], [3] fail to ascertain the stability for any time delay. Furthermore, if we set exponential convergence rate k be fixed as 0.35, none of the criteria of [4], [17] can guarantee the stability for any time delay with $\dot{\tau}(t) \neq 0$ or $\dot{\sigma}(t) \neq 0$. Set $\eta_1 = \eta_2 \geq 1$, all of the criteria given in [7], [15] fail to verify the stability for any time delay, the upper bound of allowable time delay obtained by Gau et al. [5] is 0.5110, while our method shows

Table II Calculated maximal upper bounds of time delays $\bar{\sigma}, \bar{\tau}$ for various η_1, η_2 of Example 5.2 with $k = 0.35$

$\eta_1 = \eta_2$	0.5	0.8	0.9	unknown η_1, η_2
[7], [15]	0.9215	–	–	–
[5]	1.2660	0.8495	0.6853	0.5110
Theorem 1	1.4714	1.3478	1.3459	–
Remark 3	–	–	–	1.3262

that the system is exponentially stable for any time delay with $\tau(t) \leq 1.3262, \sigma(t) \leq 1.3262$. This is much larger than the one of [5], which shows the less conservativeness of our developed method. The maximal upper bounds of time delays $\bar{\sigma}, \bar{\tau}$ for various η_1, η_2 from Theorem 1 and Remark 3 in this paper and those in [5], [7], [15] are listed in Table II. It is clear that the results in this paper are much less conservativeness than those in [2]–[5], [7], [15], [17].

Example 5.3. Consider system (1) with Assumption 1 and the following parameters:

$$A_1 = \mathrm{diag}\{2.2, 1.3\},\ A_2 = \mathrm{diag}\{1.2, 1.1\},$$
$$C_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.15 & 0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$
$$J_1 = [\ 3 \quad -2\ ]^T,\ J_2 = [\ 1 \quad -4\ ]^T,$$
$$H_0 = H_1 = I, G_1 = G_3 = G_4 = G_6 = -0.2I,$$
$$\bar{B}_1 = \bar{B}_2 = 0, E_0(t) = E_1(t) = e^{-t}I,$$
$$f_1(s) = f_2(s) = g_1(s) = g_2(s) = 0.5(|s+1| - |s-1|).$$

Obviously, the activation functions are bounded and satisfy assumptions (2) and (3) with

$$L_1 = L_3 = 0,\ L_2 = L_4 = I.$$

This model was studied in [5], [11]. For this system, the result given in [11] fails to ascertain the stability for any time delay with $\dot{\tau}(t) \neq 0$ or $\dot{\sigma}(t) \neq 0$. Furthermore, if we set exponential convergence rate k be fixed as 0.1, when $\sigma(t), \tau(t)$ are not differentiable, the upper bound of delay obtained by [5] is $\bar{\sigma} = \bar{\tau} = 1.3661$. However, by Remark 5, we can prove that the equilibrium point of this system is robust exponential stable for any time delays with $\tau(t) = \sigma(t) \leq 4.0227$. For $\tau(t) = \sigma(t) = 4.0227$, a feasible solution of LMIs (10)-(14) are given by

$$P_{11} = \begin{bmatrix} 6.2833 & -0.6337 \\ -0.6337 & 5.7874 \end{bmatrix},$$
$$Q_{11} = \begin{bmatrix} 3.6035 & -0.3516 \\ -0.3516 & 2.8841 \end{bmatrix},$$
$$Z_{11} = \begin{bmatrix} 0.8431 & -0.3499 \\ -0.3499 & 0.1529 \end{bmatrix},$$
$$Z_{12} = \begin{bmatrix} -0.0204 & 0.0198 \\ 0.0175 & -0.0173 \end{bmatrix},$$
$$Z_{13} = \begin{bmatrix} -0.0009 & -0.0020 \\ 0.0006 & 0.0002 \end{bmatrix},$$
$$Z_{22} = \begin{bmatrix} 0.0993 & -0.0159 \\ -0.0159 & 0.0814 \end{bmatrix},$$
$$Z_{23} = \begin{bmatrix} -0.0398 & 0.0087 \\ 0.0157 & -0.0435 \end{bmatrix},$$
$$Z_{33} = \begin{bmatrix} 0.4500 & 0.0333 \\ 0.0333 & 0.3318 \end{bmatrix},$$
$$W_{11} = \begin{bmatrix} 0.1578 & -0.1274 \\ -0.1274 & 0.1033 \end{bmatrix},$$
$$W_{12} = \begin{bmatrix} -0.0220 & 0.0212 \\ 0.0164 & -0.0164 \end{bmatrix},$$
$$W_{13} = \begin{bmatrix} -0.0010 & 0.0007 \\ 0.0007 & -0.0005 \end{bmatrix},$$
$$W_{22} = \begin{bmatrix} 0.0562 & -0.0326 \\ -0.0326 & 0.0797 \end{bmatrix},$$
$$W_{23} = \begin{bmatrix} -0.0944 & 0.0074 \\ -0.0101 & -0.0875 \end{bmatrix},$$
$$W_{33} = \begin{bmatrix} 0.3231 & 0.0043 \\ 0.0043 & 0.2437 \end{bmatrix},$$
$$U_1 = \begin{bmatrix} 5.0573 & 0.6059 \\ 0.6059 & 3.6529 \end{bmatrix},$$
$$U_2 = \begin{bmatrix} 0.6987 & -0.1052 \\ -0.1052 & 0.9836 \end{bmatrix},$$
$$U_3 = \begin{bmatrix} 0.0096 & -0.0097 \\ -0.0097 & 0.0097 \end{bmatrix},$$
$$U_4 = \begin{bmatrix} 2.9251 & 0.0251 \\ 0.0251 & 2.2011 \end{bmatrix},$$
$$U_5 = \begin{bmatrix} 0.6836 & -0.0375 \\ -0.0375 & 0.6165 \end{bmatrix},$$
$$U_6 = \begin{bmatrix} 0.0012 & -0.0009 \\ -0.0009 & 0.0006 \end{bmatrix},$$
$$T_1 = 10^{-3} \times \mathrm{diag}\{0.1667, 0.0826\},$$
$$T_2 = \mathrm{diag}\{0.7313, 0.6215\},$$
$$T_3 = \mathrm{diag}\{0.0073, 0.0016\},$$
$$T_4 = \mathrm{diag}\{0.2548, 0.3577\},$$
$$X_1 = \begin{bmatrix} -0.0065 & 0.0364 \\ 0.0064 & -0.0363 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.1484 & -0.0234 \\ 0.0248 & 0.1583 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} -0.1529 & 0.0264 \\ -0.0185 & -0.1632 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 0.0399 & -0.0470 \\ -0.0407 & 0.0479 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} -0.0295 & 0.0198 \\ 0.0207 & -0.0140 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0.0299 & 0.0190 \\ -0.0818 & 0.1101 \end{bmatrix},$$
$$N_1 = \begin{bmatrix} -0.0840 & -0.0214 \\ 0.0101 & -0.0960 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.2973 & -0.0208 \\ 0.0182 & 0.2568 \end{bmatrix},$$
$$\varepsilon_0 = 7.2274, \varepsilon_1 = 0.0231$$

This implies that for this example the stability conditions in this paper are less conservative than that in [5], [11].

For this model with constant delay $\tau(t) = \sigma(t) \equiv 4$, from Theorem 1 we can verify that the equilibrium point

Table III Calculated maximal upper bounds of time delays $\bar{\sigma}, \bar{\tau}$ for various η_1, η_2, k of Example 5.4

$\eta_1 = \eta_2$	0.5	0.5	unknown	unknown
k	0.1	0.2	0	0.1
[5]	2.11	0.81	1.226	0.661
Theorem 3	3.4871	1.4826	–	–
Remark 6	–	–	4.8096	2.7026

$(\frac{347}{264}, -\frac{157}{104}, \frac{11}{12}, -\frac{42}{11})^T$ is robust stable. The stability with the initial state $(2, -2, 4, -1)^T$ is shown in Fig. 1.

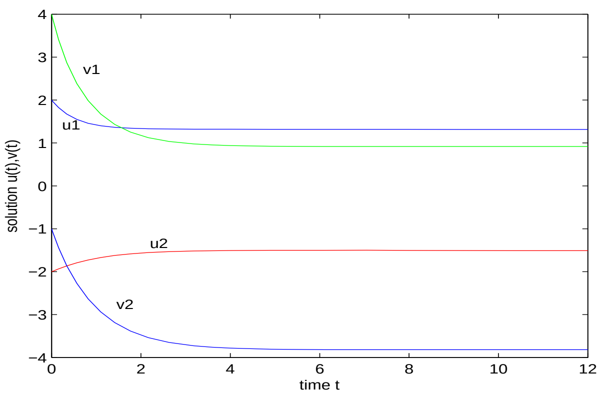

Fig. 1. The state responses of system (1) in Example 5.3.

Example 5.4. Consider system (4) with Assumption 2 and the following parameters:

$$A_1 = \text{diag}\{2.2, 1.3\}, \;\; A_2 = \text{diag}\{1.2, 1.1\},$$
$$C_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.15 & 0.1 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$
$$\rho_1 = \rho_4 = 0.5, \rho_3 = \rho_6 = 0.1,$$
$$\bar{B}_1 = \bar{B}_2 = L_1 = L_3 = 0, \;\; L_2 = L_4 = I.$$

This model was studied in [5]. Set $k = 0, \eta_1 = \eta_2 \geq 1$, the allowable time delay upper bound obtained by Gau et al. [5] is 1.226, while our method shows that the system is exponentially stable for any time delay with $\tau(t) \leq 4.8096, \sigma(t) \leq 4.8096$. This is much larger than the one of [5], which shows the less conservativeness of our developed method. The maximal upper bounds of time delays $\bar{\sigma}, \bar{\tau}$ for various η_1, η_2 from Theorem 3 and Remark 6 in this paper and those in [5] are listed in Table III. It is clear that the results in this paper are much more effectiveness than that in [5].

VI. Conclusion

In this paper we have investigated the global robust stability problem of delayed uncertain BAM neural networks. By employing new Lyapunov Krasovskii functional, we proposed several novel stability criteria for the considered systems. The obtained results are all in the form of LMI, which can be easily optimized. Finally, four examples are given to show the superiority of our proposed stability conditions to some existing ones.

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