Numerical study of double diffusive convection in a cylindrical annular geometry
Validation and mesh sensibility

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Abstract

In this paper, we consider the problem of double diffusion in an annular cylindrical space with an aspect ratio R=2. The ADI method has been used to describe the two-dimensional symmetric solutions in terms of isotherms, streamlines and iso-concentrations. This numerical study has been validated both by comparing the results to the pure thermal case and by analyzing the case of weak solutal buoyancies. The influence of the mesh on the numerical results was also studied. The influence of thermal and solutal buoyancies was investigated.

Keywords: *Natural convection, Double diffusion, Finite difference, Buoyancy ratio.*

1. Introduction

The considerable interest which has been given recently to the study of heat and mass transfer is mainly due to its presence in many industrial applications such as separation processes in chemical and oil industries, the nuclear waste storage, the dispersion of chemical contaminants in groundwater layers, the drying process, the desalination of sea water etc.

Several experimental studies have been performed, and some numerical and analytical solutions to the problems associated with this phenomenon have been proposed.

The present work is a contribution to the numerical simulation of the thermal and solutal convection in an annular space between two coaxial cylinders subjected to gradients of temperature and concentration in the radial direction (Fig.1), with a density represented by the equation:

$$\rho = \rho_0 \left[1 - \beta_T (T - T_0) - \beta_s (C - C_0)\right] \quad (1)$$

Where β_T and β_s s are respectively the coefficients of thermal and solutal expansion, T_0 and C_0 are a temperature and a concentration of reference.

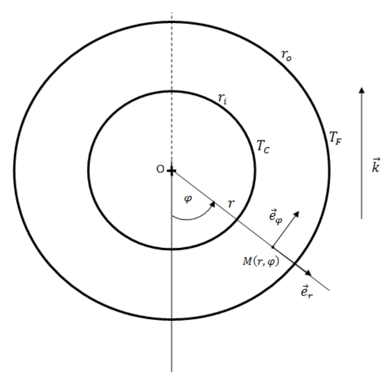

Fig.1 Definition diagram

In this geometry, the problem of thermal diffusion has been exhaustively studied [1-3]. This paper deals with the double diffusion, thermal and solutal, which has been investigated in recent years in square or rectangular cavities [4, 5].

2. Mathematical formulation and numerical method

The non-dimensional equations governing the two-dimensional problem are written in polar coordinates (r,φ), using the stream function-vorticity formulation ψ - Ω.

$$\Delta\psi + \omega = 0 \quad (2)$$

$$\frac{\partial \omega}{\partial t} + \frac{1}{r}\left[\frac{\partial \psi}{\partial \varphi}\frac{\partial \omega}{\partial r} - \frac{\partial \psi}{\partial r}\frac{\partial \omega}{\partial \varphi}\right] = Pr\Delta\omega + Ra_T Pr\left(\frac{\cos\varphi}{r}\frac{\partial T}{\partial \varphi} + \sin\varphi\frac{\partial T}{\partial r}\right) + NRa_T Pr\left(\frac{\cos\varphi}{r}\frac{\partial C}{\partial \varphi} + \sin\varphi\frac{\partial C}{\partial r}\right) \quad (3)$$

$$\frac{\partial T}{\partial t} + \frac{1}{r}\left(\frac{\partial \psi}{\partial \varphi}\frac{\partial T}{\partial r} - \frac{\partial \psi}{\partial r}\frac{\partial T}{\partial \varphi}\right) = \Delta T \quad (4)$$

$$\frac{\partial C}{\partial t} + \frac{1}{r}\left(\frac{\partial \psi}{\partial \varphi}\frac{\partial C}{\partial r} - \frac{\partial \psi}{\partial r}\frac{\partial C}{\partial \varphi}\right) = \frac{1}{Le}\Delta C \quad (5)$$

With the boundary conditions on the cylindrical frontiers, and symmetry conditions:

$$\boldsymbol{r} = \boldsymbol{1} : T = 1 ; C = 1 ; \psi = 0 ; \frac{\partial \psi}{\partial r} = 0; \frac{\partial^2 \psi}{\partial r^2} + \omega = 0 \quad \forall \varphi$$

$$\boldsymbol{r} = \boldsymbol{R} : T = 0; C = 0 ; \psi = 0 ; \frac{\partial \psi}{\partial r} = 0 ; \frac{\partial^2 \psi}{\partial r^2} + \omega = 0 \quad \forall \varphi \quad (6a)$$

$$\boldsymbol{\varphi} = \boldsymbol{0} \;\; et \;\; \boldsymbol{\varphi} = \boldsymbol{\pi} : \psi = 0; \;\; \frac{\partial T}{\partial \varphi} = 0; \;\; \frac{\partial C}{\partial \varphi} = 0; \;\; \omega = 0 \quad \forall r \quad (6b)$$

The problem is characterized by 5 dimensionless parameters. They are: the thermal Rayleigh number:

$Ra_T = g\beta_T \Delta T r_i^3 / \nu a$,the Lewis number: $Le = a/D$, the Prandtl number : $Pr = \nu/a$, the ratio: $N = \beta_s \Delta C / \beta_T \Delta T$, and the geometrical ratio (the cylindrical rays' ratio): $R = r_o/r_i$.

These equations are discretized in centered finite differences, using the Alternating Direction Implicit method ADI [6]. At every step of time, we solve algebraic systems with tridiagonal matrixes. The reversing of these matrixes is done by the Thomas algorithm.

The initial conditions correspond to the pure conduction situation (giving the initial thermal field as concentric circles). In our study, the solutions we are in search of are stationary solutions. The criterion of stopping the calculations depends on assessment of the variables residues, as well as on the calculations of the global numbers of Nusselt Nug and Sherwood Shg, characterizing the heat and mass transfer respectively. These are defined respectively by:

$$Nug = -\frac{1}{\pi} LogR \int_0^{\pi} \frac{\partial T}{\partial r}\Big|_{r=1} d\varphi \quad (7a)$$

$$Shg = -\frac{1}{\pi} LogR \int_0^{\pi} \frac{\partial C}{\partial r}\Big|_{r=1} d\varphi \quad (7b)$$

3. Validation

To validate the numerical code developed, we conducted tests for R = 2 and Pr = 0.7, considering two thermal Rayleigh numbers: $Ra_T = 3000$ and $Ra_T = 10000$. The code has been validated by comparison with the pure thermal case and then in the thermosolutal case [7].

3.1 Pure thermal case

At first, the code has been validated by comparison with the purely thermal case ($N = 0, Le = 1$). The results obtained with our code for $N = 0$, are strictly identical to those obtained with a code dealing with thermal convection .
It is observed that the flow consists of a single convective cell having a shape of crescent, occupying a half annular space and which rotates in the direction of clockwise. The fluid rises against the hot inner cylinder. At the top, it cools down against the cold cylinder (Fig. 2 and Fig. 3).

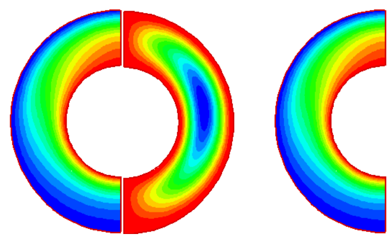

Fig.2 Isotherms, stream-lines and iso-concentrations for $Ra_T = 3000, Le = 1 \; and \; N = 0$

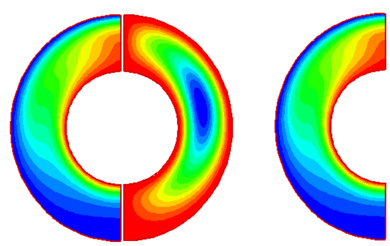

Fig. 3 Isotherms, stream-lines and iso-concentrations For $Ra_T = 10000, Le = 1 \; and \; N = 0$

3.2 Thermosolutal case

For low absolute values of N, $-0.2 \leq N \leq 0.2$, we verified that when N tends to 0 by higher or lower values, the global number of Nusselt Nug tends to the value obtained for the pure thermal case (N=0). This was done for $Ra_T = 3000$ and $Ra_T = 10000$ and different values of Lewis number (Le=1, 10 and 20) (Fig. 4 and Fig. 5). The same verifications were made for the global number of Sherwood Shg (Fig. 6 and Fig. 7) and for the maximal stream function ψ_{max}(Fig. 8 and Fig. 9).

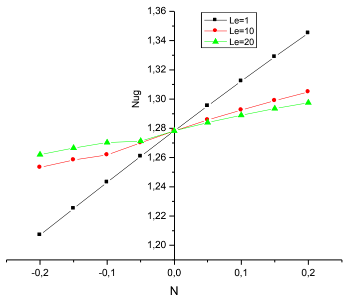

Fig. 4 Effect of N on Nug for $Ra_T = 3000$ and different values of Le (1, 10, 100)

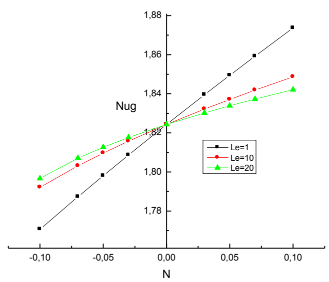

Fig. 5 Effect of N on Nug for $Ra_T = 10000$ and different values of Le (1, 10, 100)

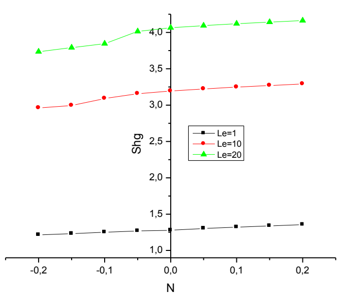

Fig.6 Effect of N on Shg for $Ra_T = 3000$ and different values of Le $(1, 10, 100)$

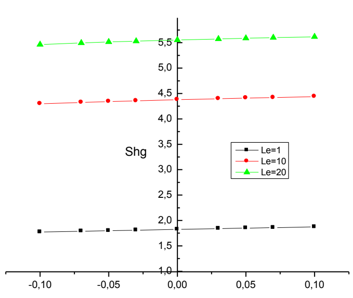

Fig.7 Effect of N on Shg for $Ra_T = 10000$ and different values of Le $(1, 10, 100)$

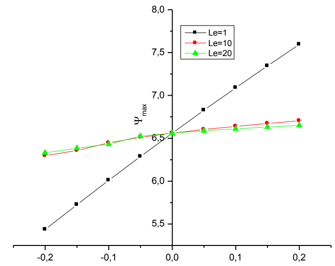

Fig. 8 Effect of N on ψ_{max} for $Ra_T = 3000$ and different values of Le $(1, 10, 100)$

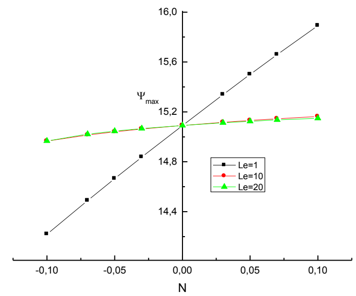

Fig.9 Effect of N on ψ_{max} for $Ra_T = 10000$ and different values of Le $(1, 10, 100)$

4. Mesh effect

The computational domain is discretized with a uniform mesh$(I_M \times J_M)$. The accuracy of the numerical results depends on the mesh. The more it is refined the better is the observation of physical phenomena.
The choice of the optimal mesh depends also on various control parameters (aspect ratio, Lewis number, Prandtl number, ...) and on numerical errors.
In our study, the results are obtained for a mesh (65 x81). The chosen mesh is a mesh that meets the criterion accuracy / computation time . It helps to have extremely small differences with the asymptotic values of Nug, Shg and ψ_{max} for a given thermal Rayleigh number(Table 1, Table 2 and Table 3). In general, we find that when the mesh is more refined, the results tend to asymptotic values. These asymptotic values were determined from the results obtained for the most refined meshes.
For $Ra_T = 3000$, $N = 1$ and $Le = 1$, this difference is about 2.47 ‰ for Nug and 0.027 ‰ for ψ_{max} (Table 1).
For $Ra_T = 10000$, $N = 1$ and $Le = 1$, it is about 3.57 ‰ for Nug and 0.0024 ‰ for ψ_{max} (Table 1).
For $Ra_T = 3000$, $N = 1$ and $Le = 10$, values of Nug, Shg , and ψ_{max} ,for the chosen mesh, differ from asymptotic values respctively by 5.02‰ ,3.89‰ and 0.21‰ (Table 2).
For $Ra_T = 10000$, $N = 1$ and $Le = 10$, These differences are about 6.59 ‰ for Nug, 4.56 ‰ for Shg and 0.4 ‰ for ψ_{max} (Table 2).

For $Ra_T = 3000$, $N = 2$ and $Le = 5$, the relative error is about 5.886‰ for Nug, 4.607‰ for Shg and 0.11‰ for ψ_{max} (Table 3).
For $Ra_T = 10000$, $N = 2$ and $Le = 5$, this error is about 7.88‰ for Nug, 5.14‰ for Shg, and 0.38‰ for ψ_{max} (Table 3).
Nug, Shg and ψ_{max} are plotted against $1/\Delta S$, where ΔS is the average spacing(Fig.10 to Fig.25).
With: $\Delta S = r\Delta r\Delta\varphi$, r is the average radius. We consider $r = \frac{1+R}{2}$

$\Delta r = \frac{R-1}{I_M - 1}$: Step of radial discretization.

$\Delta\varphi = \frac{\pi}{J_M - 1}$: Step of tangential discretization.

In the case $R = 2$:

$$\frac{1}{\Delta S} = \frac{2(I_M - 1)(J_M - 1)}{3\pi}$$

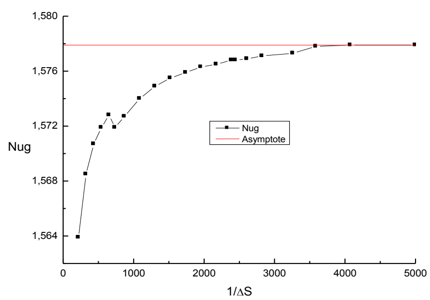

Fig.10 Effect of the mesh on Nug for $Ra_T = 3000$, $N = 1$ and $Le = 1$

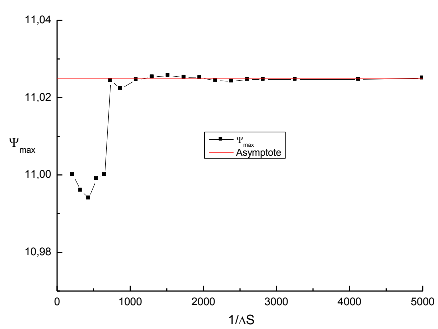

Fig.11 Effect of the mesh on ψ_{max} for $Ra_T = 3000$, $N = 1$ and $Le = 1$

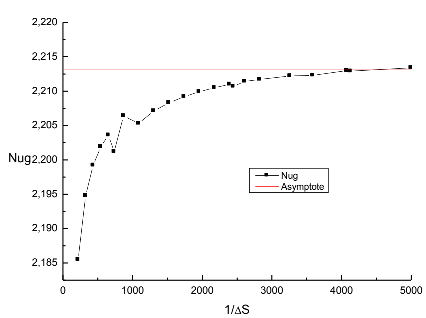

Fig.12 Effect of the mesh on Nug for $Ra_T = 10000$, $N = 1$ and $Le = 1$

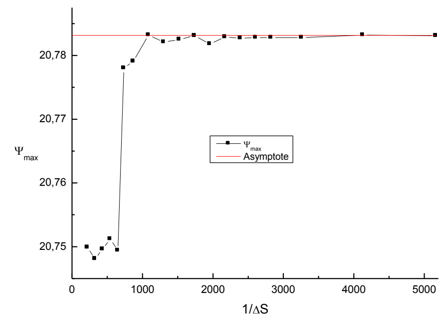

Fig.13 Effect of the mesh on ψ_{max} for $Ra_T = 10000$, $N = 1$ and $Le = 1$

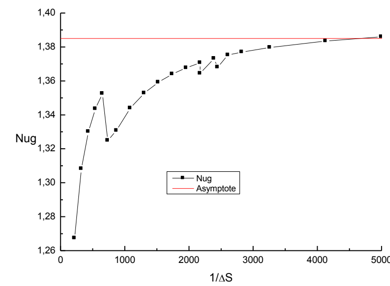

Fig.14 Effect of the mesh on Nug for $Ra_T = 3000$, $N = 1$ and $Le = 10$

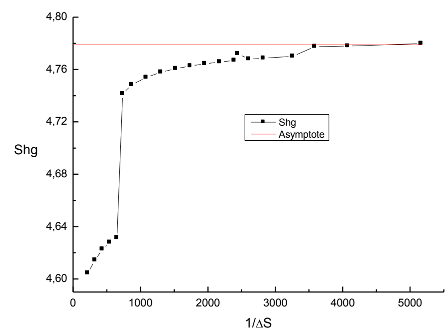

Fig.15 Effect of the mesh on Shg for $Ra_T = 3000$, $N = 1$ and $Le = 10$

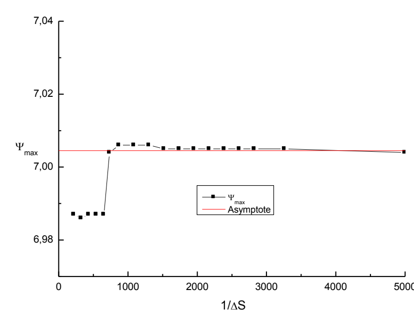

Fig.16 Effect of the mesh on ψ_{max} for $Ra_T = 3000$, $N = 1$ and $Le = 10$

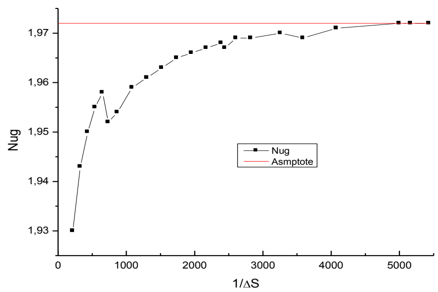

Fig.17 Effect of the mesh on Nug for $Ra_T = 10000$, $N = 1$ and $Le = 10$

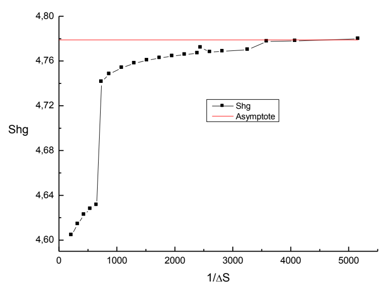

Fig.18 Effect of the mesh on Nug for $Ra_T = 10000$, $N = 1$ and $Le = 10$

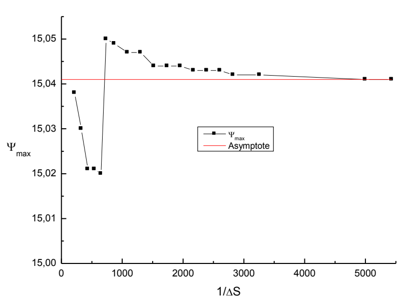

Fig.19 Effect of the mesh on ψ_{max} for $Ra_T = 10000$, $N = 1$ and $Le = 10$

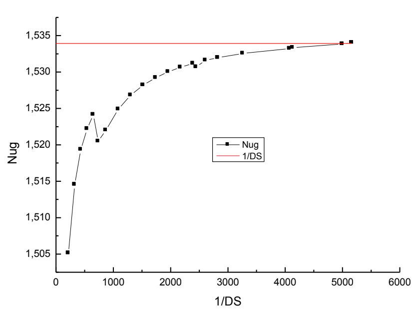

Fig.20 Effect of the mesh on Nug for $Ra_T = 3000$, $N = 2$ and $Le = 5$

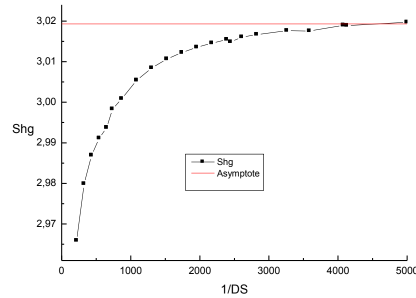

Fig.21 Effect of the mesh on Shg for $Ra_T = 3000$, $N = 2$ and $Le = 5$

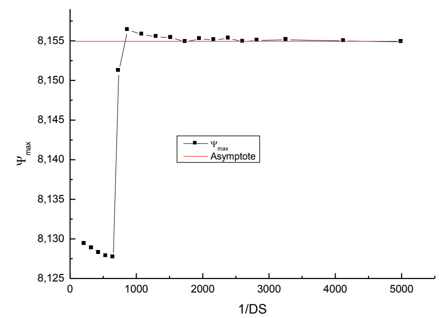

Fig.22 Effect of the mesh on ψ_{max} for $Ra_T = 3000$, $N = 2$ and $Le = 5$

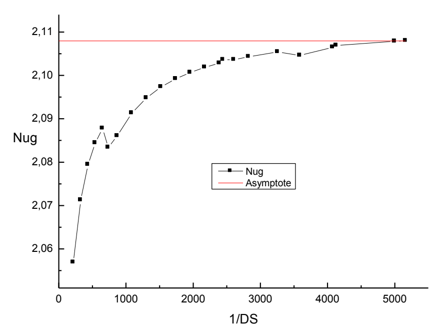

Fig.23 Effect of the mesh on Nug for $Ra_T = 10000$, $N = 2$ and $Le = 5$

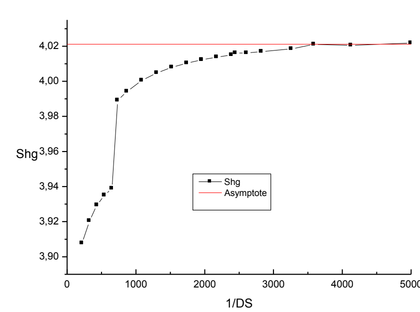

Fig.24 Effect of the mesh on Shg for $Ra_T = 10000$, $N = 2$ and $Le = 5$

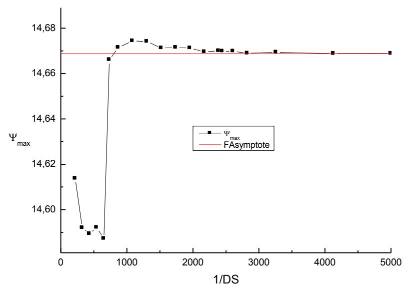

Fig.25 Effect of the mesh on ψ_{max} for $Ra_T = 10000$, $N = 2$ and $Le = 5$

Ra_T	Mesh 65X81		Asymptotic values	
	ψ_{max}	Nug	ψ_{max}	Nug
3000	11.0246 (0.027‰)	1.5740 (2.47‰)	11.0249	1.5779
10000	20.7832 (0.002‰)	2.2053 (3.57‰)	20.7832	2.2132

Table 1: Influence of mesh size on Nug and ψ_{max} for $N = 1$, $Le = 1$, $Pr = 0.7$ and $R = 2$

Ra_T	Mesh 65X81			Asymptotic Values		
	ψ_{max}	Nug	Shg	ψ_{max}	Nug	Shg
3000	7.006	1.388	3.4993	7.0045	1.395	3.5136
	0.21‰	5.02‰	3.89‰			
10000	15.047	1.959	4.7540	15.041	1.972	4.7758
	0.4 ‰	6.59‰	4.56‰			

Table 2: Influence of mesh size on ψ_{max}, Nug and Shg for $N = 1$, $Le = 10$, $Pr = 0.7$ and $R = 2$

Ra_T	Mesh 65X81			Asymptotic Values		
	ψ_{max}	Nug	Shg	ψ_{max}	Nug	Shg
3000	8.1559	1.5249	3.0054	8.1549	1.5339	3.0193
	0.11‰	5.886‰	4.607‰			
10000	14.6744	2.0913	4.0005	14.6688	2.1079	4.0219
	0.38‰	7.88‰	5.14‰			

Table 3: Influence of mesh size on ψ_{max}, Nug and Shg for $N = 2$, $Le = 5$, $Pr = 0.7$ and $R = 2$

5. Conclusion

The problem of double diffusion in an annular cylindrical space with an aspect ratio of R = 2, was considered. The ADI method has been implemented and used to describe the two-dimensional symmetric solutions in terms of isotherms, streamlines and iso-concentrations. This numerical study has been validated both by comparing the results to the pure thermal case and by analyzing the case of weak solutal buoyancy forces compared to thermal ones. Then a study of sensibility of the results to the mesh was led.

Nomenclature

a	Thermic diffusivity
C	Non –dimensional Concentration
D	Massic diffusivity
g	Gravitation acceleration
Le	Lewis number
N	Ratio of solutal and thermic buoyancies
Nug	Global number of Nusselt
Pr	Prandtl number
R	Radius ratio
r	Non-dimensional radial coordinate
r_i	Inner cylinder radius
r_o	Outer cylinder radius
Ra_T	Thermal Rayleigh number
Shg	Global number of Sherwood
T	Non-dimensional temperature
T_C	Temperature of inner wall
T_F	Temperature of outer wall
ν	Cinematic Viscosity
μ	Dynamic Viscosity
λ	Thermic conductivity
ρ	Fluid density
ψ	Stream function
β	Coefficient of thermic or solutal expansion
ω	Vorticity
φ	Polar coordinate

Biography

B. Cheddadi is a professor in Superior School of Technology, University Hassan II in Casablanca. She had Engineer degree from E.N.S.E.M. (National Superior School of Electricity and Mechanics) in 1992 and she had doctorate degree from E.M.I.(Mohammadia Shool of Engineers) in 2013. Professor R. Aboulaich and Professor A. Cheddadi are the supervisors of her research works. They are respectively the director of laboratory of studies and researchs in applied mathematics and the director of laboratory of researchs in thermal systems and real flows in Mohammadia School of Engineers, University Mohammed V in Rabat.

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