IJCSI International Journal of Computer Science Issues, Vol. 11, Issue 2, No 1, March 2014
ISSN (Print): 1694-0814 | ISSN (Online): 1694-0784
www.IJCSI.org

On Marking of Continuous Generalized Timed Events Graphs

Samir HAMACI, Karim LABADI and A.-Moumen DARCHERIF

School of Electrical Engineering and Industrial Management (EPMI)/ECS-Lab,
13, Boulevard de l'Hautil, Cergy-Pontoise, 95092, France

Abstract

We study fluid analogues of a subclass of Petri Nets, called Continuous Generalized Timed Event Graphs, which are a time extension of weighted T-Systems studied in the Petri Net literature. These event graphs can be studied in the algebraic structure called (min,+) algebra. In this paper we deal with the problem of allocating an initial marking in a Continuous Generalized Timed Event Graphs for a desired cycle time. for that, to calculate the marking of some places, we proceed by linearization of the mathematical model reflecting the behavior of these graphs in order to obtain a (min,+) linear model. From the latter, we determine the marking which satisfiers the desired cycle time fixed initially.

Keywords: *Timed Petri net, Fluid Timed Event Graphs, (min,+) algebra, Cycle time, Linearization.*

1. Introduction

Timed Event Graphs (TEG) are well adapted to model synchronization phenomena occurring in discrete event systems. Their behavior can be modelled by recurrent linear equations in (min, +) algebra [2]. The marking of a place in a TEG may correspond to the state of a device, e.g. a machine is or is not available. This marking can be compared to a boolean variable. A marking can also be associated with an integer, e.g. the number of parts in the input buffer of a machine. In this second case, the number of tokens may be a large number. This may result in such a large number of reachable markings that a limit is formed for use of TEG. When the size of the model becomes very significant, techniques of analysis developed for these graphs reach their limits. A possible alternative consists in using Continuous Generalized Timed Event Graphs, denoted CGTEG. Indeed, the use of weights associated with arcs is natural to model a large number of systems, for example, when the achievement of a specific task requires several units of a same resource, or when an assembly operation requires several units of a same part.

Synchronisation is not specific to discrete systems, and we will consider here Continuous analogues of Generalized Timed Event Graphs (CGTEG) in which fluids circulate instead of tokens. For instance, in chemical processes, synchronisation (stoichiometry here) is essential and the products used in a chemical reaction may be fluids.

In this paper we deal with the problem of allocating an initial marking for a some places of control circuit (Part the slowest of the system) in a CGTEG, strongly connected and neutral [10], for a desired cycle time. In the literature several approach have been developed to study the problem of allocating a number of tokens in a discrete Petri net. In [7] the authors deal with the problem in a Cyclic timed event-graphs so as to minimize the cycle time of the graph. Note that both the initial marking and the cycle time are decision variable in this approach. This problem has a practical relevance: as an example, in the manufacturing domain it corresponds to determining the optimal allocation of a finite set of resources so as to maximize the throughput. Another method [11] deals with the scheduling of shared resources in cyclic problem. This scheduling problem is solved by transforming a petri net with conflicts into a marked graph. this latter can be easily analyzed thank's to (min,+) dioid in order to compute analytically its behavior. To our Knowledge, no work has been done to solve the problem of allocating a marking of CGTEG. For that, we propose a method based on the linearization of the mathematical model reflecting the behavior of a CGTEG in order to obtain a model (min, +) linear. From the latter, we determine the marking which satisfiers the desired cycle time.

The fluid case, considered here, is much simpler than the discrete case considered by Munier. The linearization procedure does not increase the number of transitions of the system, while the expansion procedure of [9] results in a blow up of the number of transitions. Another linearization method was proposed in [10] when each elementary circuit of graph contains at least one *normalized* transition (*i.e.,* a transition for which its corresponding elementary T-semiflow component is equal to one). This method increases the number of transitions. Inspired by this work, a linearization method without increasing the number of transition was proposed in [6].

This paper is organized as follows. Some concepts on CGTEG and their functioning are recalled in Section 2. The method of linearization is presented in Section 3. From the equivalent, or approached, ordinary Continuous TEG of a CGTEG, we deduce the marking of some places of control circuit in a CGTEG for a desired cycle time in the Section 4. We give a short example before concluding.

2. Nonlinear Dynamic Behavior of CGTEG

Let us introduce the Continuous Timed Petri net with weights we consider here. These models were introduced by G. Cohen [3],[4].

A *Continuous Timed Petri net with weights* is a valued bipartite graph given by a five-tuple $(\mathcal{P}, \mathcal{T}, M, m, \tau)$.

- $\mathcal{P}$ and $\mathcal{T}$ represent the finite set of *places*, and *transitions* respectively;
- A *weight* ***M*** is associated with each *arc*. Given $q \in T$ and $p \in P$, the weight M_{pq} (respectively, M_{qp}) specifies the weight (in $\mathbb{R}$) of the arc from transition ***q*** to place ***p*** (respectively, from place ***p*** to transition ***q***). A zero value for M codes an absence of arc;
- With each place are associated an *initial marking* (m_p in $\mathbb{R}$) in place ***P*** and a *holding time* (τ_p gives the minimal time a token must spend in place ***p*** before it can contribute to the enabling of its downstream transitions).

These models are an approximation of discrete Petri net since they provide an upper bound for the real behavior.The firing of a transition in a CGTEG is done in a real amount [3],[4].

-Unlike the models proposed by H. Alla [1], there is no restriction at the crossing transitions.

We denote by ${}^{\bullet}q$ (resp., $q^{\bullet}$) the set of places upstream (resp., downstream) transition q. Similarly, ${}^{\bullet}p$ (resp., $p^{\bullet}$) denotes the set of transitions upstream (resp., downstream) place p.

An *event graph* is a Petri net whose each place has exactly one upstream and one downstream transition.

We denote W the incidence matrix of a Petri net. A vector $\theta \in \mathbb{R}^T$ such that $\theta \neq 0$ and $W\theta = 0$ is a T-semiflow. A T-semiflow θ has a minimal support *iff* there exists no other T-semiflow, θ', such that $\{q \in T \mid \theta'(q) > 0\} \subset \{q \in T \mid \theta(q) > 0\}$.

A vector $Y \in \mathbb{R}^P$ such that $Y \neq 0$ et $Y^t W = 0$ is a P-semiflow.

In the rest of the paper we assume that CGTEG are *consistent* (*i.e.*, there exists a T-semiflow θ covering all transitions : $\|\theta\| = T$) and are *conservative* (*i.e.*, there exists a P-semiflow Y covering all places: $\|Y\| = P$).

Defined by [9], the gain of weighted circuit γ (also called loop gain) is defined by $G(\gamma) = \prod_{P_i \in \gamma} \frac{w_i}{\nu_i}$. Where w_i the weight of the input arc of place P_i, and ν_i the weight of the output arc of P_i. we interest only to the case where $G(\gamma) = 1$, i.e, all circuits of graph is neutral. For these type of graph, there will be no blocking or divergence marking.

In the following, we restrict ourselves to strongly connected graphs. i.e. there exists an oriented directed path that connects any node (Place or transition) to any other of the graph. A non-strongly connected graph can always be decomposed into a finite number of strongly connected sub-graphs. In a more advanced manner, a strongly connected graph can always be decomposed into a finite set of elementry circuit; an elementary circuit that contains each node (place or transition) at most once. From these graph theory concepts, [10] classifies a strongly connected graph with weights as follows.

- A strongly connected graph with weights is *neutral* if and only if each circuit in graph is neutral.
- A strongly connected graph with weights is *absorbing* if it contains at least one absorbing circuit.
- A strongly connected graph with weights is *generating* if and only if it contains no absorbing circuit and has at least one generating circuit.

Theorem 1: (Liveness of CGTEG) A CGTEG is alive if and only if all circuits are non-absorbing and each circuit has at least one place with marking positive, $m \geq 0$.

To study the CGTEG, for each transition *q* is associated a *counter variable*, denoted n_q : n_q is an increasing map from $\mathbb{R}$ to $\mathbb{R} \cup \{+\infty\}$, $t \mapsto n_q(t)$ which denotes the amount of fluid having fired the transition *q* up to time *t*.

In the following, we assume that counter variables satisfy the *earliest firing* rule, *i.e.*, a transition q fires as soon as all its upstream places $\{p \in {}^\bullet q\}$ contain a positive marking ($m_p > 0$) having spent at least τ units of time in place p. When the transition q fires, it consumes $\frac{m_p}{M_{qp}}$ amount of fluid in each upstream place p and produces $M_{p'q}$ amount of fluid in each downstream place $p' \in q^\bullet$.

Assertion 1: The counter variable n_q of a CGTEG (under the earliest firing rule) satisfies the following *transition to transition* equation:

$$n_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} M_{qp}^{-1}(m_p + M_{pq'} n_{q'}(t - \tau)). \quad (1)$$

Example 1: The counter variables associated with transitions of CGTEG depicted in the figure 1 satisfy the next equations:

$$\begin{cases} n_1(t) &= \min(\frac{1}{2}n_2(t-1), 2 + n_1(t-3), \frac{1}{2}(1+3u(t))), \\ n_2(t) &= 2 + 2n_1(t-1), \\ y(t) &= n_2(t). \end{cases}$$

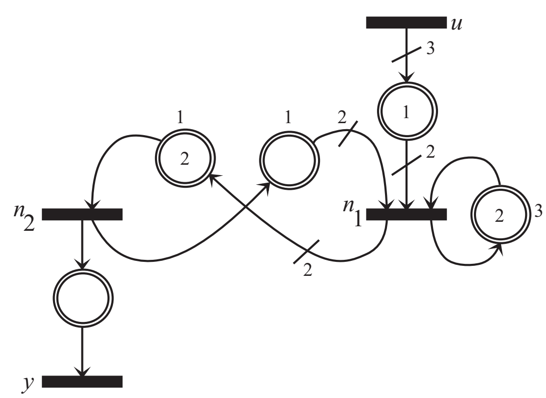

Fig. 1. Continuous Generalized Timed Events Graphs.

In the case of ordinary CTEG, the *transition to transition* equation given in Eq.(1) becomes:

$$x_q(t) = \min_{p \in {}^\bullet q,\, q' \in {}^\bullet p} (m_p + x_{q'}(t - \tau_p)). \quad (2)$$

This equation is linear in the algebraic structure called (*min*, +) algebra. This structure, denoted $\mathbb{R}_{\min}$, is defined as the set $\mathbb{R} \cup \{+\infty\}$, equipped with the *min* as additive law (denoted $\oplus$) and with the usual addition as multiplicative law (denoted $\otimes$). The neutral element of the law $\oplus$ (resp., $\otimes$) is denoted $\varepsilon = +\infty$ (resp., $e = 0$). More generally, the (*min*, +) algebra is a *dioid* [2].

A *dioid* $(\mathcal{C}, \oplus, \otimes)$ is a semiring in which $\oplus$ is *idempotent* ($\forall a,\ a \oplus a = a$). Neutral elements of $\oplus$

and $\otimes$ are denoted ε and e respectively.

Example 2: In dioid $\mathbb{R}_{\min}$, Eq.(2) is written as follows :

$$x_q(t) = \bigoplus_{p\in {}^\bullet q,\, q'\in {}^\bullet p} (m_p \otimes x_{q'}(t-\tau_p)). \tag{3}$$

From the Equation (3) obtained for each transition, one can express a TEG as the following recursive matrix equation:

$$x(t) = M \otimes x(t-1), \tag{4}$$

where M is a square matrix with coefficient in $\mathbb{R}_{\min}$, and $x(t)$ is the vector of the counter variables associated with transitions of the graph. See [2] for more details on the representation of TEG in the dioid $\mathbb{R}_{\min}$.

3. Linear Dynamic Behavior of CGTEG in (min,+) algebra

The linearization method presented here is inspired by [4]. The difference lies in the use of T-vector semiflows, in our case, instead of a vector called the potential (see [4] for details).

A CGTEG is *linearizable* if there exists a change of variable $n_q(t) = \theta_q x_q(t)$ such that $x_q(t)$ satisfies a (*min*,+) linear recurrent equation knowing that:

- $n_q(t)$ is the counter associated with transition q of CGTEG,
- θ_q is the component of T-semiflow associated with transition q ($\theta_q \in \mathbb{R}^*_+$).

Proposition 1: A CGTEG reduces to a CTEG by a change of counting units iff it has a T-semiflow.

Proof: According to assertion (1), we have for each transition q of a CGTEG:

$$n_q(t) = \min_{p\in {}^\bullet q,\, q'\in {}^\bullet p} M_{qp}^{-1}(m_p + M_{pq'} n_{q'}(t-\tau_p)).$$

Using the change of variable $n_q(t) = \theta_q x_q(t)$, and by distributivity of the multiplication with respect to the *min* operator, we have:

$$x_q(t) = \min_{p\in {}^\bullet q,\, q'\in {}^\bullet p} \tfrac{1}{\theta_q}(\tfrac{m_p}{M_{qp}} + \tfrac{M_{pq'}}{M_{qp}} n_{q'}(t-\tau_p)).$$

From relation

$\frac{\theta_q}{M_{pq'}} = \frac{\theta_{q'}}{M_{qp}}$, obtained for consistent and conservative CGTEG (see [9]), we have

$$x_q(t) = \min_{p\in {}^\bullet q,\, q'\in {}^\bullet p} \tfrac{1}{\theta_q}(\tfrac{m_p}{M_{qp}} + \tfrac{\theta_q}{\theta_{q'}} n_{q'}(t-\tau_p)),$$

i.e.,

$$x_q(t) = \min_{p\in {}^\bullet q,\, q'\in {}^\bullet p} \tfrac{1}{\theta_q}(\tfrac{m_p}{M_{qp}} + \theta_q x_{q'}(t-\tau_p)).$$

Because $\theta_q x_{q'}(t-\tau_p) \in \mathbb{R}$, we finally obtain

$$x_q(t) = \min_{p\in {}^\bullet q,\, q'\in {}^\bullet p} (\frac{1}{\theta_q}\frac{m_p}{M_{qp}} + x_{q'}(t-\tau_p)), \tag{5}$$

which corresponds to a linear recurrent equation in dioid $\mathbb{R}_{\min}$, his expression is equivalent to :

$$x_q(t) = \bigoplus_{p\in {}^\bullet q,\, q'\in {}^\bullet p} (\frac{1}{\theta_q}\frac{m_p}{M_{qp}} \otimes x_{q'}(t-\tau_p)), \tag{6}$$

4. Allocating resources of CGTEG

In this section, we focus on resource use in the control circuit to achieve the desired performance.

We recall main results characterizing an ordinary CTEG modelled in the dioid $\mathbb{R}_{\min}$[2],[6].

Definition 1 (Irreducible matrix): A matrix M is said *irreducible* if for any pair *(i,j)*, there is an integer m such that $(M^m)_{ij} \neq \varepsilon$.

Theorem 2: Let M be a square matrix with coefficient in $\mathbb{R}_{\min}$. The following assertions are equivalent:

- Matrix M is irreducible,
- The CTEG associated with matrix M is strongly connected.

One calls *eigenvalue* and *eigenvector* of a matrix M with coefficients in $\mathbb{R}_{\min}$, the scalar λ and the vector $\hbar$ such as: $M \otimes \hbar = \lambda \otimes \hbar$. When the initial vector $x(0)$ of matrix equation (4) is equal to an eigenvector of matrix M, the GTEG reaches a periodic regime from the initial state.

Theorem 3: Let M be a square matrix with coefficients in $\mathbb{R}_{\min}$. If M is irreducible, then there is a single eigenvalue denoted λ. The eigenvalue can be calculated in the following way: $\lambda = \bigoplus_{j=1}^{n}(\bigoplus_{i=1}^{n}(M^j)_{ii})^{\frac{1}{j}}$.

Regarding the GTEG strongly connected, λ corresponds to the firing rate identical for each transition. This eigenvalue λ can be directly deduced from the GTEG by

$$\lambda = \min_{c \in C} \frac{\mathcal{M}(c)}{\mathcal{T}(c)}, \tag{7}$$

where:

- C is the set of elementary circuits of the GTEG.
- $\mathcal{T}$(c) is the sum of holding times in circuit c.
- $\mathcal{M}$(c) is the number of tokens in circuit c.

Definition 2 (Cycle time): The average cycle time of a CGTEG is the average time to fire once the T-semiflow under the ealiest operational mode(i.e., transitions are fired as soon as possible) from the marking M_0.

In the case of Ordinary GTEG, The average cycle time of a CGTEG can be defined as the inverse of the eigenvalue λ.

Concerning CGTEG, the firing rate, denoted λ_{m_q}, is not identical for all transitions. It is defined for each transition n_q as the division of θ_q by TC_m. Where θ_q is the component of the T-semiflow associated with transition n_q, and TC_m is average the cycle time of the CGTEG.

If the calculation CTEG performance has been achieved through the spectral theory[2], the problem remains at our knowledge, to open the CGTEG. The difficulty of these models is the presence of the weight on arcing. These weight lead, as we saw previously, a nonlinearity in the mathematical model for the dynamic evolution of these models. This nonlinearity in algebra $(\min, +)$, prevents use the equation (7).

The firing rate λ_{m_q} of a linearizable CGTEG can be calculated from the (*min*, +) linear model by :

$$\lambda_{m_q} = \theta_q \lambda \tag{8}$$

where λ is the eigenvalue of the equivalent (*min*,+) linear model. This result is a direct consequence of the linearization proprety. This implies that $TC = TC_m$

Example 3: Two systems working in parallel, the operating speed of the system 2 and slower than the operating speed of the system 1.

The functioning of these systems can be represented by the following CGTEG. The system 1 is modelled by the circuit composed of places P_2, P_3 and the system 2 is modelled by the circuit composed of places P_1, P_4.

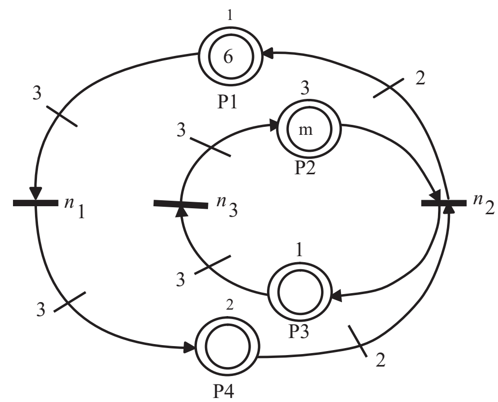

Fig. 2. Allocating resources of CGTEG.

We note that, the circuit composed of places (P_2, P_3) control the evolution of model.

For un $TC_m = 4$, One determines the initial marking associated with place P_2 of CGTEG given in the figure 2.

The counter variables associated with transitions of CGTEG depicted in the figure 2 satisfy the next equations:

$$\begin{cases} n_1(t) & = 2 + \frac{2}{3}n_2(t-1), \\ n_2(t) & = \min(\frac{3n_1(t-2)}{2}, m + 3n_4(t-3)), \\ n_3(t) & = \frac{1}{3}n_2(t-1), \end{cases}$$

These equations are nonlinear in dioid $\mathbb{R}_{min}$ because of the weight on arcing.

The CGTEG admits the elementary T-semiflow: $\theta = (3, 2, 1)$, hence, this graph is linearizable. Using the change of variables $n_i(t) = \theta_i x_i(t)$ and

thanks to Equation (6), we obtain the following equations :

$$\begin{cases} x_1(t) & = 1 + x_2(t-1), \\ x_2(t) & = \min(\frac{m}{3} + x_3(t-3), x_1(t-2)), \\ x_3(t) & = x_2(t-1). \end{cases}$$

These equations are linear in dioid $\mathbb{R}_{min}$, their expressions become :

$$\begin{cases} x_1(t) & = 1 \otimes x_2(t-1), \\ x_2(t) & = (\frac{m}{3} \otimes x_3(t-3) \bigoplus x_1(t-2)), \\ x_3(t) & = x_2(t-1). \end{cases}$$

These equations correspond to the CTEG depicted in figure 3.

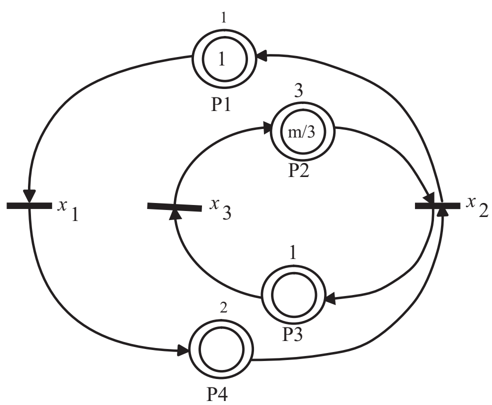

Fig. 3. CTEG obtained after the change variables.

To calculate the marking of the place P_2 of ordinary CTEG, given in figure 3, we use the expression of cycle time.

• We pose $k = \frac{m}{3}$ (marking of place P_2 of the ordinary CTEG.)

We saw previously, from the equation 8 we deduced that $TC = TC_m$. We have:

$$\lambda = min(\frac{1}{3}, \frac{k}{4}) = 1/4 \tag{9}$$

$\Rightarrow k = 1$, hence $m = 3$.

Therefore, for a cycle time $TC_m = 4$ time units, the initial marking associated with place P_2 of CGTEG, given in figure 2, equal to 3 tokens.

5. Conclusion

In this paper, we presented a method for determining the marking for a some places of control circuit in a CGTEG for a desired cycle time. For that, we proceed by linearization of the mathematical model reflecting the behavior of a CGTEG in order to obtain a model $(\min, +)$ linear. From the latter, we determine the initial marking which satisfiers the desired cycle time.

References

[1] Alla, H. and David, R. (1998). Continuous and Hybrid Petri Nets. In *Journal of Circuits, System and Computers*, pages 159-188.
[2] Baccelli, F., Cohen, G., Olsder, G., and Quadrat, J.-P. (1992). *Synchronization and Linearity: An Algebra for Discrete Event Systems*. Wiley.
[3] Cohen, G., Gaubert, S., and Quadrat, J.-P. (1998a). *"Algebraic System Analysis of Timed Petri Nets"*. pages 145-170, In: *Idempotency*, J. Gunawardena Ed., Collection of the Isaac Newton Institute, Cambridge University Press. ISBN 0-521-55344-X.
[4] Cohen, G., Gaubert, S., and Quadrat, J.-P. (1998b). "Timed-Event Graphs with Multipliers and Homogeneous Min-Plus Systems". *IEEE TAC*, 43(9):1296–1302.
[5] Cohen, G., Moller, P., Quadrat, J.-P., and Viot, M. (1989). "Algebraic Tools for the Performance Evaluation of Discrete Event Systems". *IEEE Proceedings*, 77(1):39–58.
[6] Hamaci, S., Boimond, J-L., Lahaye, S. (2008). " Modèle pour la Linéarisation de Systèmes à Evénements Discrets non Linéaires dans L'Algèbre (min,+). *MOSIM*, CDROM.
[7] Giua, A., Piccaluga, A., Seatzu, C. (2000). " Optimal Token Allocation in Timed Cyclic Event-Graphs. *WODES*, pages 209–218.
[8] Lahaye, S. (2000). " Contribution l'tude des systmes linaires non stationnaires dans l'algbre des diodes. *Thse, Universit d'Angers*.
[9] Lien, Y.L. (1976). *"Termination Properties of Generalized Petri Net"*. SIAM J.of Computers 5(2):251-265.
[10] Munier, A. (1993). Régime asymptotique optimal d'un graphe d'événements temporisé généralisé : Application à un problème d'assemblage. In *RAIPO-APII*, volume 27(5), pages 487–513.
[11] Murata, T. (1989). "Petri Nets: Properties, Analysis and Applications". *IEEE Proceedings*, 77(4):541–580.
[12] Trouillet, B. Benasser, A. (2002). Cycle Scheduling Problems with Assemblies. In *WODES*.

Samir HAMACI is currently Assistant Professor at the School of Electrical Engineering and Industrial Management (EPMI), Cergy, France. He received his PhD in Automatic degrees from the University of Angers in 2005. His research interests include modeling, performance analysis, control and resources optimization of dynamic systems modeled by discrete and hybrid Petri nets in dioid algebra.

Karim Labadi received his Master's degree in Automation and Applied Computer Science from the "Ecole Centrale de Nantes (ECN)", France, in 2001, and the Ph.D degree in Industrial Engineering from the University of Technology of Troyes (UTT), France in 2005. K. Labadi has been with Graduate School of Electrical Engineering and Industrial Management (EPMI), and System Control Laboratory (ESC-Lab ENSEA), Cergy-Pontoise, France, since 2006 as Assistant Professor. His current research interests are in Petri nets theory, discrete event systems, modelling, analysis, performance evaluation, and optimization of production and logistic systems.

A.-Moumen DARCHERIF is Accreditation to Supervise Research (HDR) and IEEE member. He received the Ph.D. degree in Electrical Engineering degree from INPG - Grenoble. Currently head of the EPMI School, author of several scientific papers, published at national and international journals, conference proceedings as well as book chapters.