Non-Iterative Three Dimensional Positioning Algorithm Based on Time Difference Of Arrival Technique

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Abstract

This paper proposes an original Three-Dimensional (3-D) positioning technique based on an enhanced Chan method by investigating the performance of position estimation method by Time Difference Of Arrival (TDOA). The studied configurations assume that each mobile locates itself and so it receives and analyses signal from multiple transmitters, namely Base Station (BS). The approach is non-iterative and gives an explicit solution which does not use intensive computing. Position estimation is made in two steps: the first step is relied on related position parameters extraction and the second step concerns position computing. Related position parameters are TDOA. They are used as input for positioning algorithm. This proposed method has been tested with simulation and experimental measurements. The test results show, on one hand, that position estimation is function of system configuration, and on the other hand, it is able to make 3-D position estimation with high accuracy in centimeter order.

Keywords: *TDOA Estimation, 3–D Positioning, Non–iterative algorithm, High accuracy.*

1. Introduction

Many applications in transport, health, mining, emergency services, require location systems and at the same time, there are no real location systems with adequate level of accuracy for indoor environments. Many reasons can explain this situation. For example, a Global Positioning System (GPS) is unusable alone indoors and there are no efficient solutions to enhance indoor location problems. Techniques used to relay GPS signal in indoor areas suffer from accuracy. The number of location systems is also another important reason. When there are many systems in the same area, it cannot be possible to locate each of them at the same time. So, the solution is to allow the mobile to locate itself, so called positioning.

Many techniques are used in practice in localization processing. There are Time Of Arrival (TOA), Time Difference Of Arrival (TDOA), Angle Of Arrival (AOA), Received Signal Strength (RSS) and hybrid techniques (TOA-TDOA, TOA-AOA, TDOA-AOA . . .). But, in indoor environment where multipath components exist, location systems need more accuracy. Among the techniques mentioned previously, TOA or TDOA techniques are widely used. A two-stage method has been suggested in [1] to estimate TOA. Non-coherent TOA estimation approach based on this two-stage TOA estimation is an alternative to avoid synchronization problem [2]. But for this proposed system, the mobile can just receive signal from base stations, it does not send any information to them. So, the two-stage technique is not adequate for our application and could be very difficult in confined areas where systems suffer from multipath propagation. The TOA technique requires also synchronization between transmitter and receiver. To avoid this TOA synchronization problem and the uncertainty of obtaining information through use of two-stage, the TDOA technique is used.

Most researches are focused on systems considering one transmitter and multiple receivers or systems for RADAR applications. These studies often refer to a 2–D studied context [3]. However, in 2-D context, all the elements are assumed to be coplanar. This consideration leads to an error when the exact vertical position (component in *z*) is unknown. The solution is to take this assumption above into account and make the 3-D location where all base station heights of the system are known. Many other considerations must be taken into account for indoor location systems as described in [4]. A 3-D location position scheme feasibility studies, for example in indoor environments, have been presented in [5].

Some research works have also shown that there are more errors in the estimated position when mobile and BS have not the same vertical position. In [6], the suggestion carrying out more investigation of 3-D study in order to better characterize component (*z*) impact on the accuracy of position location has been made. Another important point, in the location process is the positioning algorithm used to accurately find the unknown position. This step of the location system is not easy in 3-D context due to equation resolution complexity. Many studies have shown that iterative method gives better accuracy for 3-D position location [2]. But, the iterative method requires an initial start point and may suffer from convergence problems if this initial start point is not chosen accurately. Then, it can require intensive computation to determine the local least-square solution.

The purpose of this paper is to propose a positioning algorithm which improves location accuracy for 3-D context by using a non-iterative method. We have based our study on positioning algorithm developed for a 2-D position location in [7]. Authors of [7] have considered in their work that there is one transmitter and multiple receivers. We extend this approach for a 3-D context to provide a better estimate for self-location application. It means considering one receiver and multiple transmitters. This paper is organized as follows. In section 2, we describe the different kinds of positioning approaches are described, after that section 3 presents a TDOA position location technique. In section 4, 3-D positioning algorithm used to estimate position is presented. Simulation and experimental results are presented and discussed in sections 5 and 6, respectively. Finally, section 7 concludes this paper.

2. Positioning Systems Generality

This study considers one sensor (the receiver) which locates itself with signals coming from many transmitters. So, the sensor can perform its position using received signals in two main schemes. The first approach is a direct positioning where position estimation is directly performed from signals traveling between sensor and transmitters (Fig.1). This approach has been described in [8] and is based on cost-function estimation. The second approach is performed in two steps and it is the most used method such as in RADAR, GPS and others applications [9]. With this approach, in the first times, we extract parameters such as RSS, AOA, TOA or TDOA from received signals. In the second times, estimated position is then determined using information provided from estimated parameters (Fig.2). The positioning technique proposed in this paper is based on the two steps approach and uses TDOA as estimated parameters. The non-iterative location algorithm proposed to find mobile position is presented in section 4.

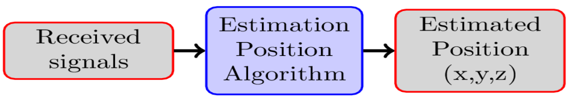

Fig. 1 Direct Positioning System.

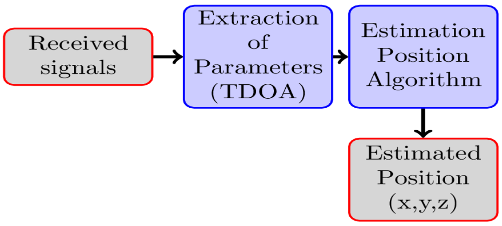

Fig. 2 Two-Step Positioning System.

3. Overview of TDOA Technique

3.1 Principle of TDOA Technique

Conventionally, many location techniques using TOA measurements require synchronization between receiver and transmitter. The TOA accuracy is affected, in None Line Of Sight (NLOS), by measurements error and by synchronization problems also. In the case of the TDOA approach, we can eliminate error affected by the synchronization problem, assuming the offset is the same from each transmitter. Estimated TDOAs are obtained without synchronization between receiver and transmitters. In this case, the difference between two TOAs from signals traveling between the receiver and two transmitters, gives one TDOA. TDOAs can also be estimated by the cross-correlation function. So, there are two ways to estimate TDOAs. The following two sub-sections present these two ways and show the relationship between TDOAs and distance measurements.

3.2 Different Techniques of TDOA Estimation

In most research works in literature based on TDOA techniques consider that the transmitter sends signal to many receivers. In this case, TDOA measurements can be obtained by performing cross-correlation of received signals and of receiver $y_i(t)$ and $y_j(t)$, respectively. In order to improve cross-correlation scheme performance, Generalized Cross-Correlation techniques (GCC) have often been used [9], [10]. GCC function is expressed as shown in Eq. (1).

$$R_{y_i,y_j}(\tau)=\frac{1}{T}\int_0^T y_i(t)y_j(t+\tau)dt . \tag{1}$$

T is the observation interval, and estimated TDOA is given by Eq. (2):

$$\hat{\tau}_{i,j}=\mathbf{arg}\ \underbrace{\mathbf{max}}_{\tau}\left|R_{y_i,y_j}(\tau)\right| . \tag{2}$$

This TDOA's approach is applied when one transmitter communicates with many receivers. But, this study concerns others cases where one receiver communicates with many transmitters. It is a self-location or positioning case. For that, the following technique is considered in the remaining of the work. So, the TDOA can be also obtained by taking the difference between arrival time of the signal at mobile from one receiver and other receiver. It is important to ensure that transmitted signals are orthogonal. For that, multi-access coding techniques are efficient to attribute unique code to each transmitted signal. Therefore, received signal is a combination of all transmitted signals and it is cross-correlated with each transmitted signal reference to estimate each arrival time. Finally, the difference between the first arrival time and each other arrival time represents a TDOA. So, with *M* transmitters, (*M-1*) TDOAs can be estimated.

Let be:

- r : the identification number of the transmitter considered as the reference,
- y_i : the received signal by mobile from the other transmitters such as $i \neq r$,
- τ_i : the time of arrival for the i^{th} transmitted signal.

We assume also that transmitters are perfectly synchronized between themselves; timing offset is the same for each TOA estimate. So TDOA measurements can be written as Eq. (3).

$$\hat{\tau}_{i,j} = \tau_i - \tau_j \tag{3}$$

3.3 Relationship between TDOA and Distance Measurements

Once TDOA parameters have been computed, the second step of the proposed method consists on estimating the mobile position with positioning algorithm. There are two estimation approaches. The first technique is a mapping (fingerprinting) approach and it is relying on the availability of database containing signal measurements at given positions [11]. In this technique, database is always obtained during a training phase performed before real-time positioning procedure starts. The second technique does not require database, but it commonly employs geometric or statistical approaches [12]. It is used estimated position-related parameters derived in the first step of the position estimation as described in section 2.

In this study, we assume that database is not always available. For that, second technique which combines geometrical and statistical methods is used in positioning algorithm. In 3-D context, each TDOA measurement determines a hyperboloid on geometrical interpretation. So, to avoid any ambiguity on the issue, at least four transmitters are necessary in order to allow the self-location of the mobile. In this case, there are at least three unknown parameters (x, y, z) , the mobile's coordinates. We consider a limited set of fixed Base Stations (BS) which represents transmitters and they are placed at known positions characterized by their coordinates (x_i, y_i, z_i).In this proposed method, among Base Stations, one is chosen as the reference as described in sub-section III-B. We assume also that the mobile moves and its coordinates (x, y, z) are the unknown parameters. Mathematically, the squared distance (d_i) between the mobile and the i^{th} transmitter is given by Eq. (4).

$$(d_i)^2 = (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 \tag{4}$$

Let be:

- c : the speed of light,
- $d_{i,r}$: the distance between the mobile and the reference transmitter, with $r \in \{1,2,3,\cdots,M\}$ and $i \in (1,2,3,\cdots,M) \setminus \{r\}$.

For (*M*) transmitters and one mobile, (*M-1*) different measured distances are expressed by Eq. (5a) and explicated by Eq. (5b).

$$d_{i,r} = d_i - d_r , \tag{5a}$$

$$d_{i,r} = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - \sqrt{(x_r - x)^2 + (y_r - y)^2 + (z_r - z)^2} . \tag{5b}$$

Finally, the relationship between the measured TDOA ($\hat{\tau}_{i,j}$) and distance ($d_{i,r}$) is given by Eq. (6).

$$d_{i,r} \approx c.\hat{\tau}_{i,r} \tag{6}$$

We can see that Eq. (5b) is not tightly equal to measurements of the right term of Eq. (6). In fact, the estimated TDOAs in Eq. (3) are affected by noise level, propagation channel errors and quality of processing equipments.

In the remaining of the paper, we assume that TDOA measurements ($\hat{\tau}_{i,r}$) are known. Then, the expression $d_{i,r}$ represents ($c.\hat{\tau}_{i,r}$). The following section 4 presents usual methods proposed to solve nonlinear equations and describe also the proposed algorithm.

4. Positioning Algorithms

4.1 Classical Methods

Equation (5b) defines a set of nonlinear hyperbolic equations whose solution gives the mobile's coordinates. Many methods have been proposed in literature to solve them. Among them, linearization method is mostly used. One way of linearizing these equations is done through the use of Taylor-series expansion [13]. A common used alternative method to Taylor-series expansion consists to transform (5b) into a set of linear equations such as it is described in [14] and [15]. So, (5a) is transformed into Eq. (7) as follow:

$$(d_i)^2 \approx (d_{i,r} + d_r)^2. \qquad (7)$$

Then, Eq. (4) can be rewritten into Eq. (8).

$$\begin{aligned}(d_{i,r})^2 + 2d_{i,r}d_r \approx\ & (x_i)^2 + (y_i)^2 + (z_i)^2 \\ & - 2x_r x - 2y_r y - 2z_r z \\ & + x^2 + y^2 + z^2. \qquad (8)\end{aligned}$$

After that, if we assume that $i = r$ in Eq. (4), and subtractration of obtained expression from Eq. (8), results in Eq. (9).

$$\begin{aligned}&(d_{i,r})^2 + 2.d_{i,r}.d_r \approx \left(x_i{}^2 + y_i{}^2 + z_i{}^2\right) - \left(x_r{}^2 + y_r{}^2 + z_r{}^2\right) \\ &- 2.(x_i - x_r).x - 2.(y_i - y_r).y - 2.(z_i - z_r).z \qquad (9)\end{aligned}$$

Let be:

$$K_i = x_i{}^2 + y_i{}^2 + z_i{}^2 ; K_r = x_r{}^2 + y_r{}^2 + z_r{}^2 \quad (10a)$$

$$x_{i,r} = x_i - x_r ; y_{i,r} = y_i - y_r ; z_{i,r} = z_i - z_r \quad (10b)$$

Then, after incorporating each expression of Eq. (10a) and Eq. (10b) in Eq. (9), yields to Eq. (11).

$$(d_{i,r})^2 + 2.d_{i,r}.d_r \approx K_i - K_r - 2.x_{i,r}x - 2.y_{i,r}y - 2.z_{i,r}z \qquad (11)$$

The set of equations in Eq. (11) are now linear with four unknown parameters which are mobile coordinates (x, y, z) and (d_r), the distance between the reference base station (x_r, y_r, z_r) and the mobile. This last equation is more easily handled comparatively to (5b).

4.2 Classical Method Limits

In literature, the solution of Eq. (11) is derived through two geometric approaches. In the first approach, transmitters are arranged linearly. It means there is a linear relation between all transmitters coordinates so that Eq. (11) resolution becomes simple. Methods which describe the situation in 2-D context with an exact solution are studied in [7] and [16].

The second approach considers that transmitters are distributed arbitrarily and in this case (11) resolution becomes more complex to solve. A straightforward approach consists in using a geometrical approach to interpret measurements and determine the true position. This approach usually gives good results if there are no measurement errors and if the system is not over-determined. The over-determination occurs when there are more measurements than unknown parameters. For example, in a 3-D context, it occurs when the estimation is made with one receiver and five transmitters. Then, there are:

- four unknown parameters which are mobile's coordinates and the distance d_r between the reference base station and the mobile,
- and three TDOA measurements determined with the four received times.

Therefore, mathematically, we have a set of equations where matrix is not squared. In this case, the set of equations cannot be solved. Also, if there are measurement errors, we cannot obtain a single intersection point. So, we have needed more transmitters to make a decision. To do so, this proposed method used five transmitters.

4.3 Proposed Method Description

Many algorithms have been developed for positioning [17], [18]. But, some of them do not solve measurement errors and over-determination problems. To overcome these limits, we present in this section an algorithm based on Chan algorithm developed for 2-D context [7]. We extend it for 3-D applications. It is important to note that in this study, we assume that receiver wants to locate itself with informations coming from many transmitters. Due to the limitations of the classical methods, statistical positioning techniques are employed. Probabilistic likelihood function is defined and we choose a correct one from the algebraic solutions derived from the TDOA equation to optimize the proposed method. The set of equations described in Eq. (11) are transformed into Eq. (12).

$$\frac{1}{2}\left(d_{i,r}^2 - K_i + K_r\right) \approx -\left(x_{i,r}x + y_{i,r}y + z_{i,r}z + d_{i,r}.d_r\right) \quad (12)$$

The left term of Eq. (12) represents known parameters, obtained after TDOA estimation, and the right term, the unknowns' parameters. Eq. (12) is the line of a matrix with (*M-1*) rows where *M* indicates total number of transmitters. This line can be written as Eq. (13).

$$\frac{1}{2}\left(d_{i,r}^2 - K_i + K_r\right) \approx -\begin{bmatrix} x_{i,r} & y_{i,r} & z_{i,r} & d_{i,r} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ d_r \end{bmatrix} \qquad (13)$$

For example, if there are:

- five transmitters (base stations),
- and the reference is the number five, then $r = 5$, $i = 1,2,3,4$ and so forth.

Eq. (13) can be expressed in compact form as Eq. (14).

$$h \approx G_a Z_a \qquad (14)$$

Consequently, if there are five transmitters with known positions and one mobile such as the reference transmitter is the number *5*, *h*, G_a and Z_a matrixes, and can be written as follow:

$$h = \frac{1}{2}\begin{bmatrix} d_{1,5}^2 - K_1 - K_5 \\ d_{2,5}^2 - K_2 - K_5 \\ d_{3,5}^2 - K_3 - K_5 \\ d_{4,5}^2 - K_4 - K_5 \end{bmatrix}, \qquad (15a)$$

$$G_a = -\begin{bmatrix} x_{1,5} & y_{1,5} & z_{1,5} & d_{1,5} \\ x_{2,5} & y_{2,5} & z_{2,5} & d_{2,5} \\ x_{3,5} & y_{3,5} & z_{3,5} & d_{3,5} \\ x_{4,5} & y_{4,5} & z_{4,5} & d_{4,5} \end{bmatrix}; \quad Z_a = \begin{bmatrix} x \\ y \\ z \\ d_5 \end{bmatrix} \qquad (15b)$$

For accurate TDOA's measurements (with less errors), left term of (14) will be equal to the term of right. Generally, TDOAs are always estimated with some errors. To do so, we assume that (14) takes into account those measurements error.

Let's consider *W* and Z_a symbolize the error vector and the unknown parameters vector, respectively. TDOA's measurements are very important to make localization with enough accuracy. If their estimations contain some errors, comparatively to their actual values, it implies that estimated position could be affected also by some errors. In the remaining of the paper, $(d_{i,r})$ represents TDOA's measurements converted into distance such as indicated by Eq. (6). So, in the presence of measurement errors, Eq. (14) can be rewritten as:

$$W = h - G_a Z_a. \qquad (16)$$

In most cases, there is no prior information about the vector (Z_a). For that, Maximum Likelihood (ML) estimation is commonly used to find (Z_a) value which maximizes likelihood function which is defined by:

$$\hat{Z}_{a_{ML}} = \arg \underbrace{\max}_{Z_a}\left(p\left(h / Z_a\right)\right), \qquad (17)$$

where $p(h / Z_a)$ represents the probability density function of (h) conditioned by (Z_a). If the noise vector is modeled as a Gaussian random variable with mean (*m*) and covariance matrix (Φ), the likelihood function can be expressed as [9]:

$$p(h / Z_a) = \frac{1}{(2\pi)^{\frac{(M-1)}{2}} |\Phi|^{\frac{1}{2}}} \times$$

$$\exp\left\{-\frac{1}{2}\left(h - G_a Z_a - m\right)^T \times \Phi^{-1}\left(h - G_a Z_a - m\right)\right\} \qquad (18)$$

Then, for a noise distribution with zero mean (*m=0*) and a known covariance matrix (Φ), Eq. (17) becomes

$$\hat{Z}_{a_{ML}} = \arg \underbrace{\mathbf{min}}_{Z_a}\left(\left(h - G_a Z_a\right)^T \times \Phi^{-1}\left(h - G_a Z_a\right)\right), \qquad (19)$$

because the "**exp**" function is increasing function. When, we replace *W* by its expression of Eq. (16) in Eq. (19), we get:

$$\hat{Z}_{a_{ML}} = \arg \underbrace{\min}_{Z_a}\left(W^T \Phi^{-1} W\right), \qquad (20)$$

where $(W)^T$ is the transpose of (W).

The relation of Eq. (19) is also called the Weighted Least Square (WLS) [9]. The elements of (Z_a) are related to Eq. (4), which means that Eq. (16) is still a set of nonlinear equations with three variables x, y and z. We apply maximum likelihood (ML) approximation in two-stage to solve nonlinear problem. As in [7], we assume, firstly that there is no relationship between (x, y, z) and (d_r). Then, the application of Weighted Least-Squares (WLS) to Eq. (19) gives:

$$\hat{Z}_{a_{ML}} = \left(G_a^T \phi^{-1} G_a\right)^{-1} \times \Phi^{-1}\left(G_a^T \phi^{-1} h\right). \qquad (21)$$

where, ϕ is the covariance matrix of W. It can be approximated to [7]:

$$\phi \approx c^2 \mathrm{B} Q B. \qquad (22)$$

B is diagonal matrix of distance measurements (d_i), except (d_r), Q is the TDOA noise vector covariance matrix of size $(M-1) \times (M-1)$ as it described by Eq. (23):

$$Q = \begin{pmatrix} a^2 & 0.5a^2 & \cdots & 0.5a^2 \\ 0.5a^2 & a^2 & \cdots & 0.5a^2 \\ \vdots & \cdots & \ddots & \vdots \\ 0.5a^2 & \cdots & \cdots & a^2 \end{pmatrix} \qquad (23)$$

where, a^2 is TDOA variance.

Incorporating (ϕ) by its expression from Eq. (22) into Eq. (18) yields to Eq. (24).

$$\hat{Z}_{a_1} = \left(G_a^T Q^{-1} G_a\right)^{-1} \times \Phi^{-1}\left(G_a^T Q^{-1} h\right) \qquad (24)$$

Equation (24) is the first estimation under assumption that (Z_a) components are independent. Let be Eq. (25) solution of Eq. (24).

$$\hat{Z}_{a_1} = \begin{bmatrix} Z_{a_x} & Z_{a_y} & Z_{a_z} & Z_{a_{d_r}} \end{bmatrix}^T \quad (25)$$

Now, a second time application of WLS is made by applying the known relationship between variables of Eq. (4), when $i = r$, into (24). This assumption shows that the (Z_a) components are dependent. In other words, it means that this relationship takes into account the fact that mobile's coordinates (x, y, z) and (d_r), the estimated range distance, between the mobile and the reference base station are dependant. We define new matrix and vectors like those in Eq. (16) as:

$$W_2 = h_2 - G_f Z_{a_2}, \quad (26)$$

where, h_2, G_f and Z_{a_2} are expressed as follow:

$$h_2 = \begin{bmatrix} (Z_{a_x} - x_r)^2 \\ (Z_{a_y} - y_r)^2 \\ (Z_{a_z} - z_r)^2 \\ \left(Z_{a_{d_r}} - d_r\right)^2 \end{bmatrix}, \quad (27a)$$

$$G_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}; \qquad Z_{a_2} = \begin{bmatrix} (x - x_r)^2 \\ (y - y_r)^2 \\ (z - z_r)^2 \end{bmatrix}. \quad (27b)$$

The equivalent diagonal matrix as B for the first step is given by D in Eq. (28). This matrix contains the estimated position of first step with respect to the reference transmitter.

$$D = diag\left(Z_{a_x} - x_r, \quad Z_{a_y} - y_r, \quad Z_{a_z} - z_r, \quad Z_{a_{d_r}} \right) \quad (28)$$

Then, the final solution is derived from Eq. (29) [7].

$$Z_f = \left(G_f^T D^{-1} Q^{-1} G_a^T D^{-1} G_a G_f\right)^{-1} \times \left(G_f^T D^{-1} Q^{-1} G_a^T D^{-1} G_a G_f\right) \times h_2 \quad (29)$$

The estimated position for the three dimension context must verify (30).

$$\left| Z_p - Z_r \right| = Z_f \quad (30)$$

where $Z_p = [x \quad y \quad z]^T$ and $Z_r = [x_r \quad y_r \quad z_r]^T$.

However, according to [7], the solution could be (31):

$$Z_p = \sqrt{Z_f} + Z_r \quad or \quad Z_p = -\sqrt{Z_f} + Z. \quad (31)$$

4.4 Performed Algorithm for 3-D Localization

But, Eq. (31) does not always give the best estimate for 3-D position localization. To overcome this problem, we propose a new approach.

Let be $\left(Z_{f_x}; Z_{f_y}; Z_{f_z}\right)$ the final solution of Eq.1 (29). We assume that each coordinate can be positive or negative. Given that, we can define $2^3 = 8$ possible solutions as described in as follow:

$$S_1 = \left(\sqrt{Z_{f_x}} + x_r \;;\; \sqrt{Z_{f_y}} + y_r \;;\; \sqrt{Z_{f_z}} + x_z\right),$$

$$S_2 = \left(-\sqrt{Z_{f_x}} + x_r \;;\; \sqrt{Z_{f_y}} + y_r \;;\; \sqrt{Z_{f_z}} + x_z\right),$$

$$S_3 = \left(\sqrt{Z_{f_x}} + x_r \;;\; -\sqrt{Z_{f_y}} + y_r \;;\; \sqrt{Z_{f_z}} + x_z\right),$$

$$S_4 = \left(\sqrt{Z_{f_x}} + x_r \;;\; \sqrt{Z_{f_y}} + y_r \;;\; -\sqrt{Z_{f_z}} + x_z\right),$$

$$S_5 = \left(-\sqrt{Z_{f_x}} + x_r \;;\; -\sqrt{Z_{f_y}} + y_r \;;\; \sqrt{Z_{f_z}} + x_z\right),$$

$$S_6 = \left(-\sqrt{Z_{f_x}} + x_r \;;\; \sqrt{Z_{f_y}} + y_r \;;\; -\sqrt{Z_{f_z}} + x_z\right),$$

$$S_7 = \left(\sqrt{Z_{f_x}} + x_r \;;\; -\sqrt{Z_{f_y}} + y_r \;;\; -\sqrt{Z_{f_z}} + x_z\right)$$

$$S_8 = \left(\sqrt{Z_{f_x}} + x_r \;;\; \sqrt{Z_{f_y}} + y_r \;;\; \sqrt{Z_{f_z}} + x_z\right).$$

These solutions include the two solutions (S_1) and (S_8) derived from Eq. (31). The choice of the best estimation is an important point of this algorithm. We have proposed an approach which consists to compute eight news TDOAs and to choose the small difference between new TDOAs and the first estimated TDOA [19]. This approach is reviewed in this work to take the best estimation and to reduce computation time. So, final result is chosen comparatively to first estimation of Eq. (24). We compare all eight solutions (S_j, j=1 . . . 8) to those of Eq. (24). The closest one from the solution of Eq. (24) will be the best estimation.

5. Simulation Results

The performance of the method is first investigated through the use of computer simulations and the criterion of performance chosen is the root mean square error (RMSE). The RMSE is plotted versus Signal to Noise Ratio (SNR) ranged from -20dB to 20dB for different mobile positions. We have considered that the mobile moves in the area delimited by:

- *x* ranges from 3 to 18 m by step of 3,
- *y* ranges from -2 to 12 m by step of 2,

- z ranges from 0 to 25 m by step of 5,
- and base stations (transmitters) positions are shown in Table 1. T_i, (i=1 . . . 5), symbolize transmitters.

Table 1: Transmitters positions for simulation

Transmitter	x_i *(m)*	y_i *(m)*	z_i *(m)*
T_1	0	0	0
T_2	15	10	20
T_3	15	0	10
T_4	15	10	0
T_5	0	10	20

From above assumptions, 288 positions have been tested (Figure3). Among these estimated positions, 203 estimated positions have a RMSE lower to 0.02 m (Figure 4). The points with circular symbol on Figure 3 represent 203 estimated positions which RMSE are lower to 0.02 m. The other 85 points without circular symbol are estimated positions which RMSE are greater than 0.02 m. This result shows that around 72% of estimated positions have RMSE lower to 0.02 m.

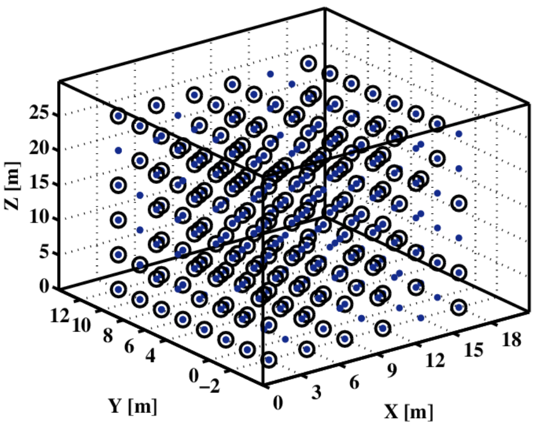

Fig. 3: Actual Mobile Positions and Estimated mobile Positions.

6. Experimental Results

6.1 Experimental Setup

The performance of this method is also investigated in guide propagation environment. This investigation allows the algorithm validation. TDOA estimates are used as in puts for positioning algorithm to find mobile position. In our experimentation, we have used five coaxial cables to

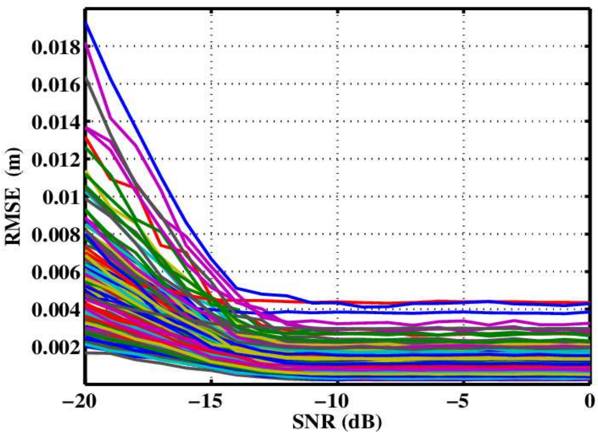

Fig. 4: Estimated positions RMSE which are lower to 0.02 m.

transmit signal to the receiver. Each coaxial cable has a different length. Arbitrary Waveform Gaussian Generator (AWG-7102) is used to send pulse into each coaxial cable. With Digital Sampling Oscilloscope (DSO), according to the length of coaxial cable, five delayed signals are computed. No modulation and no coding techniques have been applied to the pulse. We will present in a future paper the influence of modulation and coding signal to localization system. The DSO used is LecroyWave master 8620A with 20 GHz sampling frequency. The AWG-7102 has one interleave output at 20 GHz and two outputs at 10 GHz each output. So, we used the two outputs at 10 GHz to generate the five signals. For that, we have coupled two and three coaxial cables with power divider.

To determinate delayed propagation (signal arrival times), the length of coaxial cables with power divider connectors have been analyzed with a Vector Network Analyzer (VNA). Results obtained with the VNA are optimal delays of each signal and they are used as exact values for this experimentation. We apply the relation Eq. (3) to determinate TDOAs for each kind of measurements. The Table 2 and Table 3, show TDOA $\left(\hat{\tau}_{i,1}\right)$ measurements obtained from VNA and DSO, respectively.

Table 2: Vector Network Analyzer (VNA) measurements.

Transmitter	*Time Of Arrival* τ_i *(ns)*	$\hat{\tau}_{i,1}$ *(ns)*
T_1	18.70	
T_2	21.70	03.05
T_3	22.69	03.99
T_4	22.74	04.04
T_5	26.55	07.85

Table 3: Digital Sampling Oscilloscope (DSO) measurements.

Transmitters	***Time Of Arrival*** τ_i ***(ns)***	$\hat{\tau}_{i,1}$ ***(ns)***
T_1	0.17	
T_2	03.17	03.00
T_3	04.02	03.85
T_4	04.12	03.95
T_5	07.96	07.79

6.1 Measurements Exploitation

After the TDOAs have been determined, the second step converts estimated TDOA into distances. After that, the positioning algorithm is applied. The mobile position was estimated with network analyzer measurements and with Digital Sampling Oscilloscope measurements. Finally, estimation error of the two measurements was compared. We present results for the two different configurations according to the mobile height and this one of the reference transmitter.

The first configuration assumes that the mobile height is smaller than the reference transmitter height, i.e. in other words, the mobile is under the reference transmitter The second configuration assumes that the mobile is above the reference transmitter.

6.1.1 Estimation with Classical method

In this section, mobile position height is smaller than the height of the reference transmitter. This reference is the transmitter number 1 (T_1). In this position, two situations according to the transmitters' height were analyzed. For the first situation, we assume that all the transmitters are the same height and in the second situation, we consider different heights of transmitters' as we can see in Table 4.
For each situation, we have estimated the mobile position with VNA and DSO measurements. For each studied case, we compare the estimated error of VNA with DSO measurements.

Table 4: First configuration: the mobile (800; 500; 100) is under the reference transmitter (T_1).

Transmitter	***Same Height***			***Different Heights***		
	x_i ***(cm)***	y_i ***(cm)***	z_i ***(cm)***	x_i ***(cm)***	y_i ***(cm)***	z_i ***(cm)***
T_1	980	680	600	980	680	600
T_2	1096	796	600	1239	939	300
T_3	1127	173	600	1276	24	200
T_4	472	172	600	318	18	100
T_5	362	938	600	313	987	500

Tables 5 and 6 show the estimated position. The Figure 5 and Figure 6 are the illustrations from VNA and DSO measurements when transmitters have the same height. Finally, Figures 7 and 8 are the illustrations from VNA and DSO measurements when transmitters have different heights. Estimations are better with VNA than DSO measurements. The absolute error is greater when the transmitters have the same height comparatively to the different heights.

MP, EP-VNA and EP-DSO represent the actual Mobile Position, the Estimated Position with VNA measurements and the Estimated Position with DSO measurements, respectively.

Table 5: Estimations result for the first configuration but all transmitters have the same height.

	x (cm)	***y (cm)***	***z (cm)***	***Error (cm)***
MP	800	500	100	
EP-VNA	800.3	499.6	100.5	0.7
EP-DSO	801.8	497.3	86.5	13.9

Table 6: Estimations result for the first configuration but all transmitters have different heights.

	x (cm)	***y (cm)***	***z (cm)***	***Error (cm)***
MP	800	500	100	
EP-VNA	800.1	500	100.5	0.5
EP-DSO	800.3	499.5	97.4	2.7

The absolute error is around 0.7 cm for VNA against 13.9 cm for DSO when the height is the same (Table 5). When the height of each transmitter is different from one to the other, this error becomes smaller (0.5 cm or 2.7 cm) according to VNA and DSO measurements, respectively (Table 6). For this first consideration, where mobile is under the reference transmitter, the best estimation is given by the eighth solution (S_8). When transmitters have same heights Figures 5 and 6 illustrate estimations with VNA and DSO, respectively. Figures 7 and 8 illustrate estimations with VNA and DSO, respectively, when the transmitters have different heights.

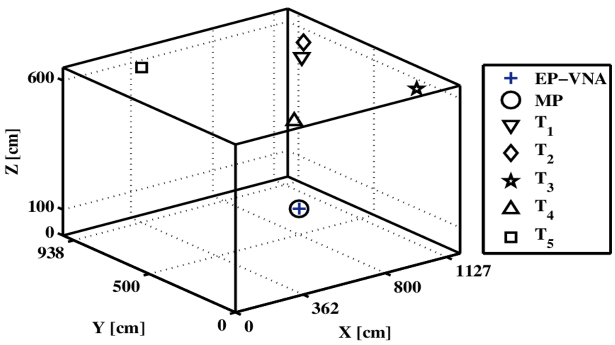

Fig. 5: Estimation positions based on Vector Network Analyzer measurements considering transmitters have the same height.

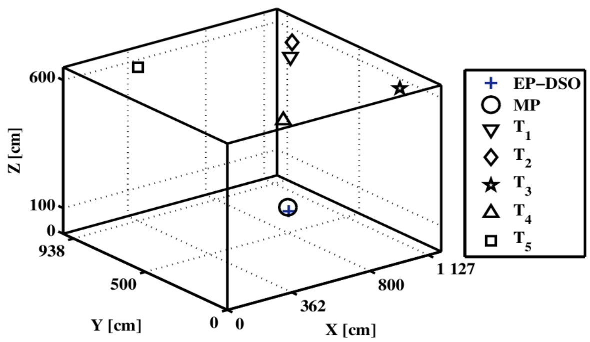

Fig. 6: Estimation positions based on Digital Sampling Oscilloscope measurements considering transmitters have the same height.

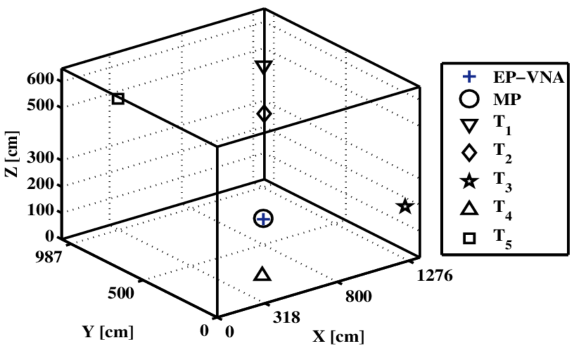

Fig. 7: Estimation positions based on Vector Network Analyzer measurements considering transmitters have different heights.

6.1.2 Estimation with Proposed Approach

In this section, an example of a case when the best estimate is not given by any of solutions S_1 and S_8 such as it has been suggested in [7], is presented. This case occurs sometimes when the mobile is above the reference transmitter. The mobile is positioned at (600; 400; 500) cm

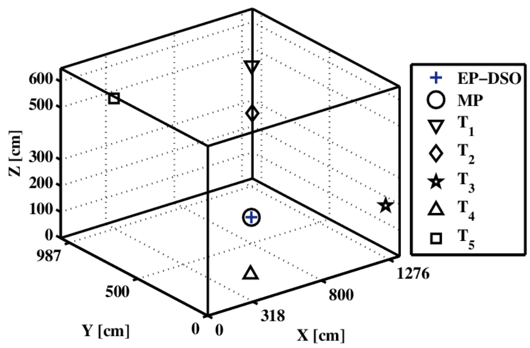

Fig. 8: Estimation positions based on Digital Sampling Oscilloscope measurements considering transmitters have different heights.

and its height is greater than the reference transmitter (T_1) height. Transmitters' positions are given in Table 7. Two situations according to the transmitters' height have been also analyzed. In the first situation, we assumed that all transmitters have the same height and in the second situation, we consider that they are different heights. For each situation, mobile position was estimated with VNA and DSO measurements. For each studied case, estimated absolute errors with VNA and DSO measurements were compared. Results of this study are given in Table 8 and Table 9. Figures 9 and 10 are illustration of estimations with VNA and DSO, respectively, when the transmitters have the same height. Figures 11 and 12 are illustrations estimations with VNA and DSO, respectively, when the transmitters have different heights.

Table 7: Second configuration: the mobile (600; 400; 500) is above the reference transmitter (T_1).

Transmitter	***Same Height***			***Different Heights***		
	x_i (cm)	***y_i (cm)***	***z_i (cm)***	***x_i (cm)***	***y_i (cm)***	***z_i (cm)***
T_1	780	580	0	878	678	100
T_2	896	696	0	1056	856	600
T_3	927	73	0	927	73	1000
T_4	272	72	0	314	114	1050
T_5	162	838	0	41	959	400

Table 8: Estimations result for the second configuration but all transmitters have the same height.

	x (cm)	***y (cm)***	***z (cm)***	***Error (cm)***
MP	600	400	500	
EP-VNA	600.3	399.6	499.5	0.7
EP-DSO	601.8	397.3	513.5	13.9

Table 9: Estimations result for the second configuration but all transmitters have different heights.

	x (cm)	*y (cm)*	*z (cm)*	*Error (cm)*
MP	600	400	500	
EP-VNA	599.8	400	500.3	0.4
EP-DSO	599.7	399.7	502.3	2.32

In term of accuracy, results are the same of case described in the previous subsection 6.1.1. When the height is the same, the absolute error is around 0.7 cm for VNA against 13.9 cm for DSO (Table 8). When the height of each transmitter is different from one to the other, this error becomes smaller or according to VNA and DSO measurements, respectively (Table 9).

For this second configuration, where mobile is above the reference transmitter, the best estimation is given by the solution four (S_4). But, this estimation would be worst if we have limited our study only on solution from Eq. (31). According to [7], the best estimation would be S_1 or S_8. Indeed, the best estimation, in this case, is given by the solution 4 (S_4).

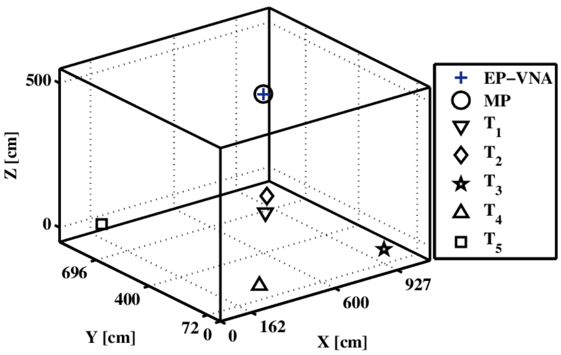

Fig. 9: Estimation positions based on Vector Network Analyzer measurements considering transmitters have the same height.

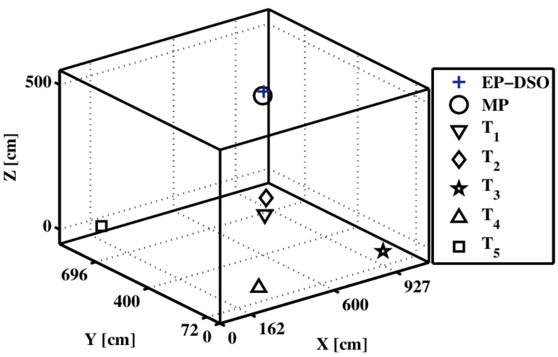

Fig. 10: Estimation positions based on Digital Sampling Oscilloscope measurements considering transmitters have the same height.

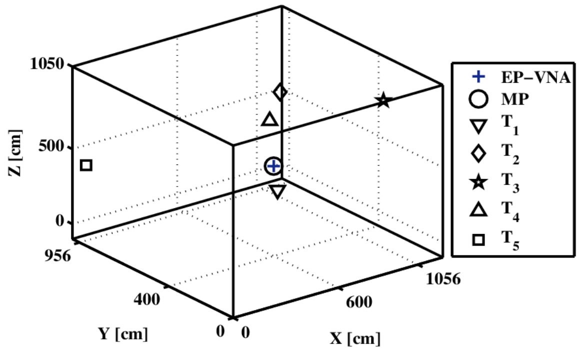

Fig. 11: Estimation positions based on Vector Network Analyzer measurements considering transmitters have different heights.

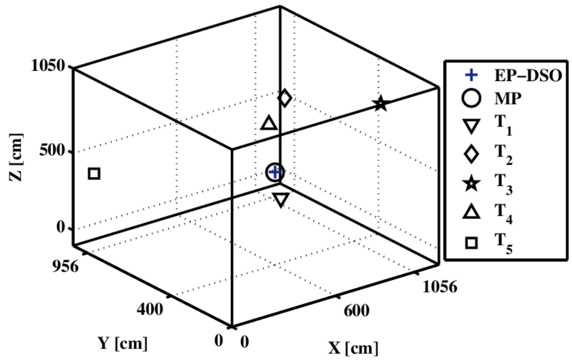

Fig. 12: Estimation positions based on Digital Sampling Oscilloscope measurements considering transmitters have different heights.

6.1.3 Discussion about experimental results

For these mobile positions studied, in each kind of measurements, according to absolute estimated error, estimated positions are the same in each situation with the same height configuration or in different heights configuration.

Estimations are better with VNA measurements than those with DSO. Estimations are also better when transmitters are not in the same plane (equivalent of 2-D context). We can remark that time measurements are the same for each mobile position and with these measurements; estimation accuracy depends on equipments used to estimate them. In view of these results, it is necessary to retain that the proposed positioning algorithm can performed 3-D localization accuracy. In the first considered position where the reference transmitter's height is much greater than those of the base station, the eighth solution (S_8), one of recommended solutions of Eq. (31), gives the best estimated position. In the other case, when the height (*z* component) of the mobile is smaller than this of the reference transmitter, the first or eighth solution did not provide good estimation. The best estimation is given by S_4, one of solutions among those proposed.

7. Conclusion and Future Works

In this paper, localization method for 3-D location system is studied. It relied on two steps approach in what, TDOA parameters are estimated firstly. After that, in the second step, non-iterative positioning algorithm is applied to TDOA parameters to make self-location. The performances of proposed method in terms of accuracy have been evaluated with simulation and experimental results. These results show that this method has accuracy in centimeters order for 3-D localization.

However, several research challenges remain to be investigated or addressed. Measurements used to evaluate positioning algorithm performance are obtained in perfect channel. So, it is important to study the impact of mobile position in different points and the channel variability effect on the method performance. For that, in our future work, we will try to bring an answer to this impact by studying coding technique effects on transmitted signal by using Impulse Radio–Ultra Wide Band (IR-UWB) transmission techniques.

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