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Overall PAPR Reduction for MIMO OFDM Systems

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Abstract

Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) systems have been receiving a great attention as one of the solutions for achieving high speed, efficient, and high quality of service wireless communications. However, the main drawback of MIMO OFDM systems is high Peak to Average Power Ratio (PAPR) because of its sensitivity to the nonlinear distortions introduced by nonlinear devices. In MIMO OFDM systems, a straightforward way for PAPR reduction is to apply existing techniques separately on each transmit antenna. Therefore, a higher overall PAPR is obtained with increasing the number of transmit antennas. In this paper, Partial Transmit Sequences (PTS) technique is modified with different circular shifting approaches. They exploit the extra degree of freedom provided by the transmit antenna array to reduce the overall PAPR even with increasing the number of transmit antennas.

Keywords: HPA, MIMO, OFDM, PAPR, PTS.

1. Introduction

The key challenge faced by future wireless communication systems is to provide high data rate wireless access at high Quality of Service (QoS) [1]. Combined with the facts that spectrum is a scarce resource, and propagation conditions are hostile due to fading caused by destructive addition of multipath components and interference from other users. Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation technique for high data rate wireless communications due to its robustness to frequency selective fading, high spectral efficiency and low computational complexity. OFDM can be used in conjunction with Multiple Input Multiple Output (MIMO) technique to increase the diversity gain and/or the system capacity by exploiting spatial domain [2]. However, the main drawback of MIMO OFDM systems is high Peak to Average Power Ratio (PAPR). Because of its sensitivity to the nonlinear distortions introduced by nonlinear devices, such as Digital to Analog Converters (DAC) and High Power Amplifiers (HPA). These distortions may severely impair system performance due to induced spectral regrowth and detection efficiency degradation [3].

Although numerous techniques have been developed to reduce the PAPR of OFDM signals, no technique can be considered the best solution [4]. The criteria for selecting a PAPR reduction technique involve many aspects such as PAPR reduction capability, transmission power, Bit Error Rate (BER), implementation complexity, and data rate. An effective PAPR reduction technique should give the best tradeoff between these factors [5]. A main consideration is the cost of extra complexity for PAPR reduction being lower than the cost of power inefficiency. It is needed to compromise the criteria to meet the system requirements.

The layout of this paper is as follows: First, Section 2 presents the MIMO OFDM system model and overall PAPR reduction in MIMO OFDM. To further enhance the overall PAPR reduction performance, the circular shifting approaches are proposed in Section 3. The performance of the proposed approaches is evaluated in Section 4 by MATLAB simulation. Finally, Section 5 presents the conclusions.

2. Overall PAPR in MIMO OFDM Systems

2.1 MIMO OFDM System Model

Consider a MIMO OFDM system with Nt transmit antennas, over which independent input vectors are transmitted. For such a system, the transmitted input vector, corresponding to the $i^{\underline{th}}$ symbol, X_i is given by stacking the Nt input vectors of the transmit antennas. Therefor $X_i=[X_{i,1}^T, X_{i,2}^T,...,X_{i,Nt}^T]^T$, where the $i^{\underline{th}}$ input vector for the $nt^{\underline{th}}$ transmit antenna is $X_{i,nt}=[X_{i,nt,0}, X_{i,nt,1}, ..., X_{i,nt,N-1}]^T$, and nt=1, 2 ...., Nt, N is the number of subcarriers. Each $X_{i,nt}$ vector is applied to Inverse Fast Fourier Transform (IFFT) to obtain the time domain vector $x_{i,nt}=F^{-1}[X_{i,nt}]$, $F^{-1}[.]$ is the IFFT operation.The PAPR of the $i^{\underline{th}}$ symbol on the $nt^{\underline{th}}$ transmit antenna is defined as

$$\mathrm{PAPR}_{\mathrm{i,nt}}(x_{\mathrm{i,nt}}) = \frac{\max[|x_{\mathrm{i,nt}}|^2]}{\mathbb{E}\left[|x_{\mathrm{i,nt}}|^2\right]} \ , \tag{1}$$

where $\mathbb{E}[.]$ and $\max[.]$ denotes the mathematical expectation and the maximum value, respectively. The Complementary Cumulative Distribution Function (CCDF) of the PAPR, which is the probability that the PAPR of OFDM signals exceeds a given threshold γ, is calculated as [4, 5]

$$\mathrm{CCDF(PAPR)} = \mathrm{P_r}(\mathrm{PAPR} > \gamma) = 1 - (1 - \mathrm{e}^{-\gamma})^{\mathrm{N}}. \tag{2}$$

For MIMO OFDM systems, the overall PAPR is defined as the maximum of PAPRs among all transmit antennas [3, 6], therefore

$$\mathrm{PAPR}_{\mathrm{i(overall)}} = \max_{1<nt<Nt} \mathrm{PAPR}_{\mathrm{i,nt}} \, . \tag{3}$$

Specifically, since NNt time domain samples are considered instead of N in Single Input Single Output (SISO) OFDM, the CCDF of the $\mathrm{PAPR}_{\mathrm{overall}}$ in MIMO OFDM is written as [6]

$$\begin{aligned} \mathrm{CCDF}(\mathrm{PAPR}_{\mathrm{overall}}) &= \mathrm{P_r}(\mathrm{PAPR}_{\mathrm{overall}} > \gamma) \\ &= 1 - (1 - \mathrm{e}^{-\gamma})^{\mathrm{NNt}}. \end{aligned} \tag{4}$$

Comparing Eq. 2 and Eq. 4, it is evident that MIMO OFDM systems result in even worse $\mathrm{PAPR}_{\mathrm{overall}}$ performance than SISO OFDM systems, specially with increasing of the number of transmit antennas [3, 6]. Therefore, PAPR reduction for MIMO OFDM signals is an important issue for high power efficiency, low power consumption, and low implementation cost.

2.2 Overall PAPR Reduction in MIMO OFDM Systems

There are many publications about PAPR reduction techniques, but most of them were developed for SISO OFDM systems [7].Generally, PAPR reduction techniques for conventional SISO OFDM systems can be directly applied to MIMO OFDM systems. It is done by applying the conventional techniques separately on each transmit antenna, then the overall PAPR is obtained as in Eq. 3 [8]. Unfortunately, the direct application of conventional techniques leads to the need for more Side Information (SI) and/or complexity. Moreover a higher overall PAPR is achieved, especially with the increase of number of transmit antennas. In literature, most modifications for PAPR reduction in MIMO OFDM focus on the optimization for overall antennas to reduce the amount of SI and/or the complexity, with slight degradation in PAPR reduction performance.

The application of Conventional Partial Transmit Sequence (CPTS) and Low Complexity Partial Transmit Sequence (LCPTS) techniques to MIMO OFDM was studied in [9, 10]. In [9] the authors presented the results for CCDF of PAPR (per transmit antenna) in the case of applying CPTS technique for each transmit antenna separately. They studied the effect of increasing the number of transmit antennas. The results show performance degradation with increasing the number of transmit antennas. LCPTS technique was presented in [10] as a simplified approach, where the optimization is carried out jointly over all transmit antennas. Although LCPTS is less complex and requires the transmission of less SI than CPTS, the achieved PAPR reduction is also smaller. In this paper, results for CCDF of overall PAPR (for the entire system) for CPTS and LCPTS with different number of transmit antennas are presented and compared with the different circular shifting approaches, which are presented in detail in the next section.

3. Circular Shifting Approaches

In this paper, among all the conventional PAPR reduction techniques, Partial Transmit Sequence (PTS) is adopted to be modified with different circular shifting approaches. They exploit the extra degree of freedom provided by the transmit antenna array, to improve the overall PAPR reduction in MIMO OFDM systems. Thus, it is not appropriate for SISO OFDM systems [9, 10].

3.1 Modified PTS with Circular Shifting (PTS-CS)

As shown in Fig. 1, in PTS-CS the input vector of the ith symbol $\mathrm{X_i}$ is partitioned into M disjoint groups.Then each $\mathrm{X}^{\mathrm{m}}_{\mathrm{i,nt}}$ vector is applied to IFFT,the resulting time domain mth group $x_{\mathrm{i}}^{\mathrm{m}}$ is given by $x_{\mathrm{i}}^{\mathrm{m}} = [x^{\mathrm{m}}_{\mathrm{i,1}}{}^{\mathrm{T}}, x^{\mathrm{m}}_{\mathrm{i,2}}{}^{\mathrm{T}}, \ldots.., x^{\mathrm{m}}_{\mathrm{i,Nt}}{}^{\mathrm{T}}]^{\mathrm{T}}$, where $x^{\mathrm{m}}_{\mathrm{i,nt}} = \mathrm{F}^{-1}[\mathrm{X}^{\mathrm{m}}_{\mathrm{i,nt}}]$. The M groups are independently shifted by a circular shifting vector $\mathrm{Cs_i} = [\mathrm{Cs_i}^1, \mathrm{Cs_i}^2, \ldots.., \mathrm{Cs_i}^{\mathrm{M}}]^{\mathrm{T}}$, where $\mathrm{Cs_i}^{\mathrm{m}} = 0, 1, \ldots.., \mathrm{Nt}-1$. The mth group vector $x_{\mathrm{i}}^{\mathrm{m}}$ is shifted by the circular shifting factor $\mathrm{Cs_i}^{\mathrm{m}}$, before combining the M groups at each antenna. The shifted mth group vector is $\dot{x}_i^m = [\dot{x}_{i,1}^{m\,T}, \dot{x}_{i,2}^{m\,T}, \ldots \ldots, \dot{x}_{i,Nt}^{m\ \ T}]^T$, where

$$\dot{x}_{i,nt}^{m} = \left\{ \begin{matrix} x_{i,nt-Cs_i^m}^{m} & nt > Cs_i^m \\ x_{i,nt+Nt-Cs_i^m}^{m} & nt \le Cs_i^m \end{matrix} \right\}. \tag{5}$$

For example, at $\mathrm{Cs_i}^{\mathrm{m}}=1$, the obtained shifted vector is $\dot{x}_i^m = \left[x_{i,Nt}^{m\ \ T}, x_{i,1}^{m\,T}, \ldots \ldots, x_{i,Nt-1}^{m\ \ \ T}\right]^T$. And for $\mathrm{Cs_i}^{\mathrm{m}}=2$, the shifted vector is $\dot{x}_i^m = [x_{i,Nt-1}^{m\ \ \ T}, x_{i,Nt}^{m\ \ T}, \ldots \ldots, x_{i,Nt-2}^{m\ \ \ T}]^T$. The optimal shifting vector $\mathrm{Cs_i}'$ is selected according to

$$\mathrm{Cs_i'} = \underset{\mathrm{Cs_i}}{\mathrm{argmin}} \left(\max_{1<nt<Nt} \mathrm{PAPR}_{i,nt}(\dot{x}_{i,nt}) \right). \tag{6}$$

In PTS-CS, there are Nt probabilities for shifting each group, $\mathrm{Cs_i}^1$ is set to 0 without any loss of performance. The complexity increases exponentially with the number of groups M, it also depends on the number of transmit antennas Nt. Since Nt^{M-1} probable Cs_i vectors are searched to get the optimum vector Cs_i', which gives the lowest overall PAPR. The number of required SI bits is $\log_2(Nt^{(M-1)})$, it increases with increasing the number of groups M, and transmit antennas Nt. The amount of overall PAPR reduction increases with increasing the number of groups M, and/or transmit antennas Nt.

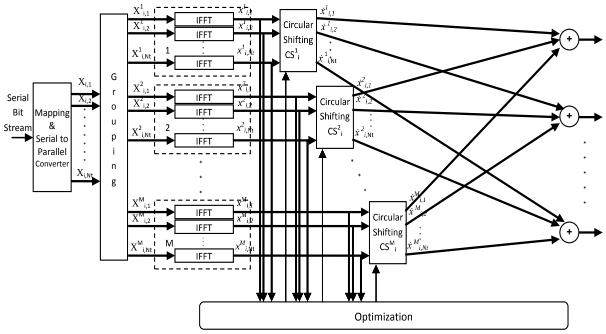

Figure 1. PTS-CS for MIMO OFDM system.

3.2 Modified PTS with Rotated Circular Shifting (PTS-RCS)

PTS-RCS is proposed, to achieve better performance in overall PAPR reduction than PTS-CS. It modifies the circular shifting by rotating independently each shifted group vector $\dot{x}_i^m = [\dot{x}_{i,1}^{m\,T}, \dot{x}_{i,2}^{m\,T}, \dots\dots, \dot{x}_{i,Nt}^{m\,T}]^T$, before combining at each transmit antenna. The M shifted groups are rotated by a phase vector $W_i=[W^{1\,T}_i, W^{2\,T}_i, \dots, W^{M\,T}_i]^T$, where $W^m_i=[W^m_{i,1}, W^m_{i,2}, \dots, W^m_{i,Nt}]$, and $W^m_{i,nt} = e^{j\theta^m_{i,nt}}, \theta^m_{i,nt} \in [0,2\pi]$. The $m^{\underline{th}}$ group vector after circular shifting and rotating is $\ddot{x}_i^m = \dot{x}_i^m \circ W_i^m$, where ∘ denotes element-wise multiplication. The final transmitted vector at the $nt^{\underline{th}}$ antenna is

$$\ddot{x}_{i,nt} = \sum_{m=1}^{M} \dot{x}_{i,nt}^m \circ W_i^m \; . \tag{7}$$

The optimal shifting vector Cs_i' and rotating vector W_i' are selected from $Nt(KNt)^{(M-1)}$ probable vectors, according to

$$\{Cs_i', W_i'\} = \underset{\{Cs_i, W_i'\}}{\operatorname{argmin}} \left(\max_{1<nt<Nt} PAPR_{i,nt}(\ddot{x}_{i,nt}) \right). \tag{8}$$

Therefore, the complexity increases with increasing the number of transmit antennas Nt, groups M, and allowed phases K. The selection of the phase factors is limited to a set with a finite number of elements, to reduce the search complexity, as in conventional PTS. The number of side information bits which must be transmitted to the receiver is $\log_2(K^{Nt}Nt)^{(M-1)}$. It increases with increasing the number of groups M, allowed phases K, and transmit antennas Nt. The amount of overall PAPR reduction increases with increasing the number of groups M, transmit antennas Nt, and the allowed phases K. Generally, PTS-RCS has better berformance, when compared with PTS-CS, LCPTS, or CPTS, with more complexity.

3.3 Modified PTS with Low Complexity Rotated Circular Shifting (PTS-LCRCS)

In order to reduce the complexity and SI of PTS-RCS, PTS-LCRCS is proposed. As shown in Fig. 2, the $m^{\underline{th}}$ group vector x_i^m is shifted by the circular shifting factor Cs_i^m to get the shifted vector $\dot{x}_i^m$. The M shifted groups are independently rotated by a phase vector$W_i=[W^{1\,T}_i, W^{2\,T}_i, \dots, W^{M\,T}_i]^T$, and $W_i^m = e^{j\theta_{i,nt}^m}, \theta_{i,nt}^m \in [0,2\pi]$. Aunique phase for each group is applied, to reduce the complexty and the SI. The $m^{\underline{th}}$ group vector after shifting and rotating is $\ddot{x}_i^m = \dot{x}_i^m W_i^m$, the final transmitted vector at the $nt^{\underline{th}}$ antenna is

$$\ddot{x}_{i,nt} = \sum_{m=1}^{M} \dot{x}_{i,nt}^m \, W_i^m \; . \tag{9}$$

The optimal shifting vector Cs_i' and rotating vector W_i' are selected from $(KNt)^{(M-1)}$ probable vectors, according to

$$\{Cs_i', W_i'\} = \underset{\{Cs_i, W_i\}}{\operatorname{argmin}} \left(\max_{1<nt<Nt} PAPR_{i,nt}(\ddot{x}_{i,nt}) \right). \tag{10}$$

The number of the transmitted SI bits is $\log_2(KNt)^{(M-1)}$. PTS-LCRCS is a compromise between complexity and performance. It has lower complexity and SI than PTS-RCS, with a slight less performance.

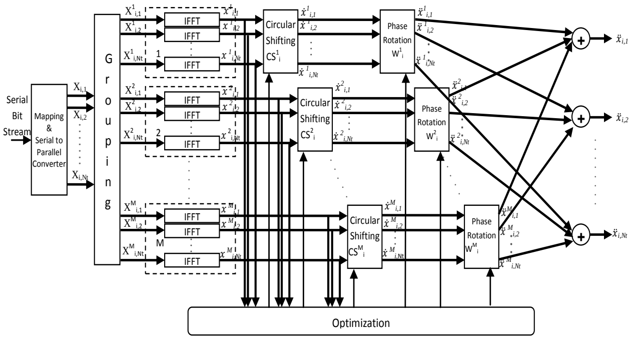

Figure 2. PTS-LCRCS for MIMO OFDM system.

4. Results

The performance of the proposed versions of PTS and modified PTS with different circular shifting approaches has been tested by means of numerical simulation using MATLAB. The parameters of the simulations are N=64 subcarriers, number of transmit antennas Nt={2,4}, the NC partitioning is applied, and Quadrature Phase Shift Keying (QPSK) modulation is used. The performance is given in terms of the CCDF of overall PAPR. When clipping occurs at power level $PAPR_0$, for a normalized signal power, the CCDF values indicate the probability of clipping. In all figures, the CCDF of PAPR without reductions are indicated by M=1. Also, curves for SISO OFDM system is indicated by Nt=1.

Figure 3 shows the CCDF of overall PAPR in CPTS for MIMO OFDM system. The results are given for M=2, K=4 and Nt={1,2,4}. It can be concluded from this figure that at a clipping probability of 10^{-3}, 2.04 dB, 1.85 dB and 1.81 dB overall PAPR reductions are achieved for Nt=1, Nt=2 and Nt=4 respectively. The amount of overall PAPR reduction decreases when Nt increases. CPTS performes worse with increasing Nt.

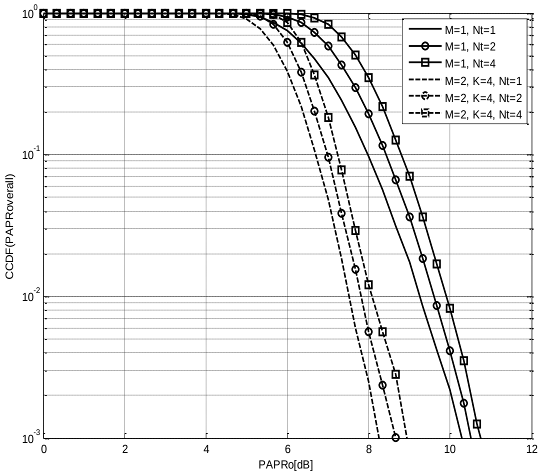

Figure 3. CCDF of overall PAPR for CPTS.

Figure 4 shows the CCDF of overall PAPR in LCPTS for MIMO OFDM system. The results are given for M=2, K=4 and Nt={1,2,4}. It can be concluded from this figure that at a clipping probability of 10^{-3}, 2.04 dB, 1.73 dB and 1.61 dB overall PAPR reductions are achieved for Nt=1, Nt=2 and Nt=4 respectively. The amount of overall PAPR reduction decreases when Nt increases. LCPTS performes worse with increasing Nt. Generally, it has worse performance than CPTS.

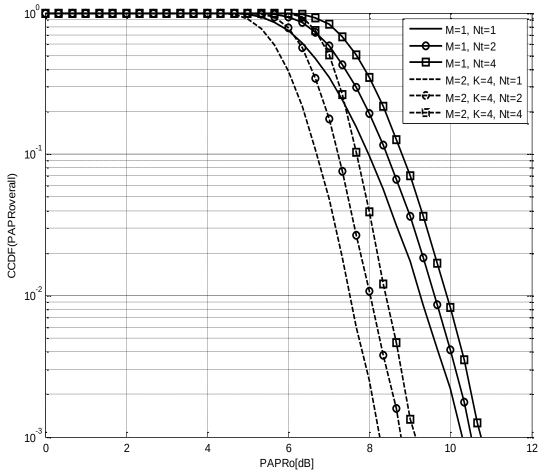

Figure 4. CCDF of overall PAPR for LCPTS.

Figure 5 shows the CCDF of overall PAPR in PTS-CS for MIMO OFDM system. The results are given for M=2, K=4 and Nt={1, 2, 4}. It can be concluded from this figure that at a clipping probability of 10^{-3}, 0 dB, 1.18 dB and 2.16 dB overall PAPR reductions are achieved for Nt=1, Nt=2 and Nt=4 respectively. The amount of overall PAPR reduction increases when Nt increases. At Nt=4, PTS-CS performes better than CPTS and LCPTS, but it isn't suitable for SISO systems (Nt=1).

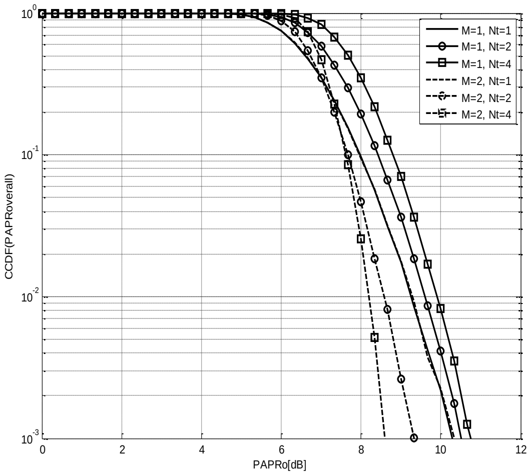

Figure 5. CCDF of overall PAPR for PTS-CS.

Figure 6 shows the CCDF of overall PAPR in PTS-RCS for MIMO OFDM system. The results are given for M=2, K=4 and Nt={1, 2, 4}. It can be concluded from this figure that at a clipping probability of 10^{-3}, 2.04 dB, 2.76 dB and 3.52 dB overall PAPR reductions are achieved for Nt=1, Nt=2 and Nt=4 respectively. The amount of overall PAPR reduction increases when Nt increases. Also, PTS-RCS has better performance than PTS-CS.

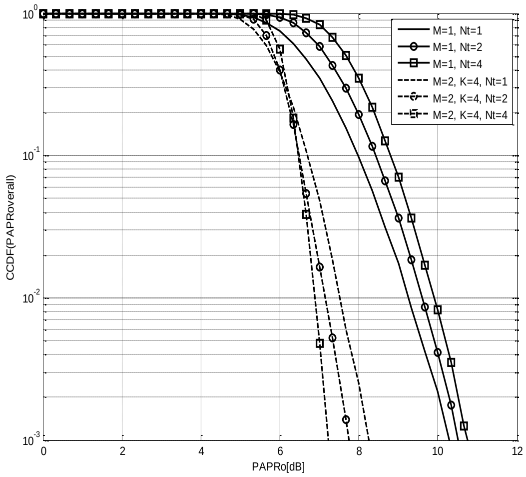

Figure 6. CCDF of overall PAPR for PTS-RCS.

Figure 7 shows the CCDF of overall PAPR in PTS-LCRCS for MIMO OFDM system. The results are given for M=2, K=4 and Nt={1, 2, 4}. It can be concludedfrom this figure that at a clipping probability of 10^{-3}, 2.04 dB, 2.65 dB and 3.10 dB overall PAPR reductions are achieved for Nt=1, Nt=2 and Nt=4 respectively. The amount of overall PAPR reduction increases when Nt increases. PTS-LCRCS performes better than PTS-CS, but worse than PTS-RCS.

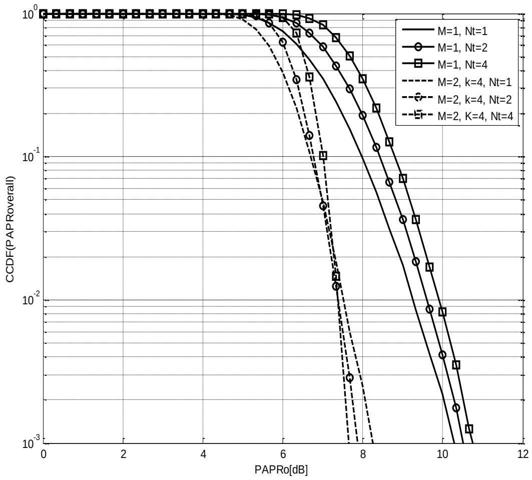

Figure 7. CCDF of overall PAPR for PTS-LCRCS.Fig.

Figure 8 shows the CCDF of overall PAPR, for the different proposed versions of PTS and modified PTS with different circular shifting approaches, for MIMO OFDM system. The results are given for M=2, K=4 and Nt=4. It can be concludedfrom this figure that at a clipping probability of 10^{-3}, 1.81 dB, 1.61 dB, 2.16 dB, 3.52 dB, and 3.10 dB overall PAPR reductions are achieved for CPTS, LCPTS, PTS-CS, PTS-RCS, and PTS-LCRCS respectively. PTS-RCS and PTS-LCRSC have better performances at Nt=4. However, CPTS and LCPTS have worse performances. PTS-CS reduces the complexity with moderate performances.

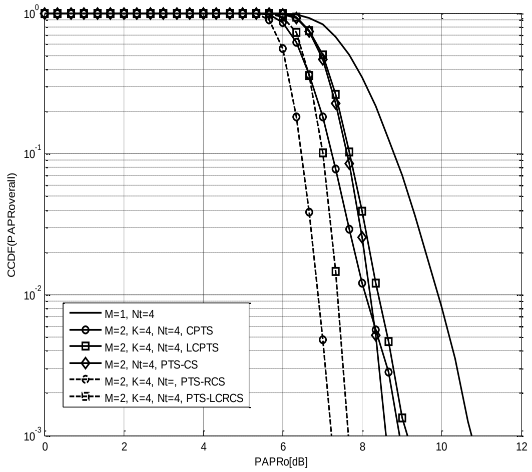

Figure 8. CCDF of overall PAPR for different schemes.

5. Conclusions

The simulation results verify the ability of modified PTS with different circular shifting approaches to reduce the overall PAPR in MIMO OFDM systems compared with the CPTS and LCPTS. LCPTS reduces the complexity of CPTS, but it performs worse than CPTS. By increasing the number of transmit antennas, CPTS and LCPTS perform worse, contrary to modified PTS with different circular shifting approaches (PTS-CS, PTS-RCS and PTS-LCRCS) which perform better. Among the circular shifting approaches, PTS-CS has the lowest overall PAPR reduction with the lowest complexity and SI. PTS-RCS introduces the largest overall PAPR reduction with more complexity and SI, compared with PTS-CS and PTS-LCRCS. PTS-LCRCS is a compromise solution between PTS-CS and PTS-RCS. It has an improved performance compared to PTS-CS with more complexity and SI. However, it reduces the complexity of PTS-RCS with less overall PAPR reduction. From all above the suitable modification can be chosen according to the system requirements. A compromise should be made between the computational complexity and the capability of overall PAPR reduction.

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