Calculation of Multiconductor transmission line capacitance of elliptical shape using numerical method

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Abstract

The first step in modeling the Multiconductor Transmission Line (MTL) is to obtain the per-unit-length inductance, resistance, and capacitance parameters; Capacitance is an important parameter in MTL equation, because inductance is usually deduced from capacitance. There are two methods to calculate the per-unit-length parameters of a MTL, the analytical and the numerical method, the analytical method is valid only for simple configurations, however for complex configurations the solution is to use a numerical method. We propose in this paper the method of moments to calculate the per-unit-length of MTL with an elliptical shape.

Keywords: *Multiconductor Transmission Line; per-unit-length; analytical method; elliptical shape.*

1. Introduction

In order to better protect and optimize the systems connected to the lines, engineers and researchers have been constantly designing new models characterizing lines and cables[1-2]. From any time the developed models are attached to reproduce as faithfully as possible the realities which they represent. Thus, the resolution of telegrapher's equations allows determining the voltage and current magnitudes at any point on the line as long as the primary parameters of the components of the connections are rigorously determined [3-4].

In this context, various formulations and methods have been developed to determine the per-unit-length parameters. The current methods for calculating linear parameters MTL can be divided into two categories, numerical techniques and analytical.

Analytical methods are valid only when certain geometry conditions are met, such as cylindrical conductors, to find simple literal expressions, thus easier to program and with a very fast calculation [5].

Moreover, for complex structures, an analytical solution is rarely trivial or impossible. The solution is the use of numerical modeling [6-7]. The advantage of this solution is that it allows the study of any configuration and this regardless of its complexity. Close to modeling errors, mainly related to the meshing structure.

In this paper we propose a numerical method based on the method of moments to calculate the per-unit-length parameters of a MTL with an elliptical section.

2. Analyzing and modeling

2.1 The theory of the transmission lines

The theory of the transmission lines is based on a system of linear equations which results from the Maxwell equations by integration of the electromagnetic field and by the assumption of quasi-stationarity. The obtained differential equations describe the evolution of a current and a voltage on the line as a function of the electromagnetic field present in the area.

Using an equation of the transmission line requires knowledge of an electrical reference [8]. Generally, this reference is given either by the cable shield or through an outer conductive element, for example a metal or a conducting ground plane. For this, we always consider, at least, that transmission line is defined by two conductors.

In the case of a line composed of N +1 conductors (N signal conductors and a grounding or return conductor), as illustrated in Figure 1, the equations are given by the collection:

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$$\left.\begin{aligned} \frac{\partial}{\partial z}V(z,t) &= -RI(\mathrm{z,t}) - \mathrm{L}\frac{\partial}{\partial t}I(\mathrm{z,t}) \\ d\frac{\partial}{\partial z}\mathrm{I}(z,t) &= -GV(\mathrm{z,t}) - C\frac{\partial}{\partial t}\mathrm{V(z,t)} \end{aligned}\right\} \quad (1)$$

Where:
Voltage and current vectors of dimension (n, 1) are simply defined by:

$$I(z)=\left[I_1(z)\ I_2(z)...\ I_N(z)\right]^T \quad (2)$$

$$V(z)=\left[V_1(z)\ V_2(z)...\ V_N(z)\right]^T \quad (3)$$

L and C are respectively matrix inductance and capacitance by unit-of-length

$$L=\begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{22} & \cdots & L_{2N} \\ \vdots & \vdots & & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{bmatrix} \quad (4)$$

$$C=\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix} \quad (5)$$

R and G are respectively matrix resistance and conductance by unit-of-length

$$R=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \quad (6)$$

$$G=\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \quad (7)$$

The resolution of the equations of the telegrapher allows determining the sizes tension and current in every respect of the line provided that the parameters of elements establishing the connections are strictly determined, these parameters are not easy to calculate, especially when it comes to a line with a complex shape. In what follows, we present an algorithm based on the method of moments to calculate the capacitance C of a matrix row with an elliptical shape.

2.2 Calculation of MTL capacitance

In a transmission line, the conductors can be sufficiently distant; the wide separation assumption provides the simplification of the charge distributions around the wire peripheries which are approximately uniform. The filament with uniform charge distribution makes the computation much easier.

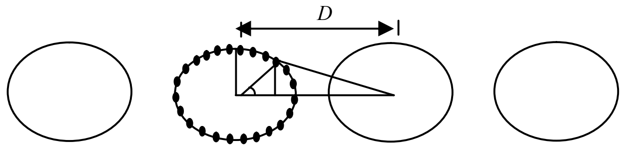

Fig.1 Systems two dimensions of elliptical conductors.

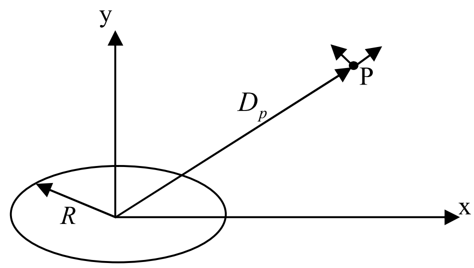

Fig. 2 The geometry of charge distribution problem

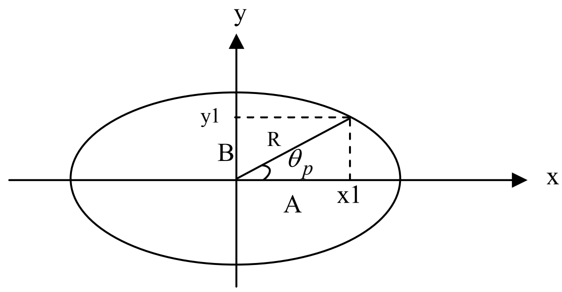

Fig. 3 Systems with two dimensions of a conductor with elliptic section.

Consider Fig 2 in which infinitesimal line charges lie on a cylindrical surface of radius r. The potential $\phi_p(r_p,\theta_p)$ at the free space point P due to the ith charge line of the nth conductor ρ_{ni} is :

$$\phi_p = -\frac{\rho_{ni}}{2\pi\varepsilon_0}\ln(\mathrm{D}_p) \quad (8)$$

Where D_p is the distance between charge line ρ_{ni} and point P. When we come to a LTM, each conductor is divided into N_k points on the periphery, the potential at a point P due to all charge distributions on this conductor and all other conductors, written as

$$\phi_p = -\sum_{n=1}^{N+1}\sum_{i=1}^{N_i}\frac{\rho_{ni}}{2\pi\varepsilon_0}\ln D_{(\rho_{ni},p)} \tag{9}$$

$D_{(\rho_{ni},p)}$ is the distance between charge point ρ_{ni} and the potential ϕ_p

In the MTL the total number of charge lines is N_{tot} points on the wires at which we enforce the potential of the wire due to all the charge distributions on this conductor and all of the other conductors as illustrated in Figure 1.

Where:

$$N_{tot} = \sum_{i=1}^{N+1} N_k \tag{10}$$

This leads to a set of N_{tot} simultaneous equations that must be solved for the expansion coefficients, written in matrix form, as

$$\Phi = \mathrm{D}\rho \tag{11}$$

$$\begin{bmatrix}\Phi_1\\ \Phi_2\\ \vdots\\ \Phi_{N+1}\end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1N+1}\\ D_{21} & D_{22} & \cdots & D_{2N+1}\\ \vdots & \vdots & & \vdots\\ D_{N+11} & D_{N+12} & \cdots & D_{N+1N+1}\end{bmatrix}\begin{bmatrix}\rho_1\\ \rho_2\\ \vdots\\ \rho_{N+1}\end{bmatrix} \tag{12}$$

Where

Φ_i is the vector of potentials at the match points on the ith conductor

ρ_i is the vector of line charge density in the ith conductor are denoted as

$$\Phi_i = \left[\phi_i \cdots \phi_i \cdots \phi_i\right]^t \tag{13}$$

$$\rho_i = \left[\rho_{i1}\ \rho_{i2}\ \cdots\ \rho_{iN_i}\right]^t \tag{14}$$

The generalized capacitance matrix $\mathscr{C}_{ij}$ can be obtained from (12), as :

$$\mathscr{C}_{ij} = AD^{-1}F \tag{15}$$

$$1 \le i \le N+1,\ 1 \le j \le N+1,$$

Where

$$F = \begin{bmatrix} F_1 & 0 & \cdots & 0\\ 0 & F_2 & \cdots & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & F_{N+1}\end{bmatrix} \tag{16}$$

F_k is $N_k \times 1$ onesvector

$$F_k = \begin{bmatrix}1 & 1 & \cdots & 1\end{bmatrix}^T \qquad 1 \le \mathrm{k} \le \mathrm{N+1} \tag{17}$$

And

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0\\ 0 & A_2 & \cdots & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & A_{N+1}\end{bmatrix} \tag{18}$$

A_k is $A_k \times 1$ onesvector

$$A_k = \begin{bmatrix}1 & 1 & \cdots & 1\end{bmatrix}^T \qquad 1 \le \mathrm{k} \le \mathrm{N+1} \tag{19}$$

We observe that the size of the matrix C is (n +1) × (n +1), whereas the parameter per-unit-length matrices L and C are of size (n × n). To obtain the matrix of the capacitance C per-unit-length we select a reference conductor as the zeroth conductor. The voltages of the line relative to the reference conductor are given by [8-11]:

$$V_i = \phi_i - \phi_0 \tag{20}$$

The matrix of the capacitance C per-unit-length is written as:

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N}\\ C_{21} & C_{22} & \cdots & C_{2N}\\ \vdots & \vdots & & \vdots\\ C_{N1} & C_{N2} & \cdots & C_{NN}\end{bmatrix} \tag{22}$$

where:

$$C_{ij} = \mathscr{C}_{ij} - \frac{\left[\sum_{m=0}^{n}\mathscr{C}_{im}\right]\left[\sum_{m=0}^{n}\mathscr{C}_{mj}\right]}{\sum_{l=0}^{n}\left[\sum_{m=0}^{n}\mathscr{C}_{lm}\right]} \tag{23}$$

3. Results and analysis

3.1 Validation

To validate our program, we propose to take as an example two configurations of three conductors of circular cross section. (fig.4). the geometrical parameters defined in Figure 4 (sectional view) are shown in Table 1

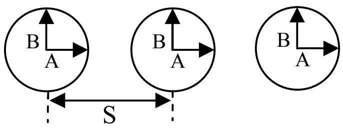

Fig. 4 Sectional view of a three-wire ribbon cable

Table1: Geometric configuration of the line

Parameters	Configuration 1 (mm)	Configuration 2 (mm)
S	10	20
A	0.1905	0.6
B	0.1905	0.6

Matrices generalized capacity were calculated using 30 points around the periphery of each surface of the conductor. The results are shown in Table 2.

Table 2: The generalized capacitances for the three-wire with circular cross-section

Config	Entry	C_{11}	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}
1	PF/m	9.67	10.79	9.67	-3.99	-2.27	-3.99
2	PF/m	10.89	12.25	10.89	-4.50	-2.35	-4.50

Matrices of capacity of transmission line per-unit-length C become as shown in Table.3.

Table3: The capacities of transmission line

Conf	Entry	C_{11}	C_{12}	C_{22}
1	C(PF/m) (MOM)	*9.971*	*-4.985*	*8.468*
	C(PF/m) (analytical method)	*9.939*	*-4.969*	*8.459*
2	C(PF/m) (MOM)	*11.399*	*-5.699*	*9.479*
	C(PF/m) (analytical method)	*11.317*	*-5.658*	*9.449*

In the table, the comparison of results obtained by different methods mentioned has allowed us to observe a good agreement between the different results. This allows us to validate the program

3.2 Application

In this section, we give the results of a 3-wire line, elliptical cross section. The geometrical parameters defined in Figure 5 (cross sectional view) are shown in Table 4:

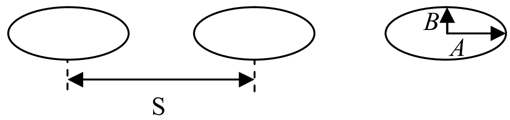

Fig. 5 Sectional view of a three-conductor line section elliptical

Table 4: Geometric configuration of the line

Parameters	Configuration 1 (mm)	Configuration 2 (mm)
S	10	20
A	0.2	0.3
B	0.1	0.1

Matrices generalized capacity were calculated using 30 points around the periphery of each surface of the conductor. The results are shown in Table 5.

Table5: The generalized capacitances for the three-wire with elliptical cross-section

Entry	C_{11}	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}
Conf 1	10.07	11.30	10.07	-4.24	-2.37	-4.24
Conf 2	11.67	12.60	11.67	-4.10	-2.44	-4.10

Matrices of capacity of transmission line per-unit-length C become as shown in Table 6.

Table 6: The capacities of transmission line

Entry	C_{11}	C_{12}	C_{22}
Configuration 1	*10.491*	*-5.245*	*8.848*
Configuration 2	*11.282*	*-5.641*	*9.882*

The convergence of the method is illustrated in the figures 6 and 7. The capacitance per-unit-length is plotted against the number chosen around the conductor.
We observe that the capacity converges to accurate values after about 5 points.

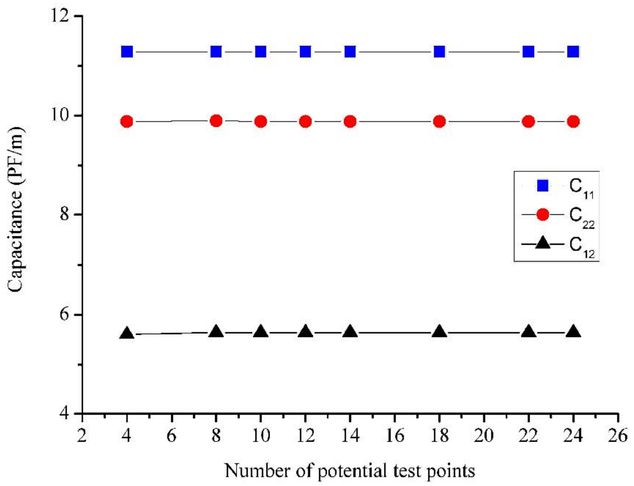

Fig. 6 The convergence of capacitance parameters for the first configuration

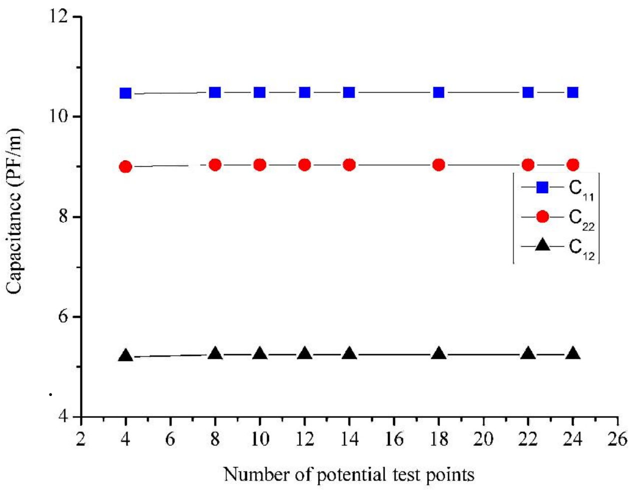

Fig. 7 The convergence of capacitance parameters for the second configuration

Conclusion

In this paper, we have presented a numerical method based on the method of moments to calculate the capacitance of a lineal MTL with an elliptical shape. Through some cases treated, we have validated our method. The results show that this method requires about 5 test points to converge to specific values, which means it does not require significant IT resources needed for the calculation and storage. In contrast, this method becomes less accurate if the wires are closely spaced, so the objective of the next contribution will be the development of a numerical technique which can be used to obtain accurate results for multiwire lines with inhomogeneous surroundings and closely spaced conductors.

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