A Three Stage Communication Network Model with Homogeneous Bulk Arrivals and Dynamic Bandwidth Allocation

G. Rajendra Kumar[1], Kuda Nageswara Rao[2], and K. Srinivasa Rao[3]

[1, 2] Department of Computer Science and Systems Engineering, College of Engineering, Andhra University, Visakhapatnam-530003, Andhra Pradesh, India

[3]Department of Statistics, Andhra University, Visakhapatnam-530003, Andhra Pradesh, India

Abstract

Recently the growth of packet switched networks lead to new traffic problems. To solve these problems, the differentiated service models have been considered as a scalable traffic management mechanism to ensure guaranteed quality of service. In this paper, a three stage/node communication network model with homogeneous bulk arrivals and dynamic bandwidth is developed and analyzed. The data packets after getting transmitted from the first node are forwarded to the second buffer connected to the second node and the packets leaving the second node are forwarded to the third node. Dynamic Bandwidth Allocation (DBA) is the strategy that the transmission rate at each node is adjusted depending upon the content of the buffer at every packet transmission. It is assumed that the arrival of packets follow compound Poisson processes and the transmission completions at each node follow Poisson processes. This model is more accurately fit into the realistic situation of the communication network having a predecessor and successor nodes for the middle node. Using the difference – differential equations, the joint probability generating function of the number of packets in each buffer is derived. The performance measures like, the probability of emptiness of the three buffers, the mean content in each buffer, mean delays in buffers, throughput etc. are derived explicitly under transient conditions. This network model is much useful in communication systems like, Telecommunications, Wireless communications, the Internet, ATM networks etc.

Keywords: *Three stage communication networks, Dynamic bandwidth allocation, Bulk arrivals, and Performance measures*

1. Introduction

Now a day the need for data/voice transformation is increasing rapidly to fulfill the needs of different users in many fields. To meet the increasing demand for data transformation, effective communication networks have been developed. In the recent past, the technological advancement and innovations in the network equipment lead to the design and development of effective communication networks with packet switching. The large volumes of data originated at different sources at different users is to be delivered with high performance rates through the network, thus the design and analysis of load dependent networks and effective utilization of transmission bandwidth on the transmission lines are major issues of the communication systems (Srinivasa Rao K. et al (2006)). The analysis of statistical multiplexing of data/voice transmission through congestion control strategies are important for efficient utilization of network resources, congestion control is achieved usually by applying bit dropping method. In order to reduce the transmission time a portion of the least significant bits are discarded in the bit dropping method when there is congestion in buffering. While maintaining quality of service expected by the end users. Input bit dropping (IBD) and output bit dropping (OBD) are the usual bit dropping methods (Kin K. Leung, (2002)).

In input bit dropping, bits may be dropped when packets are placed in the queue waiting for transmission. In output bit dropping, bits are discarded when a packet is being transmitted over the channel. This implies fluctuation in voice quality due to dynamically vary bit rate during the transmission (Karanam V.R et al (1988)). For efficient transmission some algorithms have been developed with various protocols and allocation strategies for optimum utilization of the bandwidth for an efficient transmission (bandwidth (Emre and Ezhan, 2008; Gundale and Yardi, 2008; Hongwang and Yufan, 2009; Fen Zhou et al. 2009; Stanislav, 2009). These strategies are developed based on flow control and bit dropping techniques. Some work has been initiated in literature regarding utilization of the idle bandwidth by adjusting the transmission rate instantaneously before transmission of a packet.

Dynamic bandwidth is the transmission strategy of adjusting the data transmission rate depending upon the content of the buffer connected to the node. Suresh Varma et al. (2007) have designed and developed a two node communication network with load dependent transmission their consideration of single packet arrivals to the source node is realistic. Generally, in any communication system these messages arrived to the source node are converted into a random number of packets based on the message size and thus the consideration of batch are bulk. Packet arrivals are close to the realistic situation in a communication system. Kuda Nageswara Rao et al.(2011) have developed some two node tandem communication network models with bulk arrivals at the source and dynamic bandwidth strategy. However the transmission nodes in a communication system are generally in multiple number of series or tandem between the sender and receiver. The assumption of three nodes in series having a predecessor and a successor for the middle node is more generic, appropriate and realistic to the network architectures.

In this paper three node tandem communication networks with dynamic bandwidth allocation bulk arrivals from the source connected to the first node is modeled through embedded Markov chain techniques. Using the difference differential equations the performance measures of the communication network such as the joined probability generating function of the number of packets in each buffer, the probability of emptiness of buffers, mean number of packets in the buffers, mean delays in the buffers, throughput of the nodes are derived explicitly under transient conditions. The performance evaluation of the network model is studies through numerical illustration.

2. Communication Network Model and Transient Solution

A three node tandem communication network with dynamic bandwidth allocation having bulk arrivals is modeled and analyzed. Consider the messages arrive to the first node are converted a random number of packets and stored in the first buffer connected to the first node. The packets are forwarded to the second buffer connected to the second node after transmitting from the first node. It is further considered that after transmitting from the second node the packets are forwarded to the third buffer connected to the third node. It is assumed that the arrival of packets to the first buffer is in bulk with random batch size having the probability mass function $\{\mathbf{C_x}\}$. It is considered that the random transmission is carried with dynamic bandwidth allocation in all the three nodes i.e. the transmission rate at each node is adjusted instantaneously and dynamically depending upon the content of the buffer connected to each node. This can be modeled as the transmission rates are linearly dependent on the content of the buffer. It is assumed that the arrival of packets following compound Poisson process with parameter λ and the number of transmissions at node 1, node 2 and node 3 follow Poisson process with parameters β, δ, θ, respectively.

The operating principle of the queue is First in First out (FIFO). The schematic diagram represents the proposed communication network model is shown in Figure 1. For obtaining the performance of a communication network, it is needed to know the function form of the probability mass function of the number of packets that a message can be converted (C_k). Using the difference differential equations, the Joint Probability Generating Function of the number of packets in the first, second and third buffers is derived as

$$P(Z_1, Z_2, Z_3, t) = \exp\left\{ \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f} \left(\frac{\beta}{(\delta-\beta)}\right)^{s-f}(Z_3-1)^{f}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-\delta}\right]^{s-f} \left[(Z_1-1)+\frac{\beta(Z_2-1)}{\delta-\beta}+\frac{\beta\delta(Z_3-1)}{(\delta-\beta)(\theta-\beta)}\right]^{r-s} \frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)} \right\} \text{ for } |Z_1|<1, |Z_2|<1, |Z_3|<1 \tag{1}$$

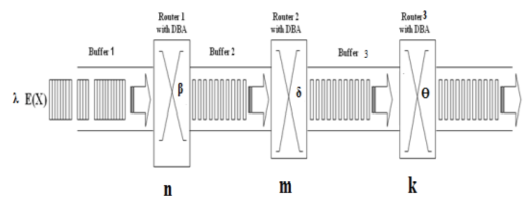

Fig.1 Three Node Tandem Communication network with dynamic bandwidth allocation and bulk arrivals

3. Performance Measures of the Proposed Communication Network

The probability of emptiness of the whole network is

$$P_{(0,0,0)}(t) = \exp\left\{ \lambda \sum_{x=1}^{\infty}\sum_{r-1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s} C_x \binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta).(\theta-\beta)}\right)^{f}\left(\frac{\beta}{\delta-\beta}\right)^{s-f}\left(\frac{-\theta}{\theta-\delta}\right)^{s-f} \left[-1-\frac{\beta}{\delta-\beta}+\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right]^{r-s} \frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)} \right\} \tag{2}$$

The probability generating function of the number of packets in the first buffer is

$$P(Z_1, t) = \exp\left\{ \lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x \binom{x}{r}(Z_1-1)^{r}\left(\frac{1-e^{-\beta r t}}{\beta r}\right)\right\} \text{ for } |Z_1|<1 \tag{3}$$

The probability that the first buffer is empty

$$P_{0..}(t) = \exp\left\{ \lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x \binom{x}{r}(Z_1-1)^{r}\left(\frac{1-e^{\beta r t}}{\beta r}\right)\right\} \tag{4}$$

The probability generating function of the number of packets in the second buffer is

$$P(Z_2,t)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}(Z_2-1)^{r}\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\};\text{for } |Z_3|<1 \quad (5)$$

The probability that the second buffer is empty

$$P_{.0.}(t)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\} \quad (6)$$

The probability generating function of the number of packets in the third buffer is

$$P(Z_3,t)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}c_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^{r}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\quad;\text{for } Z_3<1 \quad (7)$$

The probability that the third buffer is empty,

$$P_{..0}(t)=\exp.\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}c_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^{r}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \quad (8)$$

The mean number of packets in the first buffer is

$$L_1=\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty}xC_x\right)(1-e^{-\beta t}) \quad (9)$$

The mean number of packets in the second buffer is

$$L_2=\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\left(1-\frac{e^{-\delta t}}{\delta}\right)-\left(\frac{e^{-\delta t}-e^{-\beta t}}{\beta-\delta}\right)\right] \quad (10)$$

The mean number of packets in the third buffer is

$$L_3=\lambda\left(\sum_{x=1}^{\infty}xC_x\right)\left[\frac{\beta\delta\left(1-e^{-\beta t}\right)}{(\delta-\beta)(\theta-\beta)\beta}-\frac{\beta\delta(1-e^{-\delta t})}{(\delta-\beta)(\theta-\beta)\delta}+\frac{\beta\delta(1-e^{-\theta t})}{(\theta-\delta)(\theta-\beta)\theta}\right] \quad (11)$$

The probability that there is at least one packet in the first node is

$$U_1=1-P_{0..}(t)=1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(Z_1-1)^{r}\left(\frac{1-e^{-\beta rt}}{\beta r}\right)\right\} \quad (12)$$

The probability that there is at least one packet in the second node is

$$U_2=1-P_{.0.}(t)=1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\} \quad (13)$$

The probability that there is at least one packet in the third node is,

$$U_3=1-P_{..0}(t)=1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}C_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \quad (14)$$

The mean number of packets in the whole network is

$L=L_1+L_2+L_3$ (15)

Throughput of the first node is

$$Thp1=\beta U_1=\beta\left[1-P_{0..}(t)\right]=\beta\left\{1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(Z_1-1)^{r}\left(\frac{1-e^{-\beta rt}}{\beta r}\right)\right\}\right\} \quad (16)$$

Throughput of the second node is

$$Thp2=\delta U_2=\delta\left(1-P_{.0.}(t)\right)=\delta\left(1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\}\right) \quad (17)$$

Throughput of the third node is

$$Thp3=\theta U_3=\theta\left(1-P_{..0}(t)\right)=\theta\left\{1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}C_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right\} \quad (18)$$

The mean delay in the first buffer is

$$W_1=\frac{L_1}{\beta\left(1-P_{0..}(t)\right)}=\frac{\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty}xC_x\right)(1-e^{-\beta t})}{\beta\left[1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}c_x\binom{x}{r}(Z_1-1)^{r}\left(\frac{1-e^{\beta rt}}{\beta r}\right)\right\}\right]} \quad (19)$$

The mean delay in the second buffer is

$$W_2=\frac{L_2}{\delta\left(1-P_{.0.}(t)\right)}=\frac{\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty}x.C_x\right)\left[\left(1-\frac{e^{-\beta t}}{\delta}\right)-\left(\frac{e^{-\delta t}-e^{-\beta t}}{\beta-\delta}\right)\right]}{\delta\left(1-\exp.\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^{r}\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\}\right)} \quad (20)$$

The mean delay in the third buffer is

$$W_3=\frac{L_3}{\theta\left(1-P_{..0}(t)\right)}=\frac{\lambda\left(\sum_{x=1}^{\infty}xC_x\right)\left[\frac{\beta\delta\left(1-e^{-\beta t}\right)}{(\delta-\beta)(\theta-\beta)\beta}-\frac{\beta\delta(1-e^{-\delta t})}{(\delta-\beta)(\theta-\beta)\delta}+\frac{\beta\delta(1-e^{-\theta t})}{(\theta-\delta)(\theta-\beta)\theta}\right]}{\theta\left(1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}C_x\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^{f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right)} \quad (21)$$

The variance of the number of packets in the first buffer is

$$V_1=\frac{\lambda}{2\beta}\left(\sum_{x=1}^{\infty}x(x-1)C_x\right)(1-e^{-2\beta t})+\frac{\lambda}{\beta}\left(\sum_{x=1}^{\infty}xC_x\right)(1-e^{-\beta t}) \quad (22)$$

The variance of the number of packets in the second buffer is

$$V_2 = \lambda\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left(\frac{\beta}{\beta-\delta}\right)^2\left[\left(\frac{1-e^{-2\beta t}}{2\beta}\right)-2\left(\frac{1-e^{-(\beta+\delta)t}}{\beta+\delta}\right)+\left(\frac{1-e^{-2\delta t}}{2\delta}\right)\right]+ \lambda\left(\sum_{x=1}^{\infty} xC_x\right)\left[\left(\frac{1-e^{-\delta t}}{\delta}\right)-\left(\frac{e^{-\delta t}-e^{-\beta t}}{\beta-\delta}\right)\right] \quad (23)$$

The variance of the number of packets in the third buffer is

$$\begin{aligned}V_3 = {} & \lambda(\beta\delta)^2\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\Bigg\{\left(\frac{1}{(\delta-\beta)(\theta-\beta)}\right)^2\left(\frac{1-e^{-2\beta t}}{2\beta}\right)-2\left(\frac{1}{\delta-\beta}\right)^2 \\ & \left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]\left(\frac{1-e^{-(\beta+\delta)t}}{\beta+\delta}\right)+2\left(\frac{1}{\theta-\beta}\right)^2\left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right] \\ & \left(\frac{1-e^{-(\beta+\theta)t}}{\beta+\theta}\right)+\left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right]^2\left(\frac{1-e^{-2\delta t}}{2\delta}\right)-2\left(\frac{1}{\theta-\delta}\right)^2 \\ & \left[\frac{1}{(\delta-\beta)(\theta-\beta)}\right]\left(\frac{1-e^{-(\theta+\delta)t}}{\theta+\delta}\right)+\left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]^2\left(\frac{1-e^{-\theta t}}{\theta}\right) \\ & \lambda\beta\delta\left(\sum_{x=1}^{\infty} xC_x\right)\left[\left(\frac{1-e^{-\beta t}}{(\delta-\beta)(\theta-\beta)\beta}\right)-\left(\frac{1-e^{-\theta t}}{(\delta-\beta)(\theta-\delta)\delta}\right)+\left(\frac{1-e^{-\theta t}}{(\theta-\delta)(\theta-\beta)\theta}\right)\right]\end{aligned} \quad (24)$$

The coefficient of variation of the number of packets in the first node is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \quad (25)$$

The coefficient of variation of the number of packets in the second node is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \quad (26)$$

The coefficient of variation of the number of packets in the third node is

$$CV_3 = \frac{\sqrt{V_3}}{L_3} \quad (27)$$

4. Performance measures of the proposed Network model with uniform batch size distribution

It is assumed the batch size of the packets follows a uniform distribution and the probability distribution of the batch size of the packets in a message is $C_x = 1 / \{(b-a)+1\}$, for x= a, a+1, … , b. and the mean number of packets in a message is $\left(\frac{a+b}{2}\right)$ and its variance is $\frac{1}{12}\left[(b-a+1)^2-1\right]$ Substituting the value of C_x in the equation (1), we get the Joint Probability Generating Function of the number of packets in the whole networks is

$$\begin{aligned}P(Z_1,Z_2,Z_3,t) = \exp\Bigg\{ & \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta).(\theta-\beta)}\right)^f \\ & \left(\frac{\beta}{(\delta-\beta)}\right)^{s-f}(Z_3-1)^f\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-\delta}\right]^{s-f} \\ & \left[(Z_1-1)+\frac{\beta(Z_2-1)}{\delta-\beta}+\frac{\beta\delta(Z_3-1)}{(\delta-\beta)(\theta-\beta)}\right]^{r-s} \\ & \frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\Bigg\} \text{for } |Z_1|<1, |Z_2|<1, |Z_3|<1\end{aligned} \quad (28)$$

The probability of emptiness of the whole network is

$$\begin{aligned}P_{(0,0,0)}(t) = \exp\Bigg\{ & \lambda\sum_{x=a}^{b}\sum_{r-1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta).(\theta-\beta)}\right)^f\left(\frac{\beta}{\delta-\beta}\right)^{s-f}\left(\frac{-\theta}{\theta-\delta}\right)^{s-f} \\ & \left[-1-\frac{\beta}{\delta-\beta}+\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right]^{r-s}\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\Bigg\}\end{aligned} \quad (29)$$

The probability generating function of the number of packets in the first buffer is

$$P(Z_1,t) = \exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1-e^{-\beta rt}}{\beta r}\right)\right\} \text{ for } |Z_1|<1 \quad (30)$$

The probability that the first buffer is empty,

$$P_{0..}(t) = \exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1-e^{\beta rt}}{\beta r}\right)\right\} \quad (31)$$

The probability generating function of the number of packets in the second buffer is

$$\begin{aligned}P(Z_2,t) = \exp\Bigg\{ & \lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r(Z_2-1)^r \\ & \left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\Bigg\};\text{for } |Z_3|<1\end{aligned} \quad (32)$$

The probability that the second buffer is empty

$$P_{.0.}(t) = \exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\} \quad (33)$$

The probability generating function of the number of packets in the third buffer is

$$\begin{aligned}P(Z_3,t) = \exp\Bigg\{ & \lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^f\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \\ & \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^r\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\Bigg\} \quad ;\text{for } Z_3<1\end{aligned} \quad (34)$$

The probability that the third buffer is empty,

$$\begin{aligned}P_{..0}(t) = \exp\Bigg\{ & \lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^f\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f} \\ & \left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}(Z_3-1)^r\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\Bigg\}\end{aligned} \quad (35)$$

The mean number of packets in the first buffer is

$$L_1 = \frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)(1-e^{-\beta t}) \quad (36)$$

The mean number of packets in the second buffer is,

$$L_2 = \frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\left(1-\frac{e^{-\delta t}}{\delta}\right)-\left(\frac{e^{-\delta t}-e^{-\beta t}}{\beta-\delta}\right)\right] \quad (37)$$

The mean number of packets in the third buffer is

$$L_3 = \lambda\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\frac{\beta\delta(1-e^{-\beta t})}{(\delta-\beta)(\theta-\beta)\beta}-\frac{\beta\delta(1-e^{-\delta t})}{(\delta-\beta)(\theta-\beta)\delta}+\frac{\beta\delta(1-e^{-\theta t})}{(\theta-\delta)(\theta-\beta)\theta}\right] \quad 38)$$

The probability that there is at least one packet in the first node is

$$U_1 = 1 - P_{0..}(t) = 1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1-e^{-\beta r t}}{\beta r}\right)\right\} \quad (39)$$

The probability that there is at least one packet in the second node is

$$U_2 = 1 - P_{.0.}(t) = 1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\} \quad (40)$$

The probability that there is at least one packet in the third node is,

$$U_3 = 1 - P_{..0}(t) = 1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^f\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\} \quad (41)$$

The mean number of packets in the whole network is

$$L = L_1 + L_2 + L_3 \quad (42)$$

Throughput of the first node is

$$Thp1 = \beta U_1 = \beta\left[1 - P_{0..}(t)\right] = \beta\left\{1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1-e^{-\beta r t}}{\beta r}\right)\right\}\right\} \quad (43)$$

Throughput of the second node is

$$Thp2 = \delta U_2 = \delta\left(1 - P_{.0.}(t)\right) = \delta\left(1 - \exp.\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\}\right) \quad (44)$$

Throughput of the third node is

$$Thp3 = \theta U_3 = \theta\left(1 - P_{..0}(t)\right) = \theta\left\{1 - \exp.\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^f\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right\} \quad (45)$$

The mean delay in the first buffer is

$$W_1 = \frac{L_1}{\beta\left(1 - P_{0..}(t)\right)} = \frac{\frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)(1-e^{-\beta t})}{\beta\left[1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\left(\frac{1-e^{\beta r t}}{\beta r}\right)\right\}\right]} \quad (46)$$

The mean delay in the second buffer is

$$W_2 = \frac{L_2}{\delta\left(1 - P_{.0.}(t)\right)} = \frac{\frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\left(1 - \frac{e^{-\beta t}}{\delta}\right) - \left(\frac{e^{-\delta t} - e^{-\beta t}}{\beta-\delta}\right)\right]}{\delta\left(1 - \exp.\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{s-f}\left(\frac{\beta}{\delta-\beta}\right)^r\left(\frac{1-e^{-[\delta s+\beta(r-s)t]}}{\delta s+\beta(r-s)}\right)\right\}\right)} \quad (47)$$

The mean delay in the third buffer is

$$W_3 = \frac{L_3}{\theta\left(1 - P_{..0}(t)\right)} = \frac{\lambda\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\frac{\beta\delta\left(1-e^{-\beta t}\right)}{(\delta-\beta)(\theta-\beta)\beta} - \frac{\beta\delta(1-e^{-\delta t})}{(\delta-\beta)(\theta-\beta)\delta} + \frac{\beta\delta(1-e^{-\theta t})}{(\theta-\delta)(\theta-\beta)\theta}\right]}{\theta\left(1 - \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{x}\binom{s}{f}(-1)^{s-f}\left(\frac{\beta\delta}{(\theta-\delta)(\theta-\beta)}\right)^f\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\delta)}\right)^{s-f}\left(\frac{\beta\delta}{(\delta-\beta)(\theta-\beta)}\right)^{r-s}\left(\frac{1-e^{-[\theta f+\delta(s-f)+\beta(r-s)t]}}{\theta f+\delta(s-f)+\beta(r-s)}\right)\right\}\right)} \quad (48)$$

The variance of the number of packets in the first buffer is

$$V_1 = \frac{\lambda}{2\beta}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)(1-e^{-2\beta t}) + \frac{\lambda}{\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)(1-e^{-\beta t}) \quad (49)$$

The variance of the number of packets in the second buffer is

$$V_2 = \lambda\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left(\frac{\beta}{\beta-\delta}\right)^2\left[\left(\frac{1-e^{-2\beta t}}{2\beta}\right) - 2\left(\frac{1-e^{-(\beta+\delta)t}}{\beta+\delta}\right) + \left(\frac{1-e^{-2\delta t}}{2\delta}\right)\right] + \lambda\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-\delta t}}{\delta}\right) - \left(\frac{e^{-\delta t}-e^{-\beta t}}{\beta-\delta}\right)\right] \quad (50)$$

The variance of the number of packets in the third buffer is

$$V_3 = \lambda(\beta\delta)^2\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left\{\left(\frac{1}{(\delta-\beta)(\theta-\beta)}\right)^2\left(\frac{1-e^{-2\beta t}}{2\beta}\right) - 2\left(\frac{1}{\delta-\beta}\right)^2\left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]\left(\frac{1-e^{-(\beta+\delta)t}}{\beta+\delta}\right) + 2\left(\frac{1}{\theta-\beta}\right)^2\left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right]\left(\frac{1-e^{-(\beta+\theta)t}}{\beta+\theta}\right) + \left[\frac{1}{(\delta-\beta)(\theta-\delta)}\right]^2\left(\frac{1-e^{-2\delta t}}{2\delta}\right) - 2\left(\frac{1}{\theta-\delta}\right)^2\left[\frac{1}{(\delta-\beta)(\theta-\beta)}\right]\left(\frac{1-e^{-(\theta+\delta)t}}{\theta+\delta}\right) + \left[\frac{1}{(\theta-\delta)(\theta-\beta)}\right]^2\left(\frac{1-e^{-\theta t}}{\theta}\right)\right. \\ \lambda\beta\delta\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-\beta t}}{(\delta-\beta)(\theta-\beta)\beta}\right) - \left(\frac{1-e^{-\theta t}}{(\delta-\beta)(\theta-\delta)\delta}\right) + \left(\frac{1-e^{-\theta t}}{(\theta-\delta)(\theta-\beta)\theta}\right)\right] \quad (51)$$

5. Performance Evaluation of the Proposed Communication Network

The performance of the proposed network is analyzed through numerical illustration. A set of values of the input parameters are considered for allocation of bandwidth and arrival of packets. After interacting with the internet service provider, it is considered that the message arrival rate(λ) varies from $1x10^4$ messages/sec to $5x10^4$ messages/sec. the number of packets that can be converted from a message varies from 1 to 10. The message arrivals to the buffer are in batches of random size. The batch size is assumed to follow uniform distribution parameters (a, b). The transmission rate of node 1(β) varies from$1x10^4$ packets/sec to $4x10^4$ packets/sec. The packets leave the second node with a transmission rate(δ) which varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec. The packets leave the third node with a transmission rate(θ) which varies from $11x10^4$

packets/sec to $14x10^4$ packets/sec. In all the three nodes, dynamic bandwidth allocation is considered i.e. the transmission rate of each packet depends on the number of packets in the buffer connected to it at that instant.

The probability of network emptiness and different buffers emptiness are computed for different values of t, a, b, λ, β, δ, θ. It is observed that the probability of emptiness of the communication network and the two buffers are highly sensitive with respect to changes in time. As time(t) varies from 0.1 second to 0.4 second, the probability of emptiness in the network reduces from 0.1672 to 0.138 when other parameters are fixed at (2, 10, 2, 5, 10, 15) for (a, b λ, β, δ, θ). Similarly, the probability of emptiness of the three buffers reduces from 0.021 to 0.003, 0.686 to 0.388 and 0.149 to 0.071 for node 1, node 2 and node 3 respectively. When the batch distribution parameter(a) varies from$1x10^4$ packets/sec to $5x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.059 to 0.026 when other parameters are fixed at (0.5, 10, 2, 5, 10, 15) for (t, a, b, λ, β, δ, θ). The same phenomenon is observed with respect to the first and second nodes. The probability of emptiness of the first, second and third buffers decrease from 0.241 to 0.211 and 0.361 to 0.203 and 0.451 to 0.356 respectively.

When the batch size distribution parameter(b) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the probability of emptiness of the network decreases from 0.041 to 0.013 when other parameters are fixed at (0.5, 2, 2, 5, 10, 15) for (t, a, λ, β, δ, θ). The same phenomenon is observed with respect to the first, second and third node. The probability of emptiness of the first, second and third buffers decrease from 0.061 to 0.038, 0.854 to 0.506 and 0.127 to 0.078 respectively. The influence of arrival of messages on system emptiness is also studied. As the arrival rate(λ) varies from $2.5x10^4$ messages/sec to $4.0x10^4$ messages/sec, the probability of emptiness of the network decreases from 0.819 to 0.016 when other parameters are fixed at (0.5, 2, 10, 5, 10, 15) for (t, a, b, β, δ, θ). The same phenomenon is observed with respect to the first second and third nodes. The probability of emptiness of the first, the second and third buffer decrease from 0.104 to 0.064, 0.848 to 0.768 and 0.221 to 0.133 respectively. When the transmission rate (β) of node1 varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the probability of emptiness of the network increase from 0.097 to 0.132, first, second buffers are constant and third buffer is decreases from 0.326 to 0.129 when other parameters remain fixed at (0.5, 2, 10, 2, 10, 15) for (t, a, b, λ, δ, θ).

When the transmission rate of node 2**(**δ) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the probability of emptiness of the network increases from 0.013 to 0.160 the second and third buffers decreases from 0.920 to0.892 and 0.145 to 0.106 respectively when other parameters remain fixed at (0.5, 2, 10, 2, 5, 15) for (t, a, b, λ, β, θ). When the transmission rate of node 3(δ) varies from $11x10^4$ packets/sec to $14x10^4$ packets/sec, the probability of emptiness of the network increases from 0.009 to 0.039 and third buffers decreases from 0.111 to 0.105 respectively when other parameters remain fixed at (0.5, 2, 10, 2, 5, 15) for (t, a, b, λ, β, θ). The mean number of packets and the utilization of the network are computed for different values of t, a, b, λ, β, δ, θ. Values of probability of emptiness mean number packets, mean delays and throughputs in the three buffers are given in Table.1 and the relationship between mean number of packets in the three buffers and the input parameters t, a, b, λ, β, δ, θ is shown in Figures 2, 3, 4, 5, 6 and 7. It is observed that after 0.1 seconds, the first buffer is having on an average of 0.021 $x10^4$ packets, after 0.2 seconds it rapidly raised on an average of $0.029x10^4$ packets. After 0.4 seconds, the first buffer is containing an average of $0.033X10^4$ packets and there after the system stabilizes and the average number of packets remains to be the same for fixed values of other parameters (2, 10, 2.5, 10, 15) for (a, b, λ, β, δ, θ). It is also observed that as time (t) varies from 0.1second to 0.4 second, average content of the second, third buffer and the network increase from 0.008 $x10^4$ packets to 0.030 $x10^4$ packets, 0.056 $x10^4$ packets to 0.110 $x10^4$ packets and from $0.081x10^4$ packets to $0.172x10^4$ packets respectively.

It is observed that after 0.1 seconds, the first buffer is having on an average of 0.021 $x10^4$ packets, after 0.2 seconds it rapidly raised on an average of $0.029x10^4$ packets. After 0.4 seconds, the first buffer is containing an average of $0.033x10^4$ packets and there after the system stabilizes and the average number of packets remains to be the same for fixed values of other parameters (2, 10, 2.5, 10, 15) for (a, b, λ, β, δ, θ). It is also observed that as time (t) varies from 0.1second to 0.4 second, average content of the second, third buffer and the network increase from 0.008 $x10^4$ packets to $0.030x10^4$ packets, $0.056x10^4$ packets to $0.110x10^4$ packets and from $0.081x10^4$ packets to $0.172x10^4$ packets respectively. As the batch size distribution parameter (a) varies from 1to 5, the first buffer, second buffer and third buffer the network average content increase from 0.056 $x10^4$ packets to $0.072x10^4$ packets, from 0.115 $x10^4$ packets to $0.158x10^4$ packets, from 0.256 $x10^4$ packets to $0.275x10^4$ packets, from 0.182 $x10^4$ packets to $0.221x10^4$ packets respectively when other parameters remain fixed. As the batch size distribution parameter (b) varies from 6 to 9, the first buffer, second buffer, third buffer and the network average content increase from 0.037 $x10^4$ packets to 0.060 $x10^4$ packets, from 0.036 $x10^4$ packets to 0.059 $x10^4$ packets, from 0.133 $x10^4$ packets to 0.213 $x10^4$ packets, from 0.206 $x10^4$ packets to 0.331 $x10^4$ packets respectively when other parameters remain fixed.

Table 1: Values of probability of emptiness, mean number of packets and mean delays in the three buffers

t*	a	b	$\lambda^{\#}$	$\beta^{\$}$	$\delta^{\$}$	$\Theta^{\$}$	$P_{000}(t)$	$P_{0\cdot\cdot}(t)$	$P_{\cdot 0\cdot}(t)$	$P_{\cdot\cdot 0}(t)$	L_1	L_2	L_3	W_1	W_2	W_3
0.1	2	10	2	5	10	15	0.167	0.021	0.686	0.149	0.021	0.008	0.056	0.00429	0.00254	0.00438
0.2	2	10	2	5	10	15	0.161	0.019	0.584	0.097	0.029	0.016	0.080	0.00591	0.00384	0.00590
0.3	2	10	2	5	10	15	0.149	0.012	0.446	0.078	0.032	0.026	0.097	0.00647	0.00469	0.00701
0.4	2	10	2	5	10	15	0.138	0.003	0.388	0.071	0.033	0.030	0.110	0.00661	0.00490	0.00789
0.5	**1**	10	2	5	10	15	0.059	0.241	0.361	0.451	0.056	0.115	0.256	0.01475	0.01799	0.03108
0.5	**3**	10	2	5	10	15	0.045	0.235	0.303	0.403	0.061	0.128	0.261	0.01594	0.01836	0.02914
0.5	**4**	10	2	5	10	15	0.031	0.221	0.245	0.385	0.068	0.142	0.268	0.01745	0.01880	0.02905
0.5	**5**	10	2	5	10	15	0.026	0.211	0.203	0.356	0.072	0.158	0.275	0.01845	0.01982	0.02846
0.5	2	**6**	2	5	10	15	0.041	0.061	0.854	0.127	0.037	0.036	0.133	0.00788	0.02465	0.01015
0.5	2	**7**	2	5	10	15	0.030	0.051	0.776	0.105	0.043	0.045	0.152	0.00906	0.02008	0.01132
0.5	2	**8**	2	5	10	15	0.021	0.043	0.693	0.089	0.050	0.049	0.177	0.01044	0.01596	0.01295
0.5	2	**9**	2	5	10	15	0.013	0.038	0.506	0.078	0.060	0.059	0.213	0.01247	0.01194	0.01540
0.5	2	10	**2.5**	5	10	15	0.819	0.104	0.848	0.221	0.062	0.065	0.225	0.01280	0.03276	0.01925
0.5	2	10	**3**	5	10	15	0.423	0.091	0.815	0.191	0.077	0.070	0.275	0.01694	0.03783	0.02266
0.5	2	10	**3.5**	5	10	15	0.036	0.080	0.794	0.168	0.087	0.086	0.310	0.01891	0.04194	0.02483
0.5	2	10	**4**	5	10	15	0.016	0.064	0.768	0.133	0.099	0.099	0.355	0.02115	0.04267	0.02729
0.5	2	10	2	**1**	10	15	0.097	0.051	0.876	0.326	0.050	0.049	0.180	0.05268	0.03951	0.01780
0.5	2	10	2	**2**	10	15	0.110	0.051	0.876	0.224	0.050	0.042	0.167	0.02634	0.03387	0.01434
0.5	2	10	2	**3**	10	15	0.127	0.051	0.876	0.166	0.050	0.036	0.161	0.01756	0.02903	0.01286
0.5	2	10	2	**4**	10	15	0.132	0.051	0.876	0.129	0.050	0.031	0.164	0.01317	0.02500	0.01255
0.5	2	10	2	5	**6**	15	0.013	0.082	0.920	0.145	0.079	0.078	0.367	0.01721	0.16250	0.02861
0.5	2	10	2	5	**7**	15	0.046	0.072	0.912	0.117	0.069	0.065	0.246	0.01487	0.10551	0.01857
0.5	2	10	2	5	**8**	15	0.068	0.063	0.903	0.109	0.061	0.060	0.207	0.01302	0.07731	0.01548
0.5	2	10	2	5	**9**	15	0.160	0.056	0.892	0.106	0.055	0.054	0.188	0.01165	0.05555	0.01401
0.5	2	10	2	5	10	**11**	0.009	0.051	0.583	0.111	0.050	0.046	0.240	0.01053	0.01103	0.01799
0.5	2	10	2	5	10	**12**	0.013	0.051	0.750	0.107	0.050	0.047	0.218	0.01053	0.01880	0.01627
0.5	2	10	2	5	10	**13**	0.015	0.051	0.816	0.106	0.050	0.048	0.201	0.01053	0.02608	0.01498
0.5	2	10	2	5	10	**14**	0.039	0.051	0.853	0.105	0.050	0.049	0.188	0.01053	0.03333	0.014003

* = seconds, # =Multiples of 10,000 Messages/sec, $= Multiples of 10,000 Packets/sec

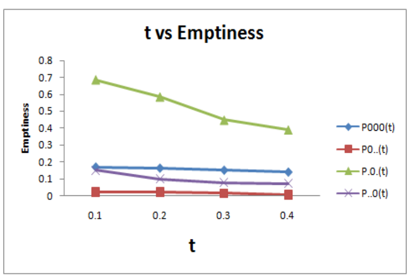

Fig. 2 Time't' Vs Emptiness of buffers at nodes 1, 2 and 3

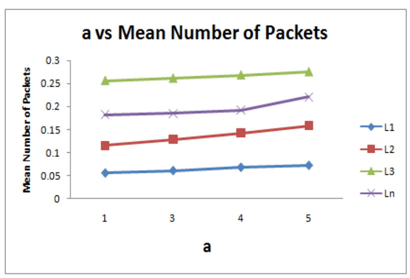

Fig. 3 Batch size distribution parameter **a** Vs Mean Number of Packets in the buffers at nodes 1, 2 and 3

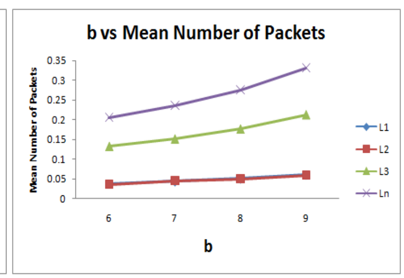

Fig.4 Batch size distribution parameter b Vs Mean Number of Packets in the buffers at nodes 1, 2 and 3

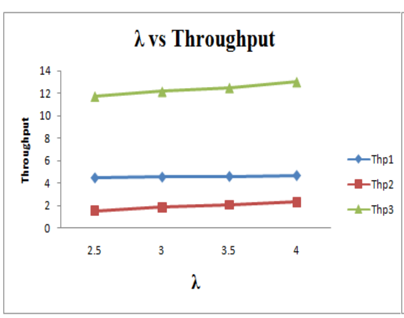

Fig. 5 Packet arrival rate **λ** Vs Throughput of the nodes 1, 2 and 3

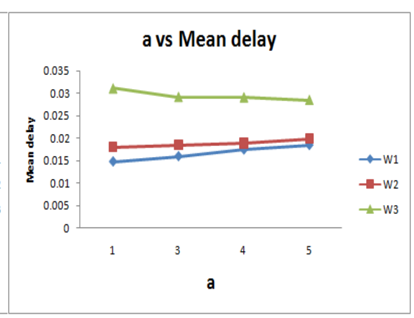

Fig. 6 Batch size distribution parameter a Vs Mean Delay in the buffers at nodes 1, 2 and 3

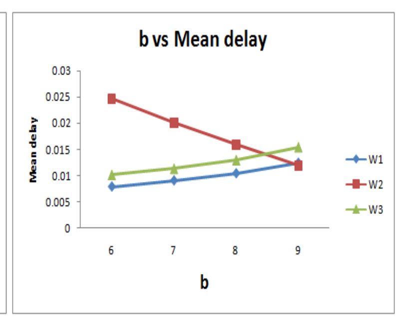

Fig. 7 Batch size distribution parameter b Vs Mean Delay in the buffers at nodes 1, 2 and 3

As the arrival rate of messages(λ) varies from $2.5x10^4$ messages/sec to $4.0x10^4$ messages/sec, the mean number of packets in the first buffer, second buffer, third buffer and in the network increase from 0.062 $x10^4$ packets to 0.099 $x10^4$ packets, from 0.065 $x10^4$ packets to 0.099 $x10^4$ packets, from $0.225x10^4$ packets to 0.355 $x10^4$ packets, from 0.345 $x10^4$packets to 0.555 $x10^4$ packets respectively when other parameters remain fixed at (0.5, 2, 10, 5, 10, 15) for (t, a, b, β, δ, θ). As the transmission rate of node 1(β) varies from$1x10^4$ packets/sec to $4x10^4$packets/sec, the first buffer is constant and the network average content decrease from 0.278 $x10^4$ packets to 0.260 $x10^4$ packets respectively when other parameters remain fixed. As the transmission rate of node 2(δ) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the second buffer and the network average content decrease from 0.078 $x10^4$ packets to 0.054 $x10^4$ packets and from 0.524 $x10^4$ packets to 0.297 $x10^4$ packets respectively when other parameters remain fixed. As the transmission rate of node 3(θ) varies from $11x10^4$ packets/sec to $14x10^4$ packets/sec, the third buffer and the network average content decrease from 0.240 $x10^4$ packets to 0.188 $x10^4$ packets and from 0.337 $x10^4$ packets to 0.286 $x10^4$ packets respectively when other parameters remain fixed.

It is revealed that the utilization characteristics are similar to mean number of packet characteristics. Here also, as the time (t) and the arrival rate of messages (λ) increase, the utilization of all the three nodes increase for fixed values of the other parameters. As the batch size distribution parameters (a) and (b) increase, the utilization of all the three nodes increase when the other parameters are fixed. It is also noticed that as the transmission rate of node 1(β), node 2 (δ) are constant and the third node increases, therefore in the communication network, dynamic bandwidth allocation strategy is necessary for control of congestion, efficient utilization of different nodes and to maintain satisfactory quality of service (QoS) with optimum speed. The throughput and the average delay of the network are computed for different values of t, a, b, λ, β, δ, θ and the values of mean delays are given in Table 1. It is observed that as the time (t) increases from 0.1 second to 0.4 seconds, the throughput of the first , second and third nodes increase from 9.790 $x10^4$ packets to 9.970 $x10^4$ packets, 4.710 $x10^4$packets to 9.180 $x10^4$ packets, 4.255 $x10^4$packets to 4.645 $x10^4$ packets respectively, when other parameters remain fixed at (2, 10, 2, 5, 10, 15) for (a, b, λ, β, δ, θ).

As the batch size distribution parameter (a) varies from 1 to 5 the throughput of the first, second and third nodes increase from 7.590 $x10^4$ packets to 7.890 $x10^4$ packets, 9.585 $x10^4$packets to 11.955 $x10^4$ packets, 2.745 $x10^4$ packets to 3.220 $x10^4$ packets respectively when other parameters remain fixed at (0.5, 10, 2, 5, 10, 15) for (t, b, λ, β, δ, θ). As the batch size distribution parameter (b) varies from 6 to 9 the throughput of the first, second and third nodes increase from 9.3900 $x10^4$ packets to 9.620 $x10^4$ packets, 2.190 $x10^4$packets to 7.410 $x10^4$ packets, $4.365x10^4$ packets to $4.610x10^4$ packets respectively when other parameters remain fixed at (0.5, 2, 2, 5, 10, 15) for (t, a, λ, β, δ, θ). As the arrival rate(λ) varies from 2.5 to 4.0 the throughput of the first, second and third nodes increase from $8.960x10^4$ packets to 9.360 $x10^4$ packets, 2.280 $x10^4$ packets to 3.480 $x10^4$ packets, 3.895 $x10^4$ packets to 4.335 $x10^4$ packets respectively when other parameters remain fixed at (0.5, 2, 10, 5, 10, 15) for (t, a, b, β, δ, θ).

As the transmission rate(β) of node1 varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the throughput of first and second nodes remains constant and for the third node it increases from $0.675x10^4$ packets to 3.484 $x10^4$ packets, when other parameters remain fixed at (0.5, 2, 10, 2, 10, 15) for (t, a, b, λ, δ, θ). As the transmission rate of node 2(β) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the throughput of first, second and third nodes increase from $5.508x10^4$ packets to 8.496 $x10^4$ packets, from $1.200x10^4$ packets to 1.620 $x10^4$ packets, from $4.275x10^4$packets to 4.470 $x10^4$ packets respectively when other parameters remain fixed at (0.5, 2, 10, 2, 5, 15) for (t, a, b, λ, β, θ). As the transmission rate of node3(θ) varies from $11x10^4$ packets/sec to $14x10^4$ packets/sec, the throughput of first node remains constant, second node decrease from $4.587x10^4$ packets to 2.058 $x10^4$ packets and third node increase from $4.445x10^4$ packets to 4.475 $x10^4$ packets respectively when other parameters remain fixed at (0.5, 2, 10, 2, 5, 10) for (t, a, b, λ, β, δ).

From Table 1, it is also observed that as time (t) varies from 0.1 second to 0.4 second, the mean delay of the first, second and third buffers increase from 0.429μs to 0.661μs and 0.254μs to 0.490μs and 0.438μs to 0.789μs respectively, when other parameters remain fixed (2, 10, 2, 5, 10,15) for (a, b, λ, β, δ, θ). As the batch size distribution parameter (a) varies from 1 to 5, the mean delay of the first, second buffers increase from 1.475μs to 01.845μs and 1.799μs to 1.982μs and third buffer decrease from 3.108μs to 2.846μs respectively, when other parameters remain fixed (0.5, 10, 2, 5, 10, 15) for (t, b, λ, β, δ, θ). As the batch size distribution parameter (b) varies from 6 to 9, the mean delay of the first increases from, 0.788μs to 1.247μs and second decrease from 2.465μs to 1.194μs and third buffers increases from 1.015μs to 1.540μs respectively, when other parameters remain fixed (0.5, 2, 2, 5, 10, 15) for (t, a, λ, β, δ, θ). When the arrival rate (λ) varies from $2.5x10^4$ messages/sec to $4.0x10^4$ messages/sec, the mean delay of the first and second and third buffers increase from 1.280μs to 2.115μs and from 3.276μs to 4.267μs and from 1.925µs to 2.729µs respectively, when other parameters remain fixed (0.5, 2, 10, 5, 10, 15) for (t, a, b, β, δ, θ).As the transmission rate of node 1 (β) varies from1x10^4 messages/sec to 4x10^4 messages/sec, the mean delay of the first node decreases from 5.268µs to 1.317µs , second and third buffer decreases from 3.951µs to 2.500µs, from 1.780µs to 1.255µs when other parameters remain fixed at (0.5, 2, 10, 2, 10, 15) for (t, a, b, λ, δ, θ). As the transmission rate of node 2 (δ) varies from 6x10^4packets/sec to 9x10^4 packets/sec, the mean delay of the first ,second buffer and third buffer decreases from 1.721µs to 1.165µs, from 1.625µs to 0.555µs and from 2.861µs to 1.401µs when other parameters remain fixed at (0.5,2, 10, 2, 5, 15) for (t, a, b, λ, β, θ). As the transmission rate of node 3 (θ) varies from 11x10^4 packets/sec to 14x10^4 packets/sec, the mean delay of the first buffer constant and second buffer increases from 1.103µs to 3.333µs and third buffer decreases from 1.799µs to 1.400µs when other parameters remain fixed at (0.5, 2, 10, 2, 5, 10) for (t, a, b, λ, β, δ).

If the variance increases then the burstness of the buffers will be high. Hence, the parameters are to be adjusted such that the variance of the buffer content in each buffer must be small. The coefficient of variation of the number of packets in each buffer will helps us to understand the consistency of the traffic flow through buffers. If coefficient variation is large then the flow is inconsistent and the requirement to search the assignable causes of high variation. It also helps us to compare the smooth flow of packets in three or more nodes. The variance of the number of packets in each buffer, the coefficient of variation of the number of packets in first, second and third buffers are computed. It is observed that, as the time (t) and the batch size distribution parameter (a) increase, the variance of first, second and third buffers increased and the coefficient of variation of the number of packet in the first, second and third buffers decreased. As the batch size distribution parameter (b) increases, the variance of first, second and third buffers increased and the coefficient of variation of the number of packets in the first buffer decreased and for the second buffer it is increased and third buffer, it is increased.

Based on the analysis, it is very clear that a dynamic bandwidth allocation strategy has high significant influence on all performance measures of the system of network. It is further observed that the measures of performance are highly sensitive towards values of time, hence it is optimal to consider dynamic bandwidth allocation and evaluate the performance under transient conditions and it is also observed that congestion in buffers and delays in transmission can be reduced to a minimum level by adopting dynamic bandwidth allocation. This phenomenon has a vital bearing on quality of data packets transmission.

6. Sensitivity Analysis

Sensitivity analysis of the model is performed with respect to t, a, b, λ, θ, β, δ on the mean number packets in the first, second and third buffers, the mean number of packets in the network, the mean delay in the first, second and third buffers, the utilization and throughput of the first, second and third nodes. The following data has been considered for the sensitivity analysis.

t = 0.1 sec, a=2 x10^4 packets/sec, b=10 x10^4 packets/sec λ = 2x10^4 packets/sec, β = 5x10^4 packets/sec, δ = 10x10^4 packets/sec and θ =15 x10^4 packets/sec.

The performance measures of the model are computed with variation of -15%, -10%, 0%, +5%, +10% and +15% on the input parameters t, a, b, λ, β δ, θ to retain them as integers. The performance measures are highly affected by time (t) and the batch size distribution of arrivals. As (t) increases to 15% the average number of packets in the three buffers and total network increases along with the average delays in buffers. Similarly, as arrival rate of messages (λ) increases by 15% the average number of packets in the three buffers and total network increases along with the average delays in buffers. The mean delays and mean content of the buffers are decreasing function of these parameters. Overall analysis of the parameters reflects that dynamic bandwidth allocation strategy for congestion control tremendously reduces the delays in communication and improves voice quality by reducing burstness in buffers.

7. Conclusions

In this paper, a three node tandem communication network with dynamic bandwidth allocation having bulk arrivals is developed and analyzed. Here, the dynamic bandwidth allocation (DBA) strategy insists for the instantaneous change in rate of transmission of the nodes depending upon the content of the buffers connected to them. The emphasis of this communication network is on the bulk or batch arrivals of packets to the initial node with random size. The performance of the statistical multiplexing is measured by approximating the arrival process with a compound Poisson process and the transmission process with Poisson process. This is chosen such that the statistical characteristics of the communication network identically matches with Poisson process and uniform distribution. A communication network model with bulk arrivals is more close to the practical transmission behavior in most of the communication systems. The sensitivity of the network with respect to input parameters is studied through numerical illustrations. It is observed that the dynamic bandwidth allocation strategy and the parameters of bulk size distribution have a significant impact on the performance measures of the network. It is further observed that transient analysis of the Communication network will approximate the performance measures more close to the practical situation. It improves the Quality of Service (QoS) by effective utilization of the bandwidth and avoids the congestion in the network.

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Mr. G. Rajendra Kumar is presently pursuing Ph.D., in the Department of Computer Science and Systems Engineering, Andhra University, Visakhapatnam. He completed M.Tech degree in Computer Science and Engineering from Acharya Nagarjuna University. His current research interests are Communication networks, Cognitive Science and Network Security.

Dr. Kuda. Nageswara Rao is presently working as Professor in the Department of Computer Science and Systems Engineering, Andhra University, Visakhapatnam. He presented several research papers in national and International conferences and seminars. He published a good number of papers in national and International journals. He guided several students for getting their M.Tech., and Ph.D., degrees in Computer Science and Engineering. His current research interests are Communication networks, Internet Technologies and Network security.

Dr. K. Srinivasa Rao is presently working as Professor and Head, Department of Statistics, Andhra University, Visakhapatnam. He is elected chief editor of Journal of ISPS and elected Vice-President of Operation Research of India. He guided 22 students for Ph.D., in Statistics, Computer Science, Electronics and Communications and Operations Research. He published 75 research papers in national and International journals with high reputation. His research interests are Communication Systems, Data Mining and Stochastic models.