Image Denoising Using Patch Based Processing With Fuzzy Triangular Membership Function

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Abstract

In this paper image denoising method based on fuzzy triangular membership function. For a given corrupted image, at first I converted to the fuzzy values using the fuzzification method. Then, I extract all patches with overlaps, after extracting all patches each patch is to be permuted and apply the fuzzy triangular membership function. I extract all patches with overlaps, refer to these as coordinates in high-dimensional space, and arrange them such that they are chained in the "shortest possible path," basically solving the traveling salesman problem. The obtained ordering I am applying the fuzzy defuzzification method to convert fuzzy value to the crisp values, to what should be a regular signal. This enables us to get good recovery of the clean image by applying comparatively simple one-dimensional smoothing operations (such as filtering or interpolation) to the reordered set of pixels. The performance of this approach is experimentally verified on a variety of images and noise levels. The results presented here demonstrate that proposed method is on par or exceeding the current state of the art, both visually and quantitatively.

Keywords—*Patch-based processing, fuzzification, defuzzification, triangular membership function, traveling salesman, pixel permutation, denoising.*

1. Introduction

Denoising of images is perhaps the most basic image restoration problem. Now days, image processing field with its local patches become highly popular and was shown highly effective foe representing work. The idea behind this is: extract all possible patches of the given image to be proceeding which are very small as compared to original image. A typical patch size would be 8×8 pixels. The processing is carried out by operating on these patches and exploiting interrelations between them. The manipulated patches are then put back into the image to form the resulting clean image.

In various ways, the relations between patches can be taken into account: weighted averaging of pixels with similar surrounding patches, as the NL-Means algorithm does [2], clustering the patches into disjoint sets and treating each set differently, as performed in [3]–[7], seeking a representative dictionary for the patches and using it to sparsely represent them, gathering groups of similar patches and applying a sparsifying transform on them. A common theme too many of these methods is the expectation that every patch taken from the image may find similar ones extracted elsewhere in the image. Put more frequently, the image patches are believed to reveal a highly-structured geometrical form in the embedding space they reside in. A joint treatment of these patches supports the restoration process by introducing a non-local force, thus enabling better recovery.

The honor of state-of-.the-art is held by block-matching with 3D filtering (BM3D) [8], aging over eight years. Levin and Nadler found that for natural images, BM3D is close to the theoretical limit of denoising, but artificial and highly correlated images still have potential for improvement. However, only a handful of methods numerically improve over BM3D, with modest increase in visual quality. State-of-the-art image denoising methods still produce visible artifacts, especially on sharp edges and in smooth regions of the original image. Such features are common for natural images, like clear sky and human skin, not to speak of synthetic images, where edges and gradients are abundant. Another nuisance is that current methods are complex and thus prohibit thorough analysis. In paper [9]-[11] shows that image denoising Based on Fuzzy and Intra-scale dependency in wavelet transform Domain.

Recently, Idan Ram, Michael Elad has been proposed image processing using smooth ordering of its patches [1]. I extend their work and propose a new fuzzy based approach for image denoisy. In this paper I propose an image denoising scheme based on fuzzy triangular membership function. For a given corrupted image, at first I converted to the fuzzy values using the fuzzification method. Then, I extract all patches with overlaps, after extracting all patches each patch is to be permuted and apply the fuzzy triangular membership function. I extract all patches with overlaps, refer to these as coordinates in high-dimensional space, and arrange them such that they are chained in the "shortest possible path," basically solving the traveling salesman problem. The obtained ordering I am applying the fuzzy defuzzification metod to convert fuzzy values to the crisp values, to what should be a regular signal. This enables us to get good recovery of the clean image by applying comparatively simple one-dimensional smoothing operations (such as filtering or interpolation) to the reordered set of pixels. The performance of this approach is experimentally verified on a variety of images and different noise levels. The results presented here demonstrate that the proposed method is better than the current state of the art, both visually and quantitatively.

2. Proposed Method

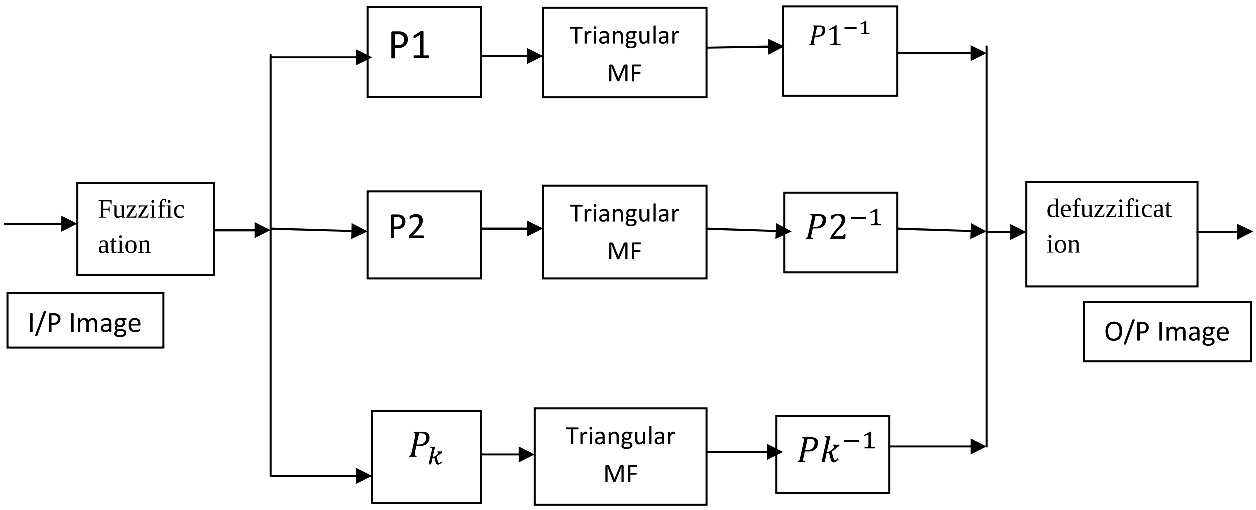

Fig.1: Proposed Method

In the Fig1 shows that the proposed method for Gaussian noise reduction consists of three parts first part is fuzzification which basically convert the image pixel values to the fuzzy values, Second part is the modification of the membership values using the triangular membership function and third is defuzification which is mainly convert the fuzzy values to the crisp pixel values. At first, I extract all patches with overlaps, after extracting all patches each patch is to be permuted and apply the fuzzy triangular membership function. I extract all patches with overlaps, refer to these as coordinates in high-dimensional space, and arrange them such that they are chained in the "shortest possible path," mainly solving the traveling salesman problem.

2.1 Proposed Scheme

Let Y be an image of size N = R × C, and let Z be a noisy or corrupted version of Y. Also, let y and z be the column stacked versions of Y and Z, respectively. A column stacked version of a matrix is a column vector, which is created by concatenating the columns of the matrix one by one. Then the corrupted image satisfies

$$z = My + v \qquad (1)$$

where the N × N matrix M denotes a linear operator which corrupts the data and v denotes an additive white Gaussian noise independent of y with zero mean and variance δ^2 . To reconstruct y from z this method employ a permutation matrix P of size N × N. It is assumed that when P is applied to the target signal, it produces a smooth signal.

This method starts by applying the permutation matrix P to z and obtain z^p = Pz. After that it applies a simple one dimensional smoothing operator H such as interpolation or filtering on z. And finally, it applies inverse permutation matrix P^{-1} to the result, and obtains the reconstructed image.

$$\hat{y} = P^{-1}H\{Pz\} \qquad (2)$$

I use concept of “cycle spinning” method to better smooth the recovered image. For that, I randomly construct K different permutation matricesP_k, utilize each to denoising the image z and then average the results. This can be expressed by

$$\hat{y} = \frac{1}{k}\sum_{k=1}^{k} P_k^{-1} H\{Pz\} \quad (3)$$

2.2 Building the Permutation Matrix P

Design a Permutation matrix P which produces a smooth signal when it is applied to the target image y. When the image Y is known, the optimal solution would be to reorder it as a vector, and then apply a simple sort operation on the obtained vector. However, I interested in the case where I only have the corrupted image Z (noisy, containing missing pixels, etc.), and any permutation defined by simply reordering the corrupted pixels to a regular signal does not necessarily smooth y. Therefore, as the pixels in the corrupted image are not helpful to us, I settle for a suboptimal ordering operation, using patches from the corrupted image.

Let $y_i$and z_i denote the i[th] samples in the vectors y and z, respectively. I denote by x_i the column stacked version of the $\sqrt{n} \times \sqrt{n}$ patch around the location of x_i in Z. I assume that under a distance measure w (x_i ,x_j), proximity between the two patches x_i and x_j suggests proximity between the uncorrupted versions of their center pixels y_i and y_j . Thus, I will try to reorder the points x_j so that they form a smooth path, hoping that the corresponding reordered 1D signal y^p will also become smooth. The “smoothness” of the reordered signal y^p can be measured using its total-variation measure

$$\|y^p\|_{TV} = \sum_{j=2}^{N} \|y^p(j) - (j+1)\| \quad (4)$$

Let $\{x^p\}_{j=1}^{N}$ denote the points $\{x_i\}_{i=1}^{N}$ in their new order. Then by analogy, I measure the “smoothness” of the path through the points the points $X_j^p$by the measureby the measure

$$X_{TV}^p = \sum_{j=2}^{N} w(x_j^p, x_{j-1}^p) \quad (5)$$

The patch reordering proposed algorithm is given below:

Algorithm: Patch Reordering Algorithm

Purpose: Reorder the image patches x_j

Parameter: Image patches $\{x_j\}_{j=1}^{N}$ the distance/weight function w and ε is the probability
Input; Taking the noisy image
Initialization:
Choose a random index j and set Ω (1) = {j}
Start: For k=1……N-1
Set A_k to be the set of indices of the B × B patches around $x_{\Omega(k)}$
If | $\frac{A_k}{\Omega}$ |=1 then Set Ω (k+1) to be $\frac{A_k}{\Omega}$
Else if | $\frac{A_k}{\Omega}$| ≥2 then
Find x_{j1} - the nearest neighbor to $x_{\Omega(k)}$ such that j1 belongs to A_k and J1 not belongs to Ω
Find x_{j2} -the second nearest neighbor to $x_{\Omega(k)}$ such that j2 belongs to A_k and j2 not belongs to Ω
Else if | $\frac{A_k}{\Omega}$| =0 then
Find x_{j1} - the nearest neighbor to $x_{\Omega(k)}$ such that J1 not belongs to Ω
Find x_{j2} -the second nearest neighbor to $x_{\Omega(k)}$ such that j2 not belongs to Ω
Set Ω (k+1) to be:
{j1} with probability p1=a exp [-w($x_{j0,}x_{j1}$)/ ε]
{j2} with probability p2= a exp [-w($x_{j0,}x_{j2}$)/ ε]
End
Output: Ω holds the proposed ordering

Minimizing X pTV comes down to finding the shortest path that passes through the set of points x_i , visiting each point only once. This can be regarded as an instance of the traveling salesman problem [13], which can become very computationally expensive for large sets of points. I choose a simple and crude approximate solution, which is to start from a random point and then continue from each point x_{j0} to its nearest neighbor x_{j1} with a probability p1 = a exp w (x_{j0} ,x_{j1}), or to its second nearest neighbor x_{j2} with a probability p2 = a exp w (x_{j0} ,x_{j2}), where a is determined such that p1+ p2 = 1, is a design parameter, and x_{j1} and x_{j2} are taken from the set of unvisited points.

I restrict the nearest neighbor search performed for each patch to a surrounding square neighborhood which contains B × B patches. When only one unvisited patch remains in that neighborhood, I simply continue to this patch, and in the case that no unvisited patches remain, I search for the first and second nearest neighbors among all the unvisited patches in the image. This restriction decreases the overall computational complexity, and this experiments show that with a proper choice of B it also leads to improved results. The permutation applied by the matrix P is defined as the order in the found path. The patch reordering scheme is summarized in Algorithm 1.

2.3 Triangular membership function

A membership function for a fuzzy set A on the universe of discourse X is defined as μ_A: X to [0, 1], where each element of X is mapped to a value between 0 and 1. This value, called membership value or degree of membership, quantifies the grade of membership of the element in X to the fuzzy set A. Membership functions allow us to graphically represent a fuzzy set. The x axis represents the universe of discourse, whereas the y axis represents the degrees of membership in the [0, 1] interval.
Triangular function: defined by a lower limit a, an upper limit b, and a value m, where a < m < b.

$$f(x{:}a,b,c)=\begin{cases} 0 & x \le a \\ \frac{x-a}{m-a} & a \le x \le m \\ \frac{c-x}{c-m} & m \le x \le c \\ 0 & c \le x \end{cases}$$

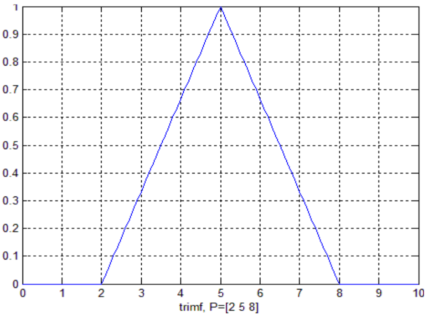

Fig.2: Triangular Membership Function

Fig.2. shows that the triangular membership functions. I am applying the triangular membership function on the permutation matrix P_k *where* $k =$ 1,2 ... k. After applying the triangular membership function I also get new values of each of the patches.

2.4 PSNR Measure

To evaluate the performance of the membership function uses the Peak Signal to Noise Ratio (PSNR) measure. The PSNR is used to evaluate the difference between the original noise free image and the denoised image. Notation "Signal" in the PSNR measure refers to noise free image. The PSNR measure is calculated using (7). Note that the high value of the PSNR shows the high quality of the reconstructed image.

$$MSE=\frac{1}{m \times n}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[\,I(i,j) - K(i,j)]^2 \qquad (6)$$

$$PSNR=10 \times log_{10}\left(\frac{MAX^2}{MSE}\right) \qquad (7)$$

Equation (6) calculates the differences between the original noiseless image and the filtered image. In (6), notations I and K refer to the input noise free image and the filtered image, respectively, and size of the image (original/filtered) is m × n. In (6), MAX^2 shows the maximum pixel value in the image. For instance, in the gray level image, MAX^2 is equal to 255.

3. Experimental Results

For this work, the four well known images have been tested such as "Lena", "Barbara", "Boat" and "House". The size of first three gray scale images is 512×512 and remaining is 256 × 256. All images are tested for different values of σ, i.e. 5, 10, 15, 20, 25, 50 and 100. The framework for implementation and testing the algorithm is Matlab7.

Table-I

DENOISING RESULTS (PSNR IN dB) OF NOISY VERSIONS OF 4 IMAGES, OBTAINED WITH BM3D ALGORITHMS AND PAPER [1] AND THE PROPOSED SCHEME. FOR EACH IMAGE AND NOISE LEVEL, THE BEST RESULT IS HIGHLIGHTED

Image	Method	σ/PSNR							
		5/34.14	10/28.12	15/24.60	20/22.10	25/20.17	50/14.14	75/10.62	100/8.12
Lena (512 × 512)									
	BM3D	38.710	36.905	35.566	34.592	33.855	31.487	30.541	29.913
	Paper[1]	38.222	36.265	34.641	33.234	32.342	31.564	30.634	29.922
	Proposed	**38.718**	**36.907**	**35.567**	**34.597**	**33.856**	**31.589**	**30.642**	**29.932**
Barbara (512 × 512)									
	BM3D	38.296	36.591	34.716	33.443	32.543	29.614	28.207	27.187
	Paper[1]	37.637	35.291	33.543	33.401	32.540	29.712	28.317	27.294
	Proposed	**38.297**	**36.598**	**34.797**	**33.512**	**32.613**	**29.783**	**28.326**	**27.306**
Boat (512 × 512)									
	BM3D	37.235	34.746	33.214	32.123	31.296	28.502	27.517	26.905
	Paper[1]	37.105	34.657	33.135	32.109	31.290	28.512	27.614	26.971
	Proposed	**37.241**	**34.751**	**33.224**	**32.129**	**31.299**	**28.517**	**27.619**	**26.978**
House (256 ×256)									
	BM3D	39.813	37.725	36.620	35.242	34.578	32.327	31.290	30.564
	Paper[1]	38.541	36.513	36.008	35.132	34.467	32.329	31.342	30.612
	Proposed	**39.814**	**37.728**	**36.628**	**35.249**	**34.612**	**32.347**	**31.349**	**30.618**

Table-I shows the PSNR values for the mentioned MFs which were tested on the different additive noises. From the Table –I shows that the highlighted part of the table are the better PSNR values and I can say that the proposed method is better than the state of the art method like BM3D,NLM and paper [1].

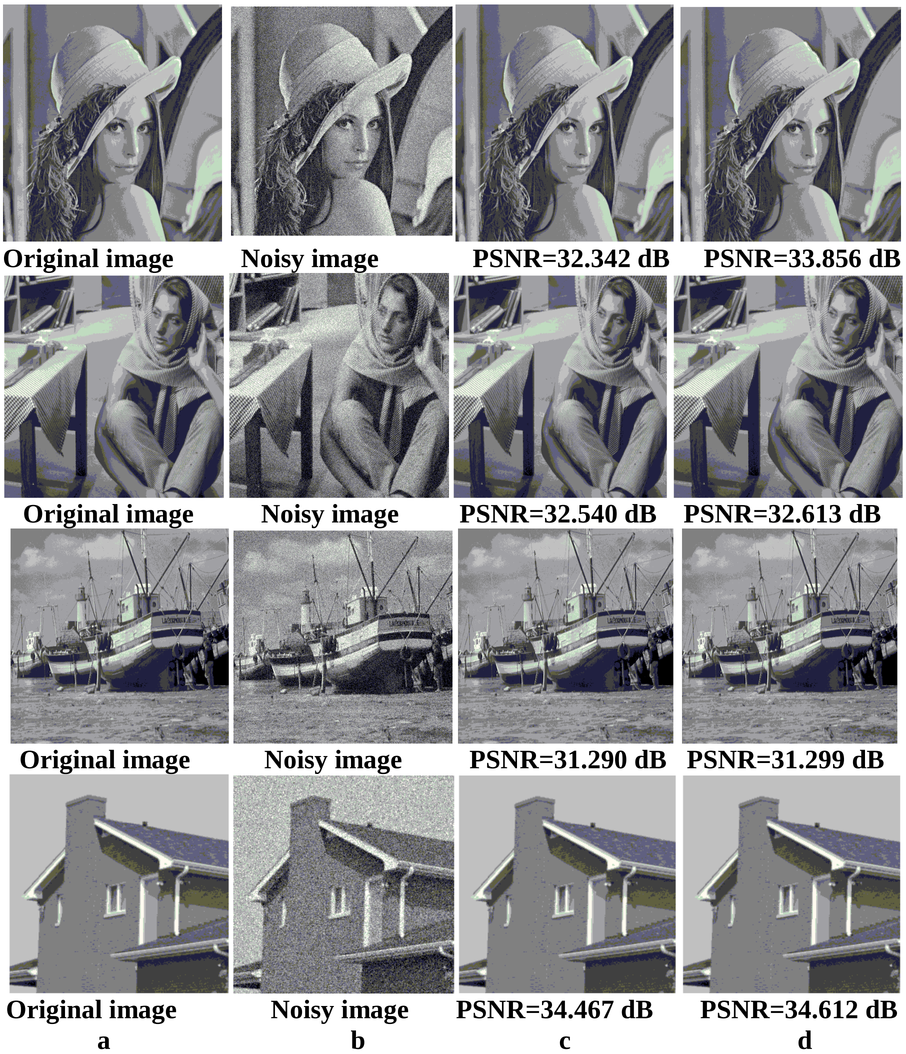

Fig.3. Comparison of denoising results on different images with noisy image corrupted by WGN of σ =25 (a) Original image, (b) noisy image, (c) Paper[1] PSNR value,(d) Proposed Method PSNR value

3.1 Discussion

Here I evaluate the proposed method for image denoising on four test images shown in Fig. 3. The related methods BM3D and paper [1] are used for comparison. To ensure a fair evaluation, I directly take the set of "standard" parameters configured by the authors of these algorithms. For this method, I use the patch of size 8 × 8, which means s = 64. The PSNR performance of BM3D, Paper [1] and proposed method are tabulated in Table-I. I also observe that the proposed method uniformly outperforms BM3D and paper [1] on these test images, and can achieve comparable PSNR performance with BM3D algorithm. The proposed method performs better on images with high repeating patterns compare the structure preserving ability of these denoising methods on both texture-less and texture rich images. I can see that in these experiments, the proposed method can preserve the local structures very well and provide quite a good visual quality.

4. Conclusion

In this paper proposed a new image processing scheme which is based on smooth 1D ordering of the pixels in the given image. A fuzzy filter is described in this paper. The proposed filter is seen to be quite effective in reducing the Gaussian noise; in addition, the proposed filter preserves the image boundaries and fine details satisfactorily. The efficacy of the proposed filter is illustrated by applying the filter on various test images contaminated by different levels of noise. This filter outperforms the existing filters in terms of qualitative and quantitative measures.

In addition, the filtered images are found to be pleasant for visual perception, since the filter is robust against Gaussian noise while preserving the image features intact. Further, the proposed filter is suitable for real-time implementation, and applications because of its adaptive in nature.

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