Application New Iterative Method For Solving Nonlinear Burger's Equation And Coupled Burger's Equations

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ABSTRACT

In the recent research , the numerical solution of Nonlinear Burger's equation and coupled Burger's equation is obtained Nonlinear using a New Iterative Method (NIM) is being proposed to obtain. We have shown that the NIM solution is more accurate as compared to the techniques like, Burger's equation and coupled Burger's equation method and HPM method. more, results also demonstrate that NIM solution is more reliable, easy to compute and computationally fast as compared to HPM method.

Keywords: Burger's equation, nonlinear partial differential equations

1. INTRODUCTION

Burger's equation emerges in a number of physically great phenomena such as model of traffic, turbulence, shock waves and fluid flow[5]. Many authors Bateman H[4], Burger J.M[5], Cole, J.D[6], Dogan A[7], Caldwell, J., P. Wanless and A.E. Cook [10] have discussed the numerical solution of Burger's equation using Finite Difference Methods and Finite Element Methods. have obtained the exact solution of Burger's equation by using Adomian Decomposition method. The real difficulty found in calculating Adomian polynomials is overcome by using He's Homotopy Perturbation Method and the solution obtained is exactly the same. [13]

New Iterative Method (NIM), improved by Daftardar Gejji and Jafari [20], to solve generalized Korteweg-de Vries equations of fifth and seventh orders [20, 21]. NIM has been used by many researchers to solve linear and nonlinear equations of integer and fractional orders [16, 3]. Benefit of NIM is that it gives highly accurate solution with comparatively much lesser number of iterations. Further, it does not contain additional overhead in computing terms such asadomian polynomials in ADM [8,19,15]

The NIM, planned by Daftardar- Gejji and Jafari in 2006 [20] and improved by Hemeda [1], was effectively applied to a variety of linear and nonlinear equations such as algebraic equations, integral equations, integro differential equations, ordinary and partial differential equations of integer and fractional order, and system of equations as well. NIM is simple to understand and easy to implement using computer packages and yields better result [16] than the existing ADM [8], Homotopy perturbation method(HPM) [11], or VIM [2].

The organization of this research is as follows: In section 2, we give the basic introduction of NIM. Solutions of the Burger's equations using is discussed in section 3. We call these solutions as NIM solutions perfect and the NIM computation technique is faster as compared to the other commonly used techniques like HPM and The comparisons between the numerical results of the proposed NIM solutions with that of HPM

2. NEW ITERATIVE METHOD (NIM)[15]

To clear the idea of the NIM, we consider the following general functional equation:

$$u = f + N(u), \cdots \ (1)$$

where N is a nonlinear operator from a Banach space B → B and f is a known function. We are seeking for a solution u of (1) having the series form

$$u = \sum_{i=0}^{\infty} u_i \,, \cdots (2)$$

The nonlinear operator N can be decomposed as:

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left[N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right] \cdots (3)$$

Now using the above eq.s (2) and (3) in (1):

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\left[N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right] \cdots (4)$$

We define the recurrence relation in the following method:

$$u_1 = N(u_0)$$

$$u_2 = N(u_0 + u_1) - N(u_0)$$

$$u_3 = N(u_0 + u_1 + u_2) - N(u_0 + u_1) \cdots (5)$$

$$u_{n+1} = N(u_0 + u_1 + \cdots + u_n) =$$

$$N(u_0 + u_1 + \cdots + u_{n-1}); \quad n = 1,2,3, \ldots.$$

Then

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$u_0 + u_1 + \cdots + u_{n+1}$

$= N(u_0 + u_1 + \cdots + u_n);\ n = 1,2,3, \ldots. \qquad \cdots (6)$

And

$$\sum_{i=0}^{\infty} u_i = f + N(\sum_{j=0}^{\infty} u_j),$$

The m-term sacrificial solution of (1) is given by $u \approx u_0 + u_1 + u_2 + \cdots + u_{m-1}$. For understanding the Convergence of this method we refer reader to [17]

3. Applications Problem

3.1 -Burger's equation Consider the one dimensional Burger's equation has the form [12,14]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \varepsilon\frac{\partial^2 u}{\partial x^2} = 0 \qquad \cdots (7)$$

With initial conditions

$$u(x,0) = \frac{\alpha+\beta+(\beta-\alpha)\exp\left(\frac{\alpha}{\varepsilon}\right)^{x-\delta}}{1+\exp\left(\frac{\alpha}{\varepsilon}\right)^{x-\delta}} \quad ,t \geq 0 \qquad \cdots (8)$$

To solve equation (7) explained the idea of the NIM method

$u = f + N(u),$

where N is a nonlinear operator from a Banach space $B \rightarrow B$ and f is a known function. We are looking for a (1).
Suppose the solution of the (NIM) given by equation (1) can be written as

$N(u) = uu_x - \varepsilon u_{xx}$

$$u(x,t) = u(x,0) - \int_0^t N(u)dt \qquad \cdots (9)$$

$$u_0 = \frac{\alpha+\beta+(\beta-\alpha)\exp\left(\frac{\alpha}{\varepsilon}\right)^{x-\delta}}{1+\exp\left(\frac{\alpha}{\varepsilon}\right)^{x-\delta}} \quad ,t \geq 0$$

And

$$u_1 = -\int_0^t N(u_0)dt = -\int_0^t u_0\frac{\partial u_0}{\partial x} - \varepsilon\frac{\partial^2 u_0}{\partial x^{u_0}}dt \qquad \cdots (10)$$

$$u_1 = \frac{2\exp x - \beta t}{(1+\exp x)^2} \qquad \cdots (11)$$

$$= -\int_0^t N(u_0+u_1) - N(u_0)dt$$

$$= -\int_0^t((u_0+u_1)\frac{\partial(u_0+u_1)}{\partial x} - \varepsilon\frac{\partial^2(u_0+u_1)}{\partial x^2} - u_0\frac{\partial u_0}{\partial x} - \varepsilon\frac{\partial^2 u_0}{\partial x^{u_0}})dt \qquad \cdots (12)$$

Solving all the above nonlinear partial differential equations we find,

$$u_2 = -\frac{1}{3}\frac{e^x\varepsilon\beta^2t^2(4e^x\varepsilon t - 4e^{2x}\varepsilon t + 3 + 3e^x - 3e^{2x} - 3e^{3x}}{(1+e^x)^5} \qquad \cdots (13)$$

And so on, in similar style more values were obtained using MAPLE13.

$u = u_0 + u_1 + u_2$

$$u = -\frac{1}{3}\frac{1}{(1+e^x)^5}(-3\varepsilon - 9e^x\varepsilon - 6\varepsilon e^{2x} + 6\varepsilon e^{3x} + 9\varepsilon e^{4x} - 3\beta - 15e^x\beta - 30e^{2x}\beta - 30e^{3x} - 15\beta e^{4x} - 3e^{5x}\beta + 3e^{5x}\varepsilon - 6e^x\beta\varepsilon t - 18e^{2x}\beta\varepsilon t - 18e^{3x}\beta\varepsilon t - 6e^{4x}\beta\varepsilon + 4e^{2x}\varepsilon^2\beta^2t^3 - 4e^{3x}\varepsilon^2\beta^2t^3 + 3e^x\varepsilon\beta^2t^2 + 3e^{2x}\varepsilon\beta^2t^2 - 3e^{3x}\varepsilon\beta^2t^2 - 3e^{4x}\varepsilon\beta^2t^2) \ldots\ldots (14)$$

The solution of $u(x,0)$ in close form is,

$$u(x,t) = \frac{\alpha+\beta+(\beta-\alpha)\exp\left(\frac{\alpha}{\varepsilon}\right)^{x-\delta}}{1+\exp\left(\frac{\alpha}{\varepsilon}\right)^{x-\delta}} \quad ,\ t \geq 0$$

which is exactly the same as solution obtained by HPM method[10].

The numerical values and behavior of the solutions obtained by of the NIM method is shown for different values of time in table 1 and figure 1 respectively.

	U_{HPM}	U_{NIM}	$EXAET$	$EROR$
x=-10	0.98662020746	0.9859462304	0.9859462610	3.060000000×10^{-8}
x=-9	0.98020254181	0.9792138377	0.9792138776	3.990000000×10^{-8}
x=-8	0.97082080578	0.9693819537	0.9693820138	6.010000000×10^{-8}
x=-7	0.95723687696	0.9551671532	0.9551672354	8.220000000×10^{-8}
x=-6	0.93783912325	0.9349112702	0.9349113685	9.830000000×10^{-8}
x=-5	0.91068025144	0.9066350758	0.9066351881	1.123000000×10^{-7}
x=-4	0.87368671730	0.8682767264	0.8682768323	1.059000000×10^{-7}
x=-3	0.82514137037	0.8182133037	0.8182133671	6.340000000×10^{-8}
x=-2	0.7644618661	0.7560678401	0.7560678146	2.550000000×10^{-8}
x=-1	0.69306405178	0.6835521525	0.6835520137	1.388000000×10^{-7}
x=0	0.61479324996	0.6048000000	0.6047997696	2.304000000×10^{-7}
x=1	0.5353713615	0.5256738597	0.5256736077	2.520000000×10^{-7}

Table 1:The values of u(x,t) evaluates by (NIM) method with $t = 0.1, \varepsilon = 1, \beta = 0.6, \alpha = 0.4.$

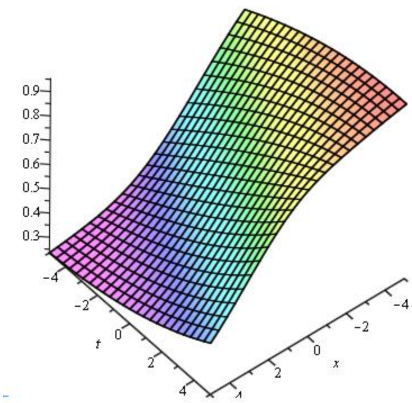

Fig 1: Graphical representation of u(x,t)

3.2. Coupled Burger's equations

To solve the homogeneous form of coupled Burger's equations by Homotopy perturbation method consider the system of equations [18]

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} + (uv)_x = 0 \quad \cdots (15)$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + u\frac{\partial v}{\partial x} + (vu)_x = 0 \quad \cdots (16)$$

With initial conditions

$$u(x,0) = sinx \quad , v(x,0) = sinx \quad \cdots (17)$$

To solve this system of equations by (NIM) method define The solution of equation (7), Burger equation is given by, $N_1(u,v): \emptyset \times [0.1] \to R$ *and* $N_1(u,v): \emptyset \times [0.1] \to R$ for equation (14) and (15) respectively that satisfy

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} + (uv)_x = 0 \quad \cdots (18)$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + u\frac{\partial v}{\partial x} + (vu)_x = 0 \quad \cdots (19)$$

Assume the solutions of the NIM (18) and (19) can be written as

$$N_1(u,v) = \frac{\partial u_0}{\partial t} - \frac{\partial^2 u_0}{\partial x^2} + u\frac{\partial u_0}{\partial x} + (uv)_x \quad \cdots (20)$$

And

$$N_2(u,v) = \frac{\partial v_0}{\partial t} - \frac{\partial^2 v_0}{\partial x^2} + u\frac{\partial v_0}{\partial x} + (vu)_x = 0 \quad \cdots (21)$$

$$u(x,t) - u(x,0) + \int_0^t N_1(u,v)dt = 0 \quad \cdots (22)$$

And

$$v(x,t) - u(x,0) + \int_0^t N_2\,(u,v)dt = 0 \quad \cdots (23)$$

$u_{0=}u(x,0) = sinx \quad , v_{0=}\, v(x,0) = sinx$

$$u_1 = -\int_0^t N_1\,(u_0,v_0)dt = 0 \quad \cdots (24)$$

$$N_1(u_0,v_0) = -2u_0 \frac{\delta}{\delta x} u_0 - \frac{\delta^2}{\delta^2 x} u_0 + u_0 \frac{\delta}{\delta x} v_0 + v_0 \frac{\delta}{\delta x} u_0$$

$$u_1 = -\int_0^t N_1\,(u_0,v_0)dt = -\sin(x)\,t \quad \cdots (25)$$

$$u_2 = -\int_0^t (N_1\,(u_1+u_0, v_1+v_0) - N_1(u_0,v_0))dt \;\cdots (26)$$

$$u_2 = -\int_0^t (-2(u_0+u_1)\frac{\delta}{\delta x}(u_0+u_1) - \frac{\delta^2}{\delta^2 x}(u_0 + u_1) + (u_0+u_1)\frac{\delta}{\delta x}(v_0+v_1) + (v_0+v_1)\frac{\delta}{\delta x}(u_0+u_1)) - (-2u_0\frac{\delta}{\delta x}u_0 - \frac{\delta^2}{\delta^2 x}u_0 + u_0\frac{\delta}{\delta x}v_0 + v_0\frac{\delta}{\delta x}u_0)dt$$

$$u_2 = \tfrac{1}{2}\sin(x)\,t \quad \cdots (27)$$

And

$$v_1 = -\int_0^t N_2\,(u_0,v_0)dt = -\sin(x)\,t \quad \cdots (28)$$

$$v_2 = \tfrac{1}{2}\sin(x)\,t \quad \cdots (29)$$

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And so on more values were obtained. The solution of coupled Burger's equations can be written as,

$u = u_0 + u_1 + \cdots + u_n \qquad n = 1,2,3, \ldots.$

$$u = sinx + (-\sin(x)\,t) + \tfrac{1}{2}\sin(x)\,t \quad \cdots (30)$$

And

$\therefore v = v_0 + v_1 + \cdots + v_n \qquad n = 1,2,3, \ldots.$

$$\therefore v = sinx + (-\sin(x)\,t) + \tfrac{1}{2}\sin(x)\,t \quad \cdots (31)$$

Which are the exact solutions.

4-Conclusion

A NEW ITERATIVE METHOD method(NIM) is successfully applied to solve non linear Burger's equation and coupled Burger's equations. The solution obtained by (NIM) method is an infinite series for suitable initial condition that can be reflexed in a closed shape, the exact solution. The solution obtained by (NIM) method is found as a powerful mathematical instrument to solve non linear partial differential equations. Comparison at the NIM with HPM method that have been advanced for solving this system, shows that the new technique is reliable , powerful ,and promising as shown in the tables (1)

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