# Performance Evaluation of Two Node Tandem Communication Network with Dynamic Bandwidth Allocation having Two Stage Direct Bulk Arrivals 

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#### Abstract

A two node tandem communication network with dynamic bandwidth allocation (DBA) having two stage direct bulk arrivals is developed and analyzed. The messages arriving to the source are packetized and stored in the buffers for forward transmission. Dynamic bandwidth allocation strategy is proposed by adjusting the transmission rate at every node just before transmission of each packet. The arrival and transmission processes at each node are characterized through compound Poisson and Poisson processes such that several of the statistical characteristics of communication networks identically matches. Using the difference differential equations, the performance measures like the joint probability generating function of the content of two buffers, average buffer content, mean delays and throughput of nodes are derived and analyzed. It is observed that the bulk arrivals at two nodes and DBA have significant influence on performance measures. This network is much useful in Tele and Satellite communications.


Keywords: Communication networks, Dynamic bandwidth allocation, Two- stage Bulk arrivals and Performance measures

## 1. Introduction

It is generally known that packet switching gives better utilization over circuit or message switching and yields relatively short network delay. In packet switching, the message is divided into a random number of small packets each having an independent header for routing. This phenomenon is visible in Tele and Satellite communications where packet switching is effectively deployed. It can be characterized through statistical multiplexing by approximating the arrival process with a compound Poisson process (Kin K. Leung, 2002; K.Srinivasa Rao et al. 2006).

To have an efficient transmission, some algorithms have been developed with various protocols and allocation strategies for optimum utilization of the bandwidth (Emre and Ezhan, 2008; Gundale and Yardi, 2008; Hongwang and Yufan, 2009; Fen Zhou et al. 2009; Stanislav, 2009). These strategies are developed based on flow control or bit dropping techniques. But utilization of the idle bandwidth by adjusting the transmission rate instantaneously just before transmission of a packet is more important to maintain quqlity of service (QoS).

Dynamic bandwidth allocation strategy of transmission considers the adjustment of transmission rate of the packet depending upon the content of the buffer connected to transmitter at that instant. Recently, P.S.Varma et al. (2007) have utilized this strategy for a
two node communication network. However, they assumed that the arrivals to the source node are single packets. But, in communication systems, the packet arrivals to the source node are
in bulk since the message is converted into a random number of packets depending upon the message size. Hence, the Poisson assumption made for arrival process of packets may lead to inaccurate prediction of performance measures in the communication networks. Therefore, it is needed to develop and analyze the tandem communication network models with bulk arrivals having dynamic bandwidth allocation. Very little work has been reported in the literature regarding tandem communication networks with bulk arrivals which are quite common in places like Tele and Satellite communications. Kuda Nageswara Rao et al. (2010) have developed a communication network with dynamic bandwidth allocation having bulk arrivals. They approximated the arrival process with a compound Poisson process which characterizes the bulk arrival nature of the communication networks. However, they assumed that the arrivals are only to the initial node. But, in Tele communication systems, the messages may arrive directly to the second node also in addition to the packets forwarded through the first node. This phenomenon of direct bulk arrivals for both nodes has significant influence on buffer management and optimal utilization of resources in general and particularly with dynamic bandwidth allocation. With this motivation, in this paper, a two node communication network with dynamic bandwidth allocation having direct bulk arrivals to two nodes is developed and analyzed using mathematical modeling. Conducting laboratory experiments with variable load conditions for communication networks are complicated and time consuming, a mathematical model provide a basic frame work for performance evaluation of communication networks (Yukuo,1993; Ushio Sunita et al. 1997; Gaujal et al. 2002; Anney et al. 2010).

Using the difference - differential equations, the performance measures of the communication network like the joint probability generating function of the content of buffers, average contents of buffers, mean delays in transmissions, throughput etc., are derived. The sensitivity
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of the performance measures with respect to the model parameters is also studied through numerical illustration.

## 2. Communication Network Model

Consider a communication network model with two nodes in tandem having bulk arrivals and dynamic bandwidth allocation. The arrivals to node 1 and node 2 are assumed to follow a compound Poisson process with parameters $\lambda_{1}$ and $\lambda_{2}$ respectively. The compound Poisson process is capable of portraying the bulk arrival nature of the communication network. Here, it is considered that the messages that arrive to both nodes are converted into random number of packets and form a batch. The batch size distribution of packets are assumed to follow rectangle ( uniform) distribution probability distribution functions $C_{k 1}$ and $C_{k 2}$ with parameters $\left(a_{1}, b_{1}\right)$ and $\left(a_{2}, b_{2}\right)$ respectively for buffer 1 and buffer 2.

It is also assumed that the number of transmissions at each transmitter follow Poisson processes with parameters $\mu_{1}$ and $\mu_{2}$ respectively. The transmission rates of packets in each node are instantaneously adjusted depending on the content of the buffers just before its transmission. The queue discipline is First-In-FirstOut (FIFO). There is no termination of packets after the transmission of first node. The schematic diagram representing the communication network is shown in Figure 1.


Fig. 1 Communication network with two stage Bulk arrivals and dynamic bandwidth allocation

Let $P_{n_{1}, n_{2}}(t)$ be the probability that there are $\mathbf{n}_{1}$ packets in the first buffer and $\mathbf{n}_{2}$ packets in the second buffer at time $t$. Then, the difference-differential equations governing the network are

$$
\begin{align*}
& \frac{\partial \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}}(\mathrm{t})}{\partial \mathrm{t}}=-\left(\lambda_{1}+\mathrm{n}_{1} \mu_{1}+\mathrm{n}_{2} \mu_{2}+\lambda_{2}\right) \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}}(\mathrm{t})+\left(\mathrm{n}_{1}+1\right) \mu_{1} \mathrm{P}_{\mathrm{n}_{1}+1, \mathrm{n}_{2}-1}(\mathrm{t})  \tag{1}\\
& +\left(\mathrm{n}_{2}+1\right) \mu_{2} \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}+1}(\mathrm{t})+\lambda_{1}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}_{1}} \mathrm{P}_{\mathrm{n}_{1}-\mathrm{i}, \mathrm{n}_{2}}(\mathrm{t}) \mathrm{C}_{\mathrm{i} 1}\right]+\lambda_{2}\left[\sum_{\mathrm{j}=1}^{\mathrm{n}_{2}} \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}-\mathrm{j}}(\mathrm{t}) \mathrm{C}_{\mathrm{j} 2}\right]  \tag{14}\\
& \frac{\partial \mathrm{P}_{\mathrm{n}_{1}, 0}(\mathrm{t})}{\partial \mathrm{t}}=-\left(\lambda_{1}+\mathrm{n}_{1} \mu_{1}+\lambda_{2}\right) \mathrm{P}_{\mathrm{n}_{1}, 0}(\mathrm{t})++\mu_{2} \mathrm{P}_{\mathrm{n}_{1}, 1}(\mathrm{t})+\lambda_{1}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}_{1}} \mathrm{P}_{\mathrm{n}_{1}-\mathrm{i}, 0}(\mathrm{t}) \mathrm{C}_{\mathrm{i} 1}\right] \\
& \frac{\partial \mathrm{P}_{0, \mathrm{n}_{2}}(\mathrm{t})}{\partial \mathrm{t}}=-\left(\lambda_{1}+\mathrm{n}_{2} \mu_{2}+\lambda_{2}\right) \mathrm{P}_{0, \mathrm{n}_{2}}(\mathrm{t})+\mu_{1} \mathrm{P}_{1, \mathrm{n}_{2}-1}(\mathrm{t})+\left(\mathrm{n}_{2}+1\right) \mu_{2} \mathrm{P}_{0, \mathrm{n}_{2}+1}(\mathrm{t})+\lambda_{2}\left[\sum_{\mathrm{j}=1}^{\mathrm{n}_{2}} \mathrm{P}_{0, \mathrm{n}_{2}-\mathrm{j}}(\mathrm{t}) \mathrm{C}_{\mathrm{j} 2}\right]  \tag{15}\\
& \frac{\partial \mathrm{P}_{1,0}(\mathrm{t})}{\partial \mathrm{t}}=-\left[\lambda_{1}+\mu_{1}+\lambda_{2}\right] \mathrm{P}_{1,0}(\mathrm{t})+\mu_{2} \mathrm{P}_{1,1}(\mathrm{t})+\lambda_{1}\left[\mathrm{P}_{0,0}(\mathrm{t}) \mathrm{C}_{1}\right]  \tag{4}\\
& \frac{\partial \mathrm{P}_{0,1}(\mathrm{t})}{\partial \mathrm{t}}=-\left(\lambda_{1}+\mu_{2}+\lambda_{2}\right) \mathrm{P}_{0,1}(\mathrm{t})+\mu_{1} \mathrm{P}_{1,0}(\mathrm{t})+2 \mu_{2} \mathrm{P}_{0,2}(\mathrm{t})+\lambda_{2}\left[\mathrm{P}_{0,0}(\mathrm{t}) \mathrm{C}_{12}\right]  \tag{5}\\
& \left.\frac{\partial \mathbf{P}_{\mathrm{O}, \mathrm{O}}(\mathrm{t})}{\partial \mathrm{t}}=-\left(\lambda_{1}+\lambda_{2}\right) \mathbf{P}_{\mathrm{o}, \mathrm{O}}(\mathrm{t})+\mu_{2} \mathbf{P}_{\mathrm{o}, 1} \text { ( } \mathrm{t}\right) \tag{6}
\end{align*}
$$

Assuming that the system is under steady state i.e. $\operatorname{Lim}_{t \rightarrow \infty} P_{n_{1}, n_{2}}(t)=P_{n_{1}, n_{2}} \quad$ and $\quad \underset{t \rightarrow \infty}{\operatorname{Lim}} \frac{\text { dP }_{n_{1}, n_{2}}(t)}{d t}=0$
The steady state equations of the model are

$$
\mathrm{P}\left(\mathrm{Z}_{1}\right)=\exp \left[\lambda_{1} \sum_{\mathrm{k}_{1}=1}^{\infty} \sum_{\mathrm{r}=1}^{\mathrm{k}_{1}}(-1)^{2 \mathrm{r}} \mathrm{C}_{\mathrm{k}_{1}}^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{r}}\left(\mathrm{Z}_{1}-1\right)^{\mathrm{r}}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right]
$$

(2) The probability that the first buffer is empty as
$\mathrm{p}_{0 .}=\exp \left[\lambda_{1} \sum_{\mathrm{k}_{1}=1}^{\infty} \sum_{\mathrm{r}=1}^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{k}_{1}}{ }^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{r}}(-1)^{3 \mathrm{r}}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right]$
(7)
$-\left(\lambda_{1}+n_{1} \mu_{1}+\lambda_{2}\right) P_{n_{1}, 0}+\mu_{2} P_{n_{1}, 1}+\lambda_{1}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}_{1}} \mathrm{P}_{\mathrm{n}_{1}-\mathrm{i}, 0} \mathrm{C}_{\mathrm{i} 1}\right]=0$
(8)
$-\left(\lambda_{1}+\mathrm{n}_{2} \mu_{2}+\lambda_{2}\right) \mathrm{P}_{0, \mathrm{n}_{2}}+\mu_{1} \mathrm{P}_{1, \mathrm{n}_{2}-1}+\left(\mathrm{n}_{2}+1\right) \mu_{2} \mathrm{P}_{0, \mathrm{n}_{2}+1}+\lambda_{2}\left[\sum_{j=1}^{\mathrm{n}_{2}} \mathrm{P}_{0, \mathrm{n}_{2}-\mathrm{j}} \mathrm{C}_{\mathrm{j} 2}\right]=0$
(9)
$-\left[\lambda_{1}+\mu_{1}+\lambda_{2}\right] \mathrm{P}_{1, \mathrm{o}}+\mu_{2} \mathrm{P}_{1,1}+\lambda_{1}\left[\mathrm{P}_{\mathrm{o}, \mathrm{o}} \mathrm{C}_{1}\right]=\mathrm{O}$
(10) $-\left(\lambda_{1}+\mu_{2}+\lambda_{2}\right) \mathrm{P}_{0,1}+\mu_{1} \mathrm{P}_{1,0}+2 \mu_{2} \mathrm{P}_{0,2}+\lambda_{2}\left[\mathrm{P}_{0,0} \mathrm{C}_{12}\right]=0$
$-\left(\lambda_{1}+\lambda_{2}\right) P_{0,0}+\mu_{2} P_{0,1}=0$
Let $\mathrm{P}\left(\mathrm{Z}_{1}, \mathrm{Z}_{2}\right)=\sum_{\mathrm{n}_{1}=0}^{\infty} \sum_{\mathrm{n}_{2}}^{\infty} \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}} \mathrm{Z}_{1}^{\mathrm{n}_{1}} \mathrm{Z}_{2}^{\mathrm{n}_{2}} \quad$ and
$C(Z)=\sum_{k=1}^{\infty} C_{k} Z^{k}$ be the probability generating functions of $P_{n_{1}, n_{2}}$ and $C_{k}$ respectively.

Multiplying the equations 7 to 12 with corresponding $Z_{1}^{\mathrm{n}_{1}}$ and $\mathrm{Z}_{2}^{\mathrm{n}_{2}}$ and summing over all $\mathrm{n} 1, \mathrm{n} 2$, we get the joint probability generating function of n 1 packets in the first buffer and n2 packets in the second buffer at any time when the system is under equilibrium as

$$
\begin{align*}
& P\left(Z_{1}, Z_{2}\right)=\exp \left\{\left[\lambda_{1} \sum_{k_{1}=1}^{\infty} \sum_{i=1}^{\infty} \sum_{j=0}^{k_{n}}(-1)^{2 r-1} C_{k_{1}}\left(k_{1} C_{r}\right)\left(C_{C}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{3}\left(Z_{2}-1\right)^{J}\left(\left(Z_{1}-1\right)+\frac{\mu_{1}\left(Z_{2}-1\right)}{\mu_{2}-\mu_{1}}\right)^{(r-1)}\right.\right. \\
& \left.\left.\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]+\lambda_{2}\left[\sum_{k_{2}=1}^{\infty} \sum_{s=1}^{k_{2}} C_{k_{2}}{ }^{k_{2}} C_{s}\left(Z_{2}-1\right)^{s}\left(\frac{1}{\mu_{2} s}\right)\right]\right\} \tag{13}
\end{align*}
$$

## 3. Performance Measures of the Communication Network

The probability generating function of the first buffer size distribution is

The mean number packets in the first buffer is
$\mathrm{L}_{1}=\frac{\lambda_{1}}{\mu_{1}}\left[\sum_{\mathrm{k}_{1}=1}^{\infty} \mathrm{C}_{\mathrm{k}_{1}} \cdot \mathrm{k}_{1}\right]$
The utilization of the first node is
$\mathrm{U}_{1}=1-\mathrm{p}_{0}$.
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$$
\begin{equation*}
=1-\exp \left[\lambda_{1} \sum_{\mathrm{k}_{1}=1}^{\infty} \sum_{\mathrm{r}=1}^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{k}_{1}}^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{r}}(-1)^{3 \mathrm{r}}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right] \tag{17}
\end{equation*}
$$

The throughput of the first node is

$$
\begin{align*}
\operatorname{Thp}_{1} & =\mu_{1} \cdot U_{1} \\
& =\mu_{1}\left[1-\exp \left[\lambda_{1} \sum_{k_{1}=1}^{\infty} \sum_{r=1}^{k_{1}} C_{k_{1}}{ }^{k_{1}} C_{r}(-1)^{3 r}\left(\frac{1}{r \mu_{1}}\right)\right]\right] \tag{18}
\end{align*}
$$

The average delay in the first buffer is

$$
\begin{equation*}
\mathrm{W}\left(\mathrm{~N}_{1}\right)=\frac{\mathrm{L}_{1}}{\mathrm{Thp}_{1}}=\frac{\frac{\lambda_{1}}{\mu_{1}}\left[\sum_{\mathrm{k}_{1}=1}^{\infty} \mathrm{C}_{\mathrm{k}_{1}} \cdot \mathrm{k}_{1}\right]}{\mu_{1}\left[1-\mathrm{ex}\left[\lambda \lambda_{\mathrm{k}_{1}=1}^{\infty} \sum_{\mathrm{r}=1}^{k_{1}} \mathrm{C}_{k_{1}}{ }^{k_{1}} C_{r}(-1)^{3 r}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right]\right]} \tag{19}
\end{equation*}
$$

The variances of the number of packets in the first buffer is

$$
\begin{array}{r}
\operatorname{Var}\left(\mathrm{N}_{1}\right)=\mathrm{E}\left[\mathrm{~N}_{1}^{2}-\mathrm{N}_{1}\right]+\mathrm{E}\left[\mathrm{~N}_{1}\right]-\left(\mathrm{E}\left[\mathrm{~N}_{1}\right]\right)^{2} \\
=\frac{\lambda_{1}}{2 \mu_{1}}\left[\sum_{\mathrm{k}_{1}=1}^{\infty} \mathrm{C}_{\mathrm{k}_{1}} \mathrm{k}_{1}\left(\mathrm{k}_{1}-1\right)\right]+\frac{\lambda_{1}}{\mu_{1}}\left[\sum_{\mathrm{k}_{1}=1}^{\infty} \mathrm{C}_{\mathrm{k}_{1}} \mathrm{k}_{1}\right] \tag{20}
\end{array}
$$

The coefficient of variation of the number of packets in the first buffer is

$$
\begin{equation*}
\operatorname{cv}\left(\mathrm{N}_{1}\right)=\frac{\sqrt{\operatorname{Var}\left(\mathrm{N}_{1}\right)}}{\mathrm{L}_{1}} \tag{21}
\end{equation*}
$$

The probability generating function of the second buffer size distribution is

$$
\begin{align*}
P\left(Z_{2}\right) & =\exp \left\{\left[\lambda_{1} \sum_{k_{1}=1}^{\infty} \sum_{=1}^{\infty} \sum_{l=0}^{k_{1}}(-1)^{r} r_{k_{1}-1}^{2 r-J} C_{k_{1}}^{k_{1}} C_{r}\right)\left({ }^{r} C_{J}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(Z_{2}-1\right)^{r}\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]  \tag{22}\\
& \left.\left.+\lambda_{2}\left[\sum_{k_{k_{2}=1}}^{\infty} \sum_{s=1}^{k_{2}} C_{k_{2}}{ }^{k_{2}} C_{s}\right)\left(Z_{2}-1\right)^{s}\left(\frac{1}{\mu_{2} s}\right)\right]\right\}
\end{align*}
$$

The probability that the second buffer is empty as

$$
\begin{array}{r}
\text { P. }=\exp \left\{\left[\lambda_{1} \sum_{k_{1}=1}^{\infty} \sum_{r=1}^{k_{1}} \sum_{J=0}^{r}(-1)^{3-5} C_{k_{1}}{ }^{k_{1}} C_{r}\right)\left({ }^{r} C_{j}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right] \\
\left.\left.+\lambda_{2}\left[\sum_{k_{2}=1}^{\infty} \sum_{s=1}^{k_{2}} C_{k_{2}}{ }^{k_{2}} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} s}\right)\right]\right\} \tag{31}
\end{array}
$$

The mean number packets in the second buffer is

$$
\begin{equation*}
\mathrm{L}_{2}=\frac{\lambda_{1}}{\mu_{2}}\left(\sum_{\mathrm{k}_{1}=1}^{\infty} \mathrm{C}_{\mathrm{k}_{1}} \mathrm{k}_{1}\right)+\frac{\lambda_{2}}{\mu_{2}}\left(\sum_{\mathrm{k}_{2}=1}^{\infty} \mathrm{C}_{\mathrm{k}_{2}} \mathrm{k}_{2}\right) \tag{24}
\end{equation*}
$$

The utilization of the second node is
The mean number packets in the network is
$\mathrm{L}_{\mathrm{N}}=\mathrm{L}_{1}+\mathrm{L}_{2}$
where,
$\mathrm{L}_{1}=$ the mean number of packets in the first node
$\mathrm{L}_{2}=$ the mean number of packets in the second node

## 4. Performance Measures with Uniform Batch Size Distribution

In this section, the performance of the communication network under steady state conditions is discussed with the assumption that the number of packets that each message can be converted follows a uniform (rectangular) distribution with parameters $\left(a_{1}, b_{1}\right)$ and $\left(a_{2}, b_{2}\right)$ for buffer 1 and buffer 2 respectively. Then the joint probability generating function of the buffers size when the system is
The throughput of second node is
(27)

The variances of the number of packets in the second buffer is

$$
\begin{array}{r}
\operatorname{Var}\left(\mathbf{N}_{2}\right)=\mathrm{E}\left[\mathbf{N}_{2}^{2}-\mathbf{N}_{2}\right]+\mathrm{E}\left[\mathrm{~N}_{2}\right]-\left(\mathrm{E}\left[\mathrm{~N}_{2}\right]\right)^{2} \\
\operatorname{var}\left(\mathrm{~N}_{2}\right)=\lambda_{1}\left\{\left(\sum_{k_{1}=1}^{\infty} \mathrm{C}_{k_{1}} \cdot \mathbf{k}_{1}\left(\mathbf{k}_{1}-1\right)\right)\left(\frac{\mu_{1}}{\mu_{1}-\mu_{2}}\right)^{2}\left[\left(\frac{1}{2 \mu_{1}}\right)-2\left(\frac{1}{\mu_{1}+\mu_{2}}\right)+\left(\frac{1}{2 \mu_{2}}\right)\right]\right\} \\
+\left\{\lambda_{2} \sum_{k_{2}=1}^{\infty} \mathrm{C}_{k_{2}} \mathbf{k}_{2}\left(\mathbf{k}_{2}-1\right)\left(\frac{1}{2 \mu_{2}}\right)\right\}+\left\{\begin{array}{l}
\left.\lambda_{1}\left(\sum_{k_{1}=1}^{\infty} \mathrm{C}_{k_{1}} \cdot \mathbf{k}_{1}\right)\left(\frac{1}{\mu_{2}}\right)\right\} \\
+\left\{\frac{\lambda_{2}}{\mu_{2}}\left(\sum_{k_{2}=1}^{\infty} \mathrm{C}_{k_{2}} \cdot \mathbf{k}_{2}\right)\right\}
\end{array}\right.
\end{array}
$$

The coefficient of variation of the number of packets in the second buffer is
$\operatorname{cv}\left(\mathrm{N}_{2}\right)=\frac{\sqrt{\mathrm{Var}\left(\mathrm{N}_{2}\right)}}{\sqrt{\digamma_{2}}}$
The probability that the network is empty is

$$
\begin{align*}
& \mathrm{P}_{00}=\exp \left\{\left[\lambda_{1} \sum_{\mathrm{k}_{1}=1}^{\infty} \sum_{\mathrm{r}=1}^{k_{1}} \sum_{\mathrm{J}=0}^{\mathrm{r}}(-1)^{2 \mathrm{r}} \mathrm{C}_{\mathrm{k}_{1}}\left(\mathrm{k}_{1} C_{r}\right)\left({ }^{\mathrm{r}} \mathrm{C}_{\mathrm{J}}\right)\left(\theta \mu_{1}\right)^{s} \frac{\left[\mu_{1}(1-\theta)-\mu_{2}\right]^{r-J}}{\left(\mu_{2}-\mu_{1}\right)^{r}}\right.\right. \\
&\left.\left.\left.\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]+\lambda_{2}\left[\sum_{k_{k_{2}}=1}^{\infty} \sum_{\mathrm{s}=1}^{k_{2}} C_{k_{2}}{ }^{k_{2}} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} \mathrm{~S}}\right)\right]\right\} \tag{30}
\end{align*}
$$

The average delay in the second buffer is

$$
\begin{aligned}
\mathrm{W}\left(\mathrm{~N}_{2}\right)=\frac{\mathrm{L}_{2}}{\operatorname{Thp}_{2}}= & \frac{\frac{\lambda_{1}}{\mu_{2}}\left(\sum_{k_{1}=1}^{\infty} C_{k_{1}} k_{1}\right)+\frac{\lambda_{2}}{\mu_{2}}\left(\sum_{k_{2}=1}^{\infty} C_{k_{2}} k_{2}\right)}{\mu_{2} \cdot\left\{1-\exp \left\{\left[\lambda_{1} \sum_{k_{1}=1}^{\infty} \sum_{1=1}^{k_{1}} \sum_{J=0}^{r}(-1)^{3 r-5} C_{k_{1}}{ }^{k_{1}} C_{r}\right)\left(C_{J}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]\right.} \\
& \left.\left.+\lambda_{2}\left[\sum_{k_{2}=1}^{\infty} \sum_{s=1}^{k_{2}} C_{k_{2}}\left(k_{2} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} k_{2}}\right)\right]\right\}\right\}
\end{aligned}
$$

$$
\begin{array}{r}
=\mu_{2} \cdot\left\{1-\exp \left\{\left[\lambda_{1} \sum_{\mathrm{k}_{1}=1}^{\infty} \sum_{\mathrm{r}=1}^{\mathrm{k}_{1}} \sum_{\mathrm{J}=0}^{\mathrm{r}}(-1)^{3 \mathrm{r}-\mathrm{J}} \mathrm{C}_{\mathrm{k}_{1}}\left(\mathrm{k}_{1} \mathrm{C}_{\mathrm{r}}\right)\left({ }^{\mathrm{r}} \mathrm{C}_{\mathrm{J}}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{\mathrm{r}}\left(\frac{1}{\mathrm{~J} \mu_{2}+(\mathrm{r}-\mathrm{J}) \mu_{1}}\right)\right]\right.\right. \\
\left.\left.\left.+\lambda_{2}\left[\sum_{\mathrm{k}_{2}=1}^{\infty} \sum_{\mathrm{s}=1}^{k_{2}} \mathrm{C}_{\mathrm{k}_{2}}{ }^{\mathrm{k}_{2}} \mathrm{C}_{\mathrm{s}}\right)(-1)^{\mathrm{s}}\left(\frac{1}{\mu_{2} \mathrm{k}_{2}}\right)\right]\right\}\right\}
\end{array}
$$

(26)
28)
under equilibrium is

$$
\begin{align*}
& \mathrm{U}_{2}=1-\mathrm{p} .0 \\
& =1-\exp \left\{\left[\lambda_{1} \sum_{k_{1}=1}^{\infty} \sum_{r=1}^{k_{1}} \sum_{J=0}^{r}(-1)^{3 r-J} C_{k_{1}}{ }^{k_{1}} C_{r}\right)\left({ }^{r} C_{J}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]  \tag{25}\\
& \left.+\lambda_{2}\left[\sum_{k_{2}=1}^{\infty} \sum_{s=1}^{k_{2}} C_{k_{2}}\left(\mu_{k_{2}} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} s}\right)\right]\right\}
\end{align*}
$$

$\mathrm{Thp}_{2}=\mu_{2} \cdot \mathrm{U}_{2}$

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$P\left(Z_{1}, Z_{2}\right)=\exp \left\{\left[\lambda_{1} \sum_{k_{1}=a_{1}}^{b_{1}} \sum_{1}=1 \sum_{j=0}^{k_{1}}(-1)^{r-r}\left(\frac{1}{b_{1}-a_{1}+1}\right)\left({ }^{k_{1}} C_{r}\right)\left({ }^{r} C_{1}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{J}\left(Z_{2}-1\right)^{J}\left(\left(Z_{1}-1\right)+\frac{\mu_{1}\left(Z_{2}-1\right)}{\mu_{2}-\mu_{1}}\right)^{(r-1)}\right.\right.$

$$
\begin{equation*}
\left.\left.\left.\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]+\lambda_{2}\left[\sum_{k_{2}=a_{2}}^{\mathrm{b}_{2}=1} \sum_{k_{2}}^{k_{2}}\left(\frac{1}{\mathrm{~b}_{2}-\mathrm{a}_{2}+1}\right) \mathrm{c}^{\mathrm{k}_{2}} C_{s}\right)\left(\mathrm{Z}_{2}-1\right)^{s}\left(\frac{1}{\mu_{2} \mathrm{~s}}\right)\right]\right\} \tag{32}
\end{equation*}
$$

The probability generating function of the first buffer size distribution is
$P\left(Z_{1}\right)=\exp \left[\lambda_{1} \sum_{k_{1}=a_{1}}^{b_{2}} \sum_{1}^{k_{1}}(-1)^{2 r}\left(\frac{1}{b_{1}-a_{1}+1}\right)^{k_{1}} C_{r}\left(Z_{1}-1\right)^{r}\left(\frac{1}{r \mu_{1}}\right)\right]$
The probability that the first buffer is empty is
$\mathrm{p}_{0}=\exp \left[\lambda_{1} \sum_{\mathrm{k}_{1}=\mathrm{a}_{1}}^{\mathrm{b}_{1}} \sum_{\mathrm{r}=1}^{\mathrm{k}_{1}}\left(\frac{1}{\mathrm{~b}_{1}-\mathrm{a}_{1}+1}\right)^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{r}}(-1)^{3 \mathrm{r}}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right]$
The mean number packets in the first buffer is
$\mathrm{L}_{1}=\frac{\lambda_{1}\left(\mathrm{a}_{1}+\mathrm{b}_{1}\right)}{2 \mu_{1}}$
The utilization of the first node is
$\mathrm{U}_{1}=1-\mathrm{p}_{0}$.
$=1-\exp \left[\lambda_{1} \sum_{k_{1}=a_{1}}^{b_{1}} \sum_{r=1}^{k_{1}}\left(\frac{1}{b_{1}-a_{1}+1}\right)^{k_{1}} C_{r}(-1)^{3 r}\left(\frac{1}{r \mu_{1}}\right)\right]$

The throughput of the first node is
$\operatorname{Thp}_{1}=\mu_{1} \cdot \mathrm{U}_{1}$
$=\mu_{1}\left\{1-\exp \left[\lambda_{1} \sum_{\mathrm{k}_{1}=\mathrm{a}_{1}}^{\mathrm{b}_{1}} \sum_{\mathrm{r}=1}^{\mathrm{k}_{1}}\left(\frac{1}{\mathrm{~b}_{1}-\mathrm{a}_{1}+1}\right)^{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{r}}(-1)^{3 \mathrm{r}}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right]\right\}$
The average delay in the first buffer is

$$
\begin{equation*}
\mathrm{W}\left(\mathrm{~N}_{1}\right)=\frac{\mathrm{L}_{1}}{\operatorname{Thp}_{1}}=\frac{\frac{\lambda_{1}\left(\mathrm{a}_{1}+\mathrm{b}_{1}\right)}{2 \mu_{1}}}{\mu_{1}\left\{1-\exp \left[\lambda_{1} \sum_{\mathrm{k}_{1}=\mathrm{a}_{1}}^{\mathrm{b}_{1}} \sum_{\mathrm{r}} \sum_{1}^{k_{1}}\left(\frac{1}{\mathrm{~b}_{1}-\mathrm{a}_{1}+1}\right) \mathrm{k}_{\mathrm{k}_{1}} \mathrm{C}_{\mathrm{r}}(-1)^{3 \mathrm{r}}\left(\frac{1}{\mathrm{r} \mu_{1}}\right)\right]\right\}} \tag{38}
\end{equation*}
$$

The variances of the number of packets in the first buffer is
$\operatorname{Var}\left(\mathrm{N}_{1}\right)=\mathrm{E}\left[\mathrm{N}_{1}^{2}-\mathrm{N}_{1}\right]+\mathrm{E}\left[\mathrm{N}_{1}\right]-\left(\mathrm{E}\left[\mathrm{N}_{1}\right]\right)^{2}$
$=\frac{\lambda_{1}}{2 \mu_{1}}\left[\sum_{k_{1}=a_{1}}^{b_{1}}\left(\frac{1}{b_{1}-a_{1}+1}\right) k_{1}\left(k_{1}-1\right)\right]+\frac{\lambda}{\mu_{1}}\left[\sum_{k_{1}=a_{1}}^{b_{1}}\left(\frac{1}{b_{1}-a_{1}+1}\right) k_{1}\right]$
The coefficient of variation of the number of packets in the first buffer is
$\operatorname{cv}\left(\mathrm{N}_{1}\right)=\frac{\sqrt{\mathrm{Var}\left(\mathrm{N}_{1}\right)}}{\mathrm{L}_{1}}$
The probability generating function of the second buffer size distribution is

$$
\begin{align*}
& P\left(Z_{2}\right)=\exp \left\{\left[\lambda_{1} \sum_{k_{1}=a_{1}, b_{r}}^{b_{1}} \sum_{1}^{k_{1}} \sum_{j=0}^{r}(-1)^{2 r-1}\left(\frac{1}{b_{1}-a_{1}+1}\right)\left({ }^{k_{1}} C_{r}\right)\left({ }^{r} C_{j}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(Z_{2}-1\right)^{r}\right.\right.  \tag{41}\\
& \left.\left.\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]+\lambda_{2}\left[\sum_{k_{2}=a_{2}}^{\mathrm{b}_{2}} \sum_{\mathrm{s}=1}^{k_{2}}\left(\frac{1}{\mathrm{~b}_{2}-\mathrm{a}_{2}+1}\right)\left(\mathrm{c}_{2} \mathrm{C}_{5}\right)\left(\mathrm{Z}_{2}-1\right)^{s}\left(\frac{1}{\mu_{2} \mathrm{~s}}\right)\right]\right\}
\end{align*}
$$

The probability that the second buffer is empty is

$$
\begin{aligned}
& \mathrm{p}_{.0}=\exp \left\{\left[\lambda_{1} \sum_{k_{1}=a_{1}=1}^{b_{1}} \sum_{1=1}^{k_{1}} \sum_{j=0}^{r}(-1)^{3 r-1}\left(\frac{1}{b_{1}-a_{1}+1}\right)\left(^{k_{1}} C_{r}\right)\left({ }^{(C} C_{j}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]\right. \\
& \left.+\lambda_{2}\left[\sum_{k_{2}=a_{2}=1}^{b_{2}} \sum_{1}^{k_{2}}\left(\frac{1}{b_{2}-a_{2}+1}\right)\left(\mathrm{c}_{2} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} \mathrm{~s}}\right)\right]\right\}
\end{aligned}
$$

The mean number packets in the second buffer is
$\mathrm{L}_{2}=\frac{\lambda_{1}\left(\mathrm{a}_{1}+\mathrm{b}_{1}\right)}{2 \mu_{2}}+\frac{\lambda_{2}\left(\mathrm{a}_{2}+\mathrm{b}_{2}\right)}{2 \mu_{2}}$

The utilization of the second node is
$\mathrm{U}_{2}=1-\mathrm{p} .0$
(44)

The throughput of second node is
$\operatorname{Thp}_{2}=\mu_{2} . \mathrm{U}_{2}$

(45)

The average delay in the second buffer is

(46)

The variances of the number of packets in the second buffer is
$\operatorname{Var}\left(\mathbf{N}_{2}\right)=\mathrm{E}\left[\mathrm{N}_{2}^{2}-\mathrm{N}_{2}\right]+\mathrm{E}\left[\mathrm{N}_{2}\right]-\left(\mathrm{E}\left[\mathrm{N}_{2}\right]\right)^{2}$
$=\lambda_{1}\left\{\left(\sum_{k_{1}=a_{1}}^{b_{1}}\left(\frac{1}{b_{1}-a_{1}+1}\right) \cdot k_{1}\left(k_{1}-1\right)\right)\left(\frac{\mu_{1}}{\mu_{1}-\mu_{2}}\right)^{2}\left[\left(\frac{1}{2 \mu_{1}}\right)-2\left(\frac{1}{\mu_{1}+\mu_{2}}\right)+\left(\frac{1}{2 \mu_{2}}\right)\right]\right\}$

$$
+\left\{\lambda_{2} \sum_{k_{2}=a_{2}}^{b_{2}}\left(\frac{1}{b_{2}-a_{2}+1}\right) k_{2}\left(k_{2}-1\right)\left(\frac{1}{2 \mu_{2}}\right)\right\}+\left\{\lambda_{1}\left(\frac{a_{1}+b_{1}}{2}\right)\left(\frac{1}{\mu_{2}}\right)\right\}
$$

$$
\begin{equation*}
+\left\{\frac{\lambda_{2}}{\mu_{2}}\left(\frac{\mathrm{a}_{2}+\mathrm{b}_{2}}{2}\right)\right\} \tag{47}
\end{equation*}
$$

The coefficient of variation of the number of packets in the second buffer is
$\operatorname{cv}\left(N_{2}\right)=\frac{\sqrt{\mathrm{Var}\left(\mathrm{N}_{2}\right)}}{\mathrm{L}_{2}}$
The probability that the network is empty is

$\left.\left.+\lambda_{2}\left[\sum_{k_{2}=a_{2}=1}^{b_{2}} \sum_{k_{2}}^{k_{2}}\left(\frac{1}{b_{2}-a_{2}+1}\right){ }^{k_{2}} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} s}\right)\right]\right\}$
The mean number packets in the network is
$\mathrm{L}_{\mathrm{N}}=\mathrm{L}_{1}+\mathrm{L}_{2}$
Where,

$$
\begin{aligned}
& =1-\exp \left\{\left[\lambda_{1} \sum_{k_{1}=1}^{b_{1}} \sum_{r=1}^{k_{1}} \sum_{1=0}^{r}(-1)^{3 r-5}\left(\frac{1}{b_{1}-a_{1}+1}\right)\left(^{k_{1}} C_{r}\right)\left(C_{j} C_{j}\right)\left(\frac{\mu_{1}}{\mu_{2}-\mu_{1}}\right)^{r}\left(\frac{1}{J \mu_{2}+(r-J) \mu_{1}}\right)\right]\right. \\
& \left.+\lambda_{2}\left[\sum_{k_{2}=2_{2}}^{b_{s}=1} \sum_{2}^{k_{2}}\left(\frac{1}{b_{2}-a_{2}+1}\right)\left({ }^{k_{2}} C_{s}\right)(-1)^{s}\left(\frac{1}{\mu_{2} s}\right)\right]\right\}
\end{aligned}
$$

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$\mathrm{L}_{1}=$ the mean number of packets in the first node
$L_{2}=$ the mean number of packets in the second node

## 5. Performance Evaluation of the Communication Network

The performance of the proposed network is discussed through numerical illustration. Different values of the parameters are considered for bandwidth allocation and arrival of packets. $\lambda_{1}$ and $\lambda_{2}$ are the message arrival rates at node 1 and node 2 respectively. The number of packets that can be converted into a message varies from 1 to 10 depending on the length of the message. The number of arrivals of packets to the buffers is in batches of random size. The batch size is assumed to follow uniform (rectangle) distribution with parameters $\left(a_{1}, b_{1}\right)$ and $\left(a_{2}, b_{2}\right)$ for first and second buffers respectively. $\mu_{1}$ is the transmission rate of node 1 which varies from $10 \times 10^{4}$ packets $/ \mathrm{sec}$ to $14 \times 10^{4}$ packets $/ \mathrm{sec}$. The packets leave the second node with a transmission rate of $\mu_{2}$ which varies from $16 \times 10^{4}$ packets/sec to $20 \times 10^{4}$ packets/sec. In both the nodes, dynamic bandwidth allocation is considered i.e. the transmission rate of each packet depends on the number of packets in the buffer connected to it at that instant.

The following set of values of the model parameters are considered in computing the performance measures like, Probabilities of emptiness of the network, first and second buffers, Mean number of packets in first and second buffers, Utilization of the nodes, Throughput of the nodes, Mean delays in first and second buffers and are given tables.
$a_{1}=1,2,3,4,5 ; b_{1}=6,7,8,9,10 ; a_{2}=1,2,3,4,5$;
$b_{2}=6,7,8,9,10 ; \lambda_{1}=0.5,1.5,2.0,2.5$ (with multiplication of $10^{4}$ messages $/ \mathrm{sec}$ ), $\lambda_{2}=0.5,1.5,2.0,2.5$ (with multiplication of
$10^{4}$ messages $/ \mathrm{sec}$ ), $\mu_{1}=10,11,12,13,14$ (with multiplication of $10^{4}$ packets $/ \mathrm{sec}$ ) and $\mu_{2}=16,17,18,19$, 20 (with multiplication of $10^{4}$ packets $/ \mathrm{sec}$ )
From equations (34), (42) and (49), the probability of network emptiness and buffers emptiness are computed for different values of $a_{1}, b_{1}, a_{2}, b_{2}, \lambda_{1}, \lambda_{2}, \mu_{1,} \mu_{2}$ observed that when the values of the network parameters $a_{1}, b_{1}, a_{2}, b_{2}, \lambda_{1}$ and $\lambda_{2}$ increase, there is a decrease in the emptiness of the network, first and the second buffers. The emptiness of the first buffer remains constant for an increase in the parameter values $\mathrm{a}_{2}, \mathrm{~b}_{2}$ and $\lambda_{2}$. When the transmission rates of first node $\left(\mu_{1}\right)$ and second node $\left(\mu_{2}\right)$ increase, the network and second buffer emptiness decrease.

From equations (35), (36), (43), 442) and (50), mean number of packets of the network are computed for different values of $\mathrm{a}_{1}, \mathrm{~b}_{1}$,
$a_{2}, b_{2}, \lambda_{1}, \lambda_{2}, \mu_{1}$ and $\mu_{2}$ and are given in Table 1. The relationship between mean number of packets in the network, buffers and the parameters $\mathrm{a}_{1}, \mathrm{~b}_{1}, \mathrm{a}_{2}, \mathrm{~b}_{2}, \lambda_{1}, \lambda_{2}, \mu_{1}$ and $\mu_{2}$ is shown in Figures2.

It is observed that when the values of network parameters $\mathrm{a}_{1}, \mathrm{~b}_{1}, \mathrm{a}_{2}, \mathrm{~b}_{2}, \lambda_{1}$, and $\lambda_{2}$ increase, the mean number of packets in the network, mean number of packets in the second buffer and the utilization of the second node increase. The mean number of packets in the first buffer and utilization of the first node increase when $a_{1}, b_{1}$ and $\lambda_{1}$ increase and remain constant when $a_{2}, b_{2}$ and $\lambda_{2}$ vary. When the transmission rate of node $1\left(\mu_{1}\right)$ varies from 10 to 14, the mean number of packets in the first buffer, utilization of the first and second

Table 1: Values of Mean Number of Packets, Average Delay and Throughput of Nodes

| $a_{1}$ | $\mathrm{b}_{1}$ | $a_{2}$ | $\mathrm{b}_{2}$ | $\lambda_{1}{ }^{\text {\# }}$ | $\lambda_{2}{ }^{\text {\# }}$ | $\mu_{1}{ }^{\text {S }}$ | $\mu_{2}{ }^{\text {S }}$ | $L_{1}$ | $L_{2}$ | $L_{N}$ | Thp ${ }_{1}$ | $W\left(N_{1}\right)$ | Thp $_{2}$ | $W\left(\mathrm{~N}_{2}\right)$ | Thp ${ }_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 5 | 10 | 1 | 2 | 5 | 20 | 0.7 | 0.925 | 1.625 | 1.55208 | 0.45101 | 6.76047 | 0.13682 | 1.55208 |
| 2 | 6 | 5 | 10 | 1 | 2 | 5 | 20 | 0.8 | 0.950 | 1.750 | 1.66845 | 0.47949 | 7.02045 | 0.13532 | 1.66845 |
| 3 | 6 | 5 | 10 | 1 | 2 | 5 | 20 | 0.9 | 0.975 | 1.875 | 1.75558 | 0.51265 | 7.25833 | 0.13433 | 1.75558 |
| 4 | 6 | 5 | 10 | 1 | 2 | 5 | 20 | 1.0 | 1.000 | 2.000 | 1.82600 | 0.54765 | 7.47674 | 0.13375 | 1.82600 |
| 5 | 6 | 5 | 10 | 1 | 2 | 5 | 20 | 1.1 | 1.025 | 2.125 | 1.88539 | 0.58343 | 7.67794 | 0.13350 | 1.88539 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.925 | 1.275 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |
| 1 | 7 | 5 | 10 | 1 | 2 | 10 | 20 | 0.40000 | 0.950 | 1.35 | 1.78256 | 0.22440 | 6.73770 | 0.14100 | 1.78256 |
| 1 | 8 | 5 | 10 | 1 | 2 | 10 | 20 | 0.45000 | 0.975 | 1.425 | 1.85971 | 0.24197 | 6.88857 | 0.14154 | 1.85971 |
| 1 | 9 | 5 | 10 | 1 | 2 | 10 | 20 | 0.50000 | 1.000 | 1.500 | 1.92918 | 0.25918 | 7.02588 | 0.14233 | 1.92918 |
| 1 | 10 | 5 | 10 | 1 | 2 | 10 | 20 | 0.55000 | 1.025 | 1.575 | 1.99234 | 0.27606 | 7.15145 | 0.14333 | 1.99234 |
| 1 | 6 | 1 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.725 | 1.075 | 1.69588 | 0.20638 | 6.00647 | 0.12070 | 1.69588 |
| 1 | 6 | 2 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.775 | 1.125 | 1.69588 | 0.20638 | 6.19517 | 0.12510 | 1.69588 |
| 1 | 6 | 3 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.825 | 1.175 | 1.69588 | 0.20638 | 6.34237 | 0.13008 | 1.69588 |
| 1 | 6 | 4 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.875 | 1.225 | 1.69588 | 0.20638 | 6.46503 | 0.13534 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.925 | 1.275 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |
| 1 | 6 | 5 | 6 | 1 | 2 | 10 | 20 | 0.35000 | 0.725 | 1.075 | 1.69588 | 0.20638 | 6.20764 | 0.11679 | 1.69588 |
| 1 | 6 | 5 | 7 | 1 | 2 | 10 | 20 | 0.35000 | 0.775 | 1.125 | 1.69588 | 0.20638 | 6.31124 | 0.12280 | 1.69588 |
| 1 | 6 | 5 | 8 | 1 | 2 | 10 | 20 | 0.35000 | 0.825 | 1.175 | 1.69588 | 0.20638 | 6.40529 | 0.12880 | 1.69588 |
| 1 | 6 | 5 | 9 | 1 | 2 | 10 | 20 | 0.35000 | 0.875 | 1.225 | 1.69588 | 0.20638 | 6.49147 | 0.13479 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 20 | 0.35000 | 0.925 | 1.275 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |
| 1 | 6 | 5 | 10 | 0.5 | 2 | 10 | 20 | 0.17500 | 0.83750 | 1.0125 | 0.88731 | 0.19723 | 5.63362 | 0.14866 | 0.88731 |
| 1 | 6 | 5 | 10 | 1.0 | 2 | 10 | 20 | 0.35000 | 0.92500 | 1.2750 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |
| 1 | 6 | 5 | 10 | 1.5 | 2 | 10 | 20 | 0.52500 | 1.01250 | 1.5375 | 2.43271 | 0.21581 | 7.44725 | 0.13596 | 2.43271 |
| 1 | 6 | 5 | 10 | 2.0 | 2 | 10 | 20 | 0.70000 | 1.10000 | 1.8000 | 3.10416 | 0.2255 | 8.26631 | 0.13307 | 3.10416 |
| 1 | 6 | 5 | 10 | 2.5 | 2 | 10 | 20 | 0.87500 | 1.18750 | 2.0625 | 3.71603 | 0.23547 | 9.03192 | 0.13148 | 3.71603 |
| 1 | 6 | 5 | 10 | 1 | 0.5 | 10 | 20 | 0.35000 | 0.36250 | 0.7125 | 1.69588 | 0.20638 | 3.63835 | 0.09963 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 1.0 | 10 | 20 | 0.35000 | 0.55000 | 0.9000 | 1.69588 | 0.20638 | 4.68093 | 0.11750 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 1.5 | 10 | 20 | 0.35000 | 0.73750 | 1.0875 | 1.69588 | 0.20638 | 5.65708 | 0.13037 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2.0 | 10 | 20 | 0.35000 | 0.92500 | 1.2750 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |


| 1 | 6 | 5 | 10 | 1 | 2.5 | 10 | 20 | 0.35000 | 1.11250 | 1.4625 | 1.69588 | 0.20638 | 7.42672 | 0.14980 | 1.69588 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 20 | 0.35 | 0.925 | 1.275 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 11 | 20 | 0.31818 | 0.925 | 1.24318 | 1.70984 | 0.18609 | 6.54281 | 0.14138 | 1.70984 |
| 1 | 6 | 5 | 10 | 1 | 2 | 12 | 20 | 0.29167 | 0.925 | 1.21667 | 1.72159 | 0.16942 | 6.51695 | 0.14194 | 1.72159 |
| 1 | 6 | 5 | 10 | 1 | 2 | 13 | 20 | 0.26923 | 0.925 | 1.19423 | 1.73162 | 0.15548 | 6.49316 | 0.14246 | 1.73162 |
| 1 | 6 | 5 | 10 | 1 | 2 | 14 | 20 | 0.25 | 0.925 | 1.175 | 1.74028 | 0.14366 | 6.47121 | 0.14294 | 1.74028 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 16 | 0.35 | 1.15625 | 1.50625 | 1.69588 | 0.20638 | 6.2151 | 0.18604 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 17 | 0.35 | 1.08824 | 1.43824 | 1.69588 | 0.20638 | 6.31524 | 0.17232 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 18 | 0.35 | 1.02778 | 1.37778 | 1.69588 | 0.20638 | 6.40731 | 0.16041 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 19 | 0.35 | 0.97368 | 1.32368 | 1.69588 | 0.20638 | 6.49229 | 0.14998 | 1.69588 |
| 1 | 6 | 5 | 10 | 1 | 2 | 10 | 20 | 0.35 | 0.925 | 1.275 | 1.69588 | 0.20638 | 6.57102 | 0.14077 | 1.69588 |

\# = Multiples of 10,000 messages/Second, \$ = Multiples of 10,000 Packets
nodes and the mean number of packets in the network decrease while the mean number of packets in the second buffer remain constant. As the transmission rate of node $2\left(\mu_{2}\right)$ varies from 16 to 20, the mean number of packets in the second node, utilization of the second node and the mean number of packets in the network decrease and the mean number of packets in the first buffer and utilization of the first node remain constant.
From equations (37), (38), (45) and (46), mean delays in the buffers and throughput of the nodes are computed for different values of $a_{1}, b_{1}, a_{2}, b_{2}, \lambda_{1}, \lambda_{2}, \mu_{1}$ and $\mu_{2}$ and are given in Table 1. The relationship between throughput of the nodes, mean delays in the buffers and the parameters $a_{1}, b_{1}, a_{2}, b_{2}, \lambda_{1}, \lambda_{2}, \mu_{1}$ and $\mu_{2}$ is shown in Figure 3 and Figure 4 respectively. From Table1, it is observed that when the batch size distribution parameters $a_{1}$ and $\mathrm{b}_{1}$ varies from 1 to 5 and 6 to 10 respectively, the throughput of the first and


Fig. 2 Relation between Mean number of Packets and various input Parameters
second nodes and mean delay in the first buffer increase and the mean delay in second buffer decrease. The throughput of the first node and mean delay in the first buffer remain constant and the throughput of the second node and mean delay in the second buffer increase when the batch size distribution parameters $\mathrm{a}_{2}$ and
$\mathrm{b}_{2}$ increase from 1 to 5 and 6 to 10 respectively. Similarly, when the message arrival rate $\lambda_{1}$ varies from $0.5 \times 10^{4}$ messages/sec to $2.5 \times 10^{4}$ messages, the throughput of first and second nodes and also the mean delay in first buffer increase.


Fig. 3 Relation between Throughput of the nodes and various input Parameters

When the message arrival rate $\lambda_{2}$ varies from $0.5 \times 10^{4}$ messages/sec to $2.5 \times 10^{4}$ messages, the throughput of the first node and mean delay in first buffer remain constant while the throughput of the second node and mean delay in the second buffer increase. Similarly, the impact of variation in other parameters on throughput and mean delay can be observed from the Table 1.


Sensitivity analysis of the network model is performed with respect to the parameters $\mathrm{a}_{1}, \mathrm{~b}_{1}, \mathrm{a}_{2}, \mathrm{~b}_{2}, \lambda_{1}, \lambda_{2}, \mu_{1}$ and $\mu_{2}$ on the mean number packets in the first and second buffers, the mean delays in the first and second buffers and also throughput of the first and second nodes. The computed values of the performance measures are given in Table 2. The following data has been considered for the sensitivity analysis.
$a_{1}=5, a_{2}=5, b_{1}=10, b_{2}=10, \lambda_{1}=1 \times 10^{4}$ messages $/ \mathrm{sec}$, $\lambda_{2}=2 \times 10^{4}$ messages $/ \mathrm{sec} \mu_{1}=10 \times 10^{4}$ packets $/ \mathrm{sec}$, $\mu_{2}=20 \times 10^{4}$ packets/sec.

The performance measures of the model are computed with variation of $-15 \%,-10 \%, 0 \%,+5 \%,+10 \%$ and $+15 \%$ on the input parameters $\lambda_{1}, \lambda_{2}, \mu_{1}$ and $\mu_{2}$. A variation of $60 \%,-40 \%,-20 \%, 0 \%,+20 \%,+40 \%$ and $+60 \%$ on the batch size distribution parameters $a_{1}$ and $a_{2}$ and $-30 \%$, $20 \%,-10 \%, 0 \%,+10 \%,+20 \%$ and $+30 \% b_{2}$ to retain them as integers. As $\lambda_{1}$ increases to $15 \%$, the average number of packets in the two buffers increasing, the the average delay in the first buffer is increasing and the the average delay in the second buffer is decreasing. Similarly, as the batch size distribution parameter $a_{1}$ increase by $60 \%$, the average number of

Fig. 4 Relation between Mean delays in the buffers and various input Parameters

## 6. Sensitivity Analysis

Table 2: Sensitivity Analysis

| Parameter | Perform ance Measure $S$ | \% of Change in Parameters |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | -15 | -10 | -5 | 0 | +5 | +10 | +15 |
| $\lambda_{1}(=1)$ | $\mathrm{L}_{1}$ | 0.595 | 0.63 | 0.665 | 0.7 | 0.735 | 0.77 | 0.805 |
|  | $\mathrm{L}_{2}$ | 0.89875 | 0.9075 | 0.91625 | 0.925 | 0.93375 | 0.9425 | 0.95125 |
|  | $\mathrm{Thp}_{1}$ | 1.35441 | 1.42152 | 1.48741 | 1.55208 | 1.61556 | 1.67788 | 1.73904 |
|  | $\mathrm{Thp}_{2}$ | 6.46091 | 6.56151 | 6.66136 | 6.76047 | 6.85884 | 6.95648 | 7.0534 |
|  | $\mathrm{W}\left(\mathrm{N}_{1}\right)$ | 0.43931 | 0.44319 | 0.44709 | 0.45101 | 0.45495 | 0.45891 | 0.4629 |
|  | $\mathrm{W}\left(\mathrm{N}_{2}\right)$ | 0.13911 | 0.13831 | 0.13755 | 0.13682 | 0.13614 | 0.13549 | 0.13486 |
| $\lambda_{2}(=2)$ | $\mathrm{L}_{1}$ | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 |
|  | $\mathrm{L}_{2}$ | 0.8125 | 0.85 | 0.8875 | 0.925 | 0.9625 | 1 | 1.0375 |
|  | $\mathrm{Thp}_{1}$ | 1.69588 | 1.69588 | 1.69588 | 1.69588 | 1.69588 | 1.69588 | 1.69588 |
|  | $\mathrm{Thp}_{2}$ | 6.02989 | 6.21265 | 6.39301 | 6.57102 | 6.7467 | 6.92008 | 7.09119 |
|  | $\mathrm{W}\left(\mathrm{N}_{1}\right)$ | 0.20638 | 0.20638 | 0.20638 | 0.20638 | 0.20638 | 0.20638 | 0.20638 |
|  | $\mathrm{W}\left(\mathrm{N}_{2}\right)$ | 0.13475 | 0.13682 | 0.13882 | 0.14077 | 0.14266 | 0.14451 | 0.14631 |
| $\mu_{1}(=10)$ | $\mathrm{L}_{1}$ | 0.41176 | 0.38889 | 0.36842 | 0.35 | 0.33333 | 0.31818 | 0.30435 |
|  | $\mathrm{L}_{2}$ | 0.925 | 0.925 | 0.925 | 0.925 | 0.925 | 0.925 | 0.925 |
|  | $\mathrm{Thp}_{1}$ | 1.66922 | 1.67903 | 1.68787 | 1.69588 | 1.70317 | 1.70984 | 1.71596 |
|  | $\mathrm{Thp}_{2}$ | 6.61841 | 6.60187 | 6.58609 | 6.57102 | 6.55661 | 6.54281 | 6.52961 |
|  | $\mathrm{W}\left(\mathrm{N}_{1}\right)$ | 0.24668 | 0.23162 | 0.21828 | 0.20638 | 0.19571 | 0.18609 | 0.17736 |
|  | $\mathrm{W}\left(\mathrm{N}_{2}\right)$ | 0.13976 | 0.14011 | 0.14045 | 0.14077 | 0.14108 | 0.14138 | 0.14166 |
| $\mu_{2}(=20)$ | $\mathrm{L}_{1}$ | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 |
|  | $\mathrm{L}_{2}$ | 1.08824 | 1.02778 | 0.97368 | 0.925 | 0.88095 | 0.84091 | 0.80435 |
|  | $\mathrm{Thp}_{1}$ | 1.69588 | 1.69588 | 1.69588 | 1.69588 | 1.69588 | 1.69588 | 1.69588 |
|  | $\mathrm{Thp}_{2}$ | 6.31524 | 6.40731 | 6.49229 | 6.57102 | 6.64418 | 6.71238 | 6.77613 |
|  | $\mathrm{W}\left(\mathrm{N}_{1}\right)$ | 0.20638 | 0.20638 | 0.20638 | 0.20638 | 0.20638 | 0.20638 | 0.20638 |
|  | $\mathrm{W}\left(\mathrm{N}_{2}\right)$ | 0.17232 | 0.16041 | 0.14998 | 0.14077 | 0.13259 | 0.12528 | 0.1187 |
| Parameter | Performan ce <br> Measures | \% Change in Parameters |  |  |  |  |  |  |
|  |  | -60 | -40 | -20 | 0 | +20 | +40 | +60 |
|  | $\mathrm{L}_{1}$ | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
|  | $\mathrm{L}_{2}$ | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 | 1.2 |


packets in the two buffers increasing and the the average delay in the first buffer is increasing. Overall analysis of the parameters reflects that dynamic bandwidth allocation strategy for congestion control tremendously reduces the delays in communication and improves quality of service by reducing burstiness in buffers.

## 7. Conclusions

In this paper, we developed and analyzed a two node Communication network model with Dynamic Bandwidth

Allocation (DBA) for bulk arrivals at both buffers. The statistical multiplexing of the Communication network is developed by characterizing the arrivals at both buffers connected to the two nodes in tandem as compound Poisson processes and transmission times with Poisson processes. This representation accurately matches the arrival and service process at Internet and Telecommunication processes. Using the Chapman Kulmogrov transition equations, the joint probability generating function of the number of packets in each buffer is derived. The behavior of the network is analyzed by obtaining the system performance measures for any general bulk size arrival distribution and in particular uniformly distributed bulk arrivals of packets. It is observed that the bulk size arrival distribution parameters are significantly influencing the congestion, mean delays in buffers and throughput of the transmitters. The sensitivity analysis through the numerical studies reveals that the DBA strategy can reduce the burstness in buffers and improves the quality of service (QoS). This numerical model also includes several of the earlier Communication network models as particular cases for specific values of the input parameters. This Communication network model is much useful in analyzing the performance of several communication networks at Tele and Satellite communications, Computer communications, ATM scheduling, Bandwidth allocation etc. It is also possible to extend this network with nonMarchovian transmission times and priority structures which requires further investigation.

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