Optimization of LSE and LMMSE Channel Estimation Algorithms based on CIR Samples and Channel Taps

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Abstract-- For spectrally efficient transmission over time-varying channels, the use of Adaptive Coding and Modulation (AMC) in wireless OFDM systems requires the estimation of radio channel at the receiver. This paper focuses on the use of time domain channel statistics, mainly concentrating on two schemes: Linear Minimum Mean Square Estimation (LMMSE) and Least Square Estimation (LSE) and their variants. LMMSE performs better than LSE but at the cost of computational complexity. The performance of LSE can be improved by increasing CIR samples and channel taps. To avoid the matrix inversion lemma, the channel matrix can be downsampled or regularized. Theoretical analysis and computer simulations are used for performance and complexity comparisons.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM), a multicarrier modulation method, is considered an essential technique for a variety of high data rate communication systems like 4G WiMAX and LTE-Advanced due to its efficient management of ISI in frequency selective fading channels. OFDM can also be used as a modulation technique because of the simple equalizer design and spectrum efficiency. The combination of OFDM with Multi-Input Multi-Output (MIMO) provides the increased data rate and improved quality of service. That is why MIMO-OFDM is adopted in B3G (Beyond 3rd Generation) mobile communication systems.

Coherent OFDM, which has 3-4 dB performance gain more than non-coherent OFDM, requires channel state information (CSI) at the receiver and/or transmitter.CSI only at the transmitter is usually preferred to make the receiver design simple. Data throughput of channel depends on the quality of the channel estimator. For channel estimation there are mainly two methods proposed as, first is decision directed channel estimation and other one is pilot-assisted channel estimation. In decision directed method, the modulation is removed from subcarriers using the previously demodulated symbol, thus all subcarriers can be used for channel estimation. This method requires a large amount of data and its convergence rate is also very slow, that is why it is not well suited for real time systems. In pilot assisted method there are two modes, if all subcarriers have known pilots then it is called block pilot mode while in comb pilot mode only a few subcarriers carry known pilots.

Channel can be estimated in time domain or frequency domain. In frequency domain two algorithms are proposed Least Square Estimation (LSE) and Linear Minimum Mean Square Estimation (LMMSE). LSE algorithm is relatively easy to implement due to its less complexity and it also does not require any channel apriority probability. To achieve better performance LMMSE is proposed. LMMSE is optimum in minimizing Mean Square Error (MSE) as it uses addition information of operating SNR and the channel statistics. But it complexity is higher due to the channel correlation and the matrix inversion lemma. There can be a compromise of complexity and performance by taking the effect of the channel taps and channel impulse response (CIR) samples. By assuming the impulse response of finite length, these two algorithms can be modified having less complexity. In mobile wireless links the channel statistics are not known, in these cases it is robust to consider the uniform Power delay profile (PDP), which also reduces complexity than LMMSE. The complexity of LSE can be reduced by regularizing the Eigen values of the matrix being inverted or by down-sampling the channel vector.

The rest of the paper is organized as: Section 2 describes OFDM signal and channel model, in Section 3, LMMSE, LSE and their different variants are discussed, followed by the simulation results in Section 4 and in the last section conclusions are drawn.

II. OFDM SIGNAL AND CHANNEL MODEL

In OFDM, the transmitted bit stream is divided into many different sub-streams and send them over many orthogonal sub-channels. Suppose the transmitted data at k-th subcarrier is $d(k)$. Then the multicarrier modulated signal will be

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} d(k)\, e^{\frac{j2\pi k}{N}}, \quad n = 0,1,2,\dots,N-1$$

Where N is total number of sub-carriers. Before transmitting x(n), guard interval (GI) is inserted to avoid Inter-symbol interference (ISI) and inter-carrier interference (ICI). This signal is then passed through a time-varying multipath channel whose impulse response is characterized by

$$g(t,\tau) = \sum\nolimits_{i=0}^{L-1} \alpha_i \delta(t - \tau_i)$$

where L is total number of multi-paths and $\{\alpha_i\}$ is a complex Gaussian random variable of zero mean having a power delay profile: $Ce^{\tau_i/\tau_{rms}}$. $\{\tau_i\}$ represents time delay between different multi-paths, whose maximum value is not supposed to exceed the guard interval length.

After passing this fading channel and removing GI, the received OFDM signal in frequency domain will be

$$Y = HX + W$$

W is the complex-valued additive Gaussian noise having zero mean and σ^2variance. H is the channel frequency response, that is DFT of the channel impulse response $g(t,\tau)$.

III. CHANNEL ESTIMATION ALGORITHMS

A. LMMSE Channel Estimation

In presence of channel noise**,** LMMSE estimation of the uncorrelated Gaussian channel vector g is given by [1]

$$\hat{g} = \boldsymbol{\Gamma}_{gy}\boldsymbol{\Gamma}_{yy}^{-1} y$$

Where

$$\boldsymbol{\Gamma}_{gy} = \boldsymbol{\Gamma}_{gg} F^H X^H$$

$$\boldsymbol{\Gamma}_{yy} = XF\boldsymbol{\Gamma}_{gg} F^H X^H + \sigma_n^2 I_N$$

$\boldsymbol{\Gamma}_{yy}$ is the auto-covariance matrix of y and $\boldsymbol{\Gamma}_{gy}$ is the cross co-variance matrix between g and y. σ_n^2 is variance of noise. For unique minimum MSE, these co-variance matrices should be positive definite,

In frequency domain the channel estimate $\hat{h}_{mmse}$ is given by

$$\hat{h}_{mmse} = F\hat{g} = FQF^H X^H y$$

Where F is orthonormal DFT-matrix and Q is given by [1]

$$Q = \boldsymbol{\Gamma}_{gg}\left[(F^H X^H XF)^{-1}\sigma_n^2 + \boldsymbol{\Gamma}_{gg}\right]^{-1}(F^H X^H XF)^{-1}$$

B. Modified LMMSE Channel Estimation

For large N the calculation of Q matrix implies high complexity. To reduce the size of Q, we can take only first L taps having significant energy. Using this approximation $\boldsymbol{\Gamma}_{gg}$ is reduced to L × L matrix. So modified LMMSE estimation becomes [1]

$$\hat{h}_{mmse} = TQ'T^H X^H y$$

Where T have only first L columns of DFT matrix and Q' is

$$Q' = \boldsymbol{\Gamma}'_{gg}\left[(T^H X^H XT)^{-1}\sigma_n^2 + \boldsymbol{\Gamma}'_{gg}\right]^{-1}(T^H X^H XT)^{-1}$$

$\boldsymbol{\Gamma}'_{gg}$ denotes the upper left L × L matrix of $\boldsymbol{\Gamma}_{gg}$.

C. Low Complex LMMSE Channel Estimation

In LMMSE channel estimation, a matrix inversion is needed as the input data X is changed which results in high complexity. This complexity can be reduced by averaging the transmitted data x i.e. $E(XX^H)^{-1}$. If we assume same signal constellation for all frequencies, then

$E(XX^H)^{-1} = E\left|\frac{1}{x_k}\right|^2$.

The simplified LMMSE estimation will be [2]

$$\hat{h}_{mmse} = \mathbf{\Gamma}_{gg}(\mathbf{\Gamma}_{gg} + \frac{\beta}{SNR}I)^{-1}X^{-1}y$$

Where β depends upon the signal constellation.

D. Robust LMMSE Channel Estimation

In mobile wireless links, the channel changes with time depending on the particular environment. It is not possible to know the channel PDP at the design time [3]. Identical MSE performance can be obtained for all PDPs with same maximum delay. So it is robust to design the channel co-variance matrix with a uniform PDP [4].

E. LSE Channel Estimation

A prior knowledge of second order channel statistics is required for LMMSE estimator, which is not possible in many practical situations. We can design an estimator filter which is a function of available data only [5]. In LSE estimation, we use only signal model, no probabilistic assumptions are required.

LSE estimation of channel is given by

$$\hat{h}_{ls} = FQ_{ls}F^HX^Hy$$

where

$$Q_{ls} = (F^HX^HXF)^{-1}$$

$\hat{h}_{ls}$ can also be written as [1]

$$\hat{h}_{ls} = X^{-1}y$$

F. Modified LSE Channel Estimation

Though no modification are needed because of less complexity of LSE estimator but performance can be improved by considering only first L high energy channel taps. The modified LSE estimator becomes

$$\hat{h}_{ls} = TQ_{ls}^{'}T^HX^Hy$$

where

$$Q_{ls}^{'} = (T^HX^HXT)^{-1}$$

G. Regularized LSE Channel Estimation

The problem of inversion of $N \times N$ matrix can be solved by regularizing the Eigen values of the matrix by adding a constant term to the diagonal elements. In this case, the matrix Q_{ls} will be [6]

$$Q_{reg,ls} = (\alpha I + F^HX^HXF)^{-1}$$

Where off-line constant α is chosen such that the matrix $Q_{reg,ls}$ is least perturbed.

H. Down-Sampled Impulse Response LSE Channel Estimation

The inversion of $N \times N$ matrix can be simplified by decreasing the sampling frequency, but ensuring the absence of aliasing. Only 2 out of 3 channel taps are used and the discarded taps are set to zero.

The down-sampled version of channel vector g can be [6]

$$\bar{g} = (g_0 \;\; g_1 \;\; 0 \;\; g_3 \;\; g_4 \;\; 0 \;\; \dots \;\; g_{L-1})^T$$

The channel transfer function can be written as

$$H^{DS} = F\bar{g}$$

Which is equivalent to

H^{DS}

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & w^1 & w^3 & \dots & w^{(L-1)} \\ 1 & w^2 & w^6 & \dots & w^{2(L-1)} \\ 1 & w^3 & w^9 & \dots & w^{3(L-1)} \\ 1 & w^4 & w^{12} & \dots & w^{4(L-1)} \\ 1 & w^5 & w^{15} & \dots & w^{5(L-1)} \\ 1 & \dots & \dots & \dots & \dots \\ 1 & w^{N-1} & w^{3(N-1)} & \dots & w^{(N-1)(L-1)} \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_3 \\ g_4 \\ \vdots \\ g_{L-1} \end{bmatrix}$$

The estimated channel in this case will be

$$\hat{h}_{DS} = (F^{DS,H}X^HXF^{DS})^{-1}F^{DS,H}X^Hy$$

IV. SIMULATION RESULTS

To demonstrate the effectiveness of the discussed algorithms, Matlab Simulations are provided in this section. All simulations have been performed for OFDM signal in Rayleigh Fading Channel with BPSK modulation scheme and FFT size is kept 64. To illustrate the performance of the estimators, the widely used Mean Square Error (MSE) has been used as a function of SNR, Channel Taps and Channel Impulse Response (CIR) samples. The complexity of the estimators is compared in terms of computational time.

a. Comparison of LMMSE Channel Estimators

The performance of LMMSE with its variants i.e. Modified LMMSE with 10 taps, 40 taps, Robust LMMSE and Low Complex LMMSE is shown in Fig.1. The difference between LMMSE and Modified LMMSE estimators is due to the fact that some parts of the channel statistics are not taken into account in the former estimators. For low SNR values, the performance of LMMSE is better than R.LMMSE but for higher SNRs R.LMMSE outperforms LMMSE. The performance of both LMMSE and Low Complex LMMSE is same and the difference lies in the complexity as the computational time of Low Complex LMMSE is less than that of LMMSE. The comparison of computational time of LMMSE estimators is given in Table 1. Table 1 indicates that there is a wide gap of time between LMMSE while using covariance matrix and correlation matrix.

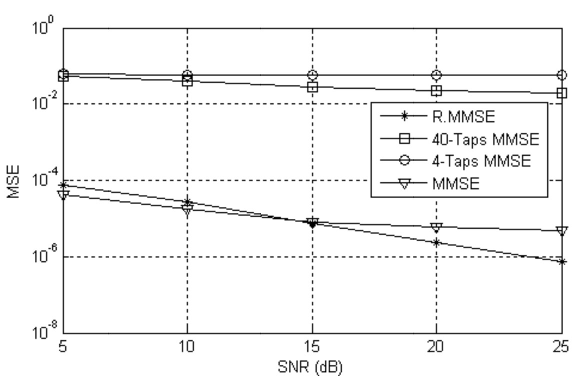

Figure 1. MSE v/s SNR for LMMSE Estimators

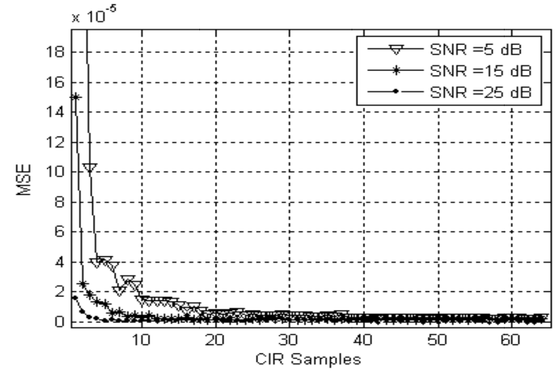

Figure 3. MSE v/s CIR Samples for LMMSE Estimator

TABLE 1 COMPUTATIONAL TIME FOR LMMSE ESTIMATORS

Estimator	5000 Simulatio (sec	1 OFDM (mSec	1 Bit (mSec)
LMMSE Modified-10	208.278		0.651
Low Complex LMMSE	320.713		1.003
LMMSE (Corr Mtx)	346.8		1.084
LMMSE Modified-40	440.945		1.378
R.LMMSE	528.133		1.651
LMMSE (Cov Mtx)	529.319	105.864	1.65

TABLE 2 TIME V/S CIR SAMPLES FOR LMMSE ESTIMATOR

CIR Samples	Time (mSec)
30	1
40	1.25
50	1.5
60	1.75

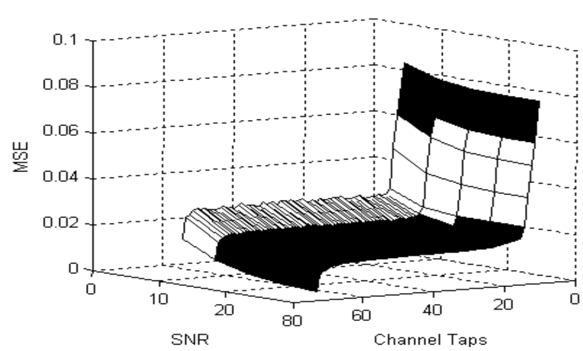

Figure 4. MSE v/s SNR v/s Channel Taps for Modified LMMSE Estimator

TABLE 3 TIME V/S CHANNEL TAPS FOR MODIFIED LMMSE ESTIMATOR

Channel Taps	Time (mSec)
30	5
40	6
50	10
60	12

The performance of LMMSE estimator in terms of CIR samples for different values of SNR is shown in Fig.3. As we notice that after a certain number of CIR samples we have the same MSE for all values of SNR. The effect of increasing CIR samples on time is shown in Table 2.

The effect of channel taps and SNR on MSE is shown in Fig.4. By increasing channel taps up to 10, there is a significant improvement in MSE but from 10 to 60, the MSE behavior remains same and after 60 we get further improvement. Since there is no improvement in MSE by increasing channel taps from 10 up to 60 as the disadvantage only comes in form of more time of computation as shown in Table 3.

b. Comparison of LSE Channel Estimators

Fig.5 shows the MSE verus SNR for LSE, Modified LS, Regularized LS and Downsampled LS estimators. Contrary to the modification of LMMSE estimator, the modification of LS estimator reduces MSE for a range of SNRs. However the same approximation effect, as in the modified LMMSE estimators, shows up at high SNRs. For every SNR, there exists an estimator which gives the smallest MSE. The effect of regularized LS is same to LSE but at higher SNR the performance of regularized LS degrades. Downsampled LS is exaclty same to that of LSE, advantage of former is only less complexity. The effect of CIR samples on MSE of LS estimator is shown in Fig.6. For CIR samples 0 to 10, there is a rapid improvement in performance specially at low SNRs, but by increasing samples further there is no further improvement in terms of MSE but the cost comes in more computational complexity that is shown in Table 4. It is clear from Table 4 that by increasing number of samples, there is a gradual increment in computational time, that is a drawback of increasing samples without improving performance. The effect of CIR samples and SNR on MSE is shown in Fig.7. The combined effect of channel taps and SNR on MSE is shown in Fig.8. For specific channel taps, the effect of CIR samples on MSE is demonstrated in Fig.9. By increasing samples from1 to 2, there is a dominant improvement in MSE but beyond this value of samples the performance satuarates. The effect of channel taps for certain values of CIR samples on MSE is shown in Fig.10.

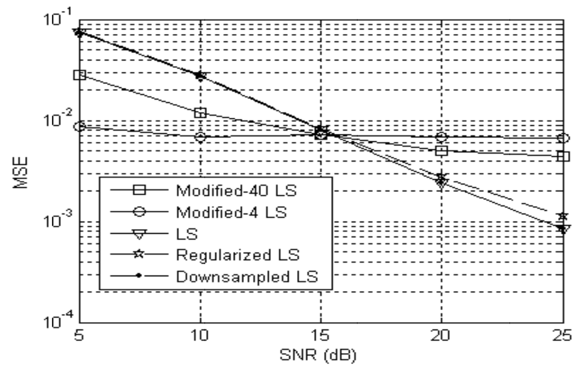

Figure 5. MSE v/s SNR for LS Estimators

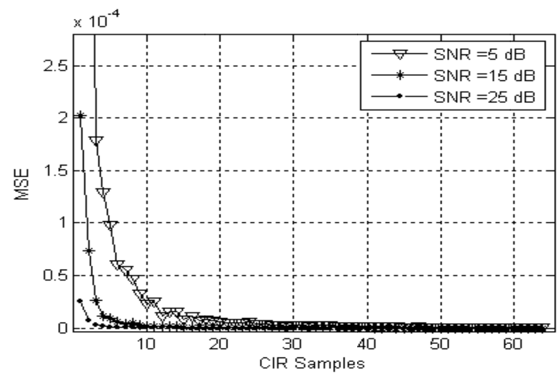

Figure 6. MSE v/s CIR Samples for LS Estimator

TABLE 4 TIME V/S CIR SAMPLES FOR LS ESTIMATOR

CIR Samples	Time (mSec)
30	0.5
40	1
50	1.25
60	1.5

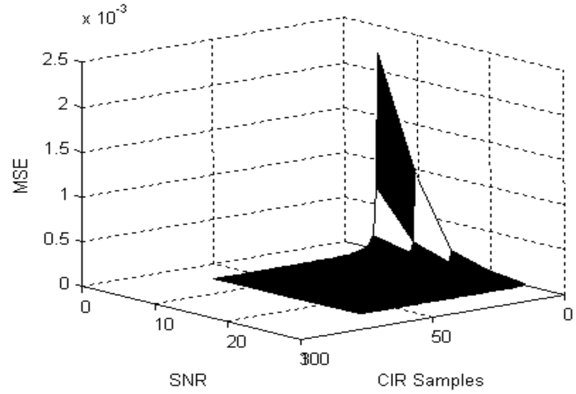

Figure 7. MSE v/s SNR v/s CIR Samples for LS Estimator

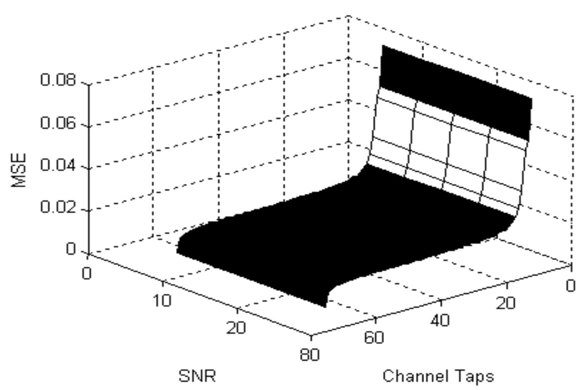

Figure 8. MSE v/s SNR v/s Channel Taps for Modified LS Estimator

The different downsampling rate versus corresponding MSE is shown in Fig.11. By increasing the downsampling rate, the performance is degraded while there is no significant effect on complexity.

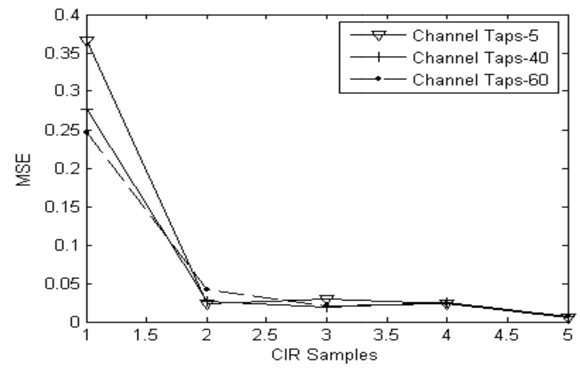

Figure 9. MSE v/s CIR Samples for Modified LS Estimator

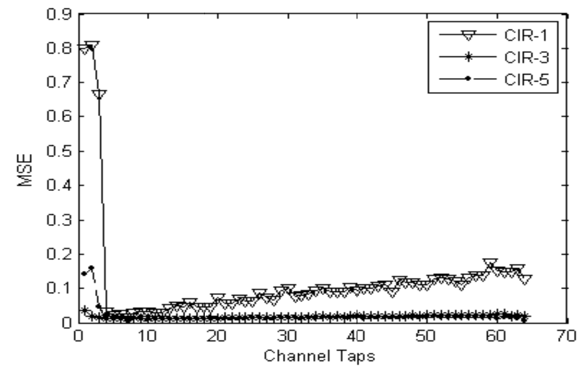

Figure 10. MSE v/s Channel Taps for Modified LS Estimator

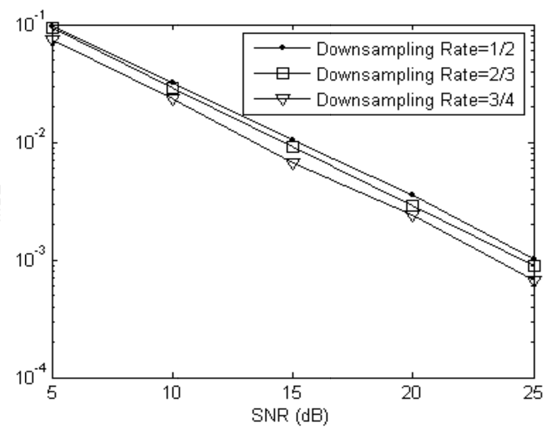

Figure 11. MSE v/s SNR for Downsampled LS Estimators

c- Comparison of LSE and LMMSE Channel Estimators

The performance comparison between LSE and LMMSE estimator is shown in Fig.12. When the channel has less number of CIR samples, then LMMSE is better to use than LSE due to less MSE, not in terms of time. But as CIR samples increases, for lower SNR values LMMSE is better in terms of MSE than LSE but for higher SNR values later one is better to use. But if we increase CIR samples further, then after certain number of CIR samples, LSE outperforms LMMSE for whole range of SNR values. The computation of both LSE and LMMSE with the increasing number of CIR samples is shown in Table 5. It is evident from Table 5 that LSE takes always less time than LMMSE, as it does not account for the channel statistics.

TABLE 5 TIME V/S CIR SAMPLES FOR LMMSE AND LS ESTIMATOR

CIR Samples	**Time (mSec)**	
	LS	***LMMSE***
30	0.5	1
40	1	1.25
50	1.25	1.5
60	1.5	1.75

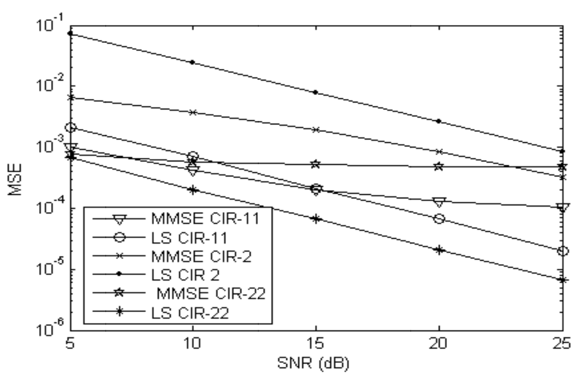

Figure 12. MSE v/s SNR for LMMSE and LS Estimators with different CIR Samples

V. CONCLUSIONS

In this paper we present LMMSE and LSE channel estimators based on CIR samples and channel taps and evaluated their comparison in terms of performance and complexity. The performance of LMMSE is better than LSE as it assumes the channel statistics which results in high complexity.The performance can be improved by increasing either CIR samples or channel taps but after a certain limit there is no prominent impact on performance while the complexity goes on increasing. . As we go on increasing CIR samples, after a certain value LSE degrades LMMSE both in performance and copmplexity. We also noticed that the channel taps have no effect on the performance of LSE estimator for different SNR values. So if we use a channel filter of more length then we can improve the channel estimator performance even without having a prior channel information.

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