# On Comparison of DFT-Based and DCT-Based Channel Estimation for OFDM System

**Saqib Saleem[1], Qamar-ul-Islam**
**Department of Communication System Engineering**
**Institute of Space Technology**
**Islamabad, Pakistan**

### Abstract

For high data rate communication with the required Quality of Service (QoS) in 3G and 4G systems, Orthogonal Frequency Division Multiplexing (OFDM) is proposed, which is capable to resist the channel impairments caused by high mobility conditions, by dividing the frequency-selective fading channel into narrowband flat fading channels. In this paper two time-domain channel estimation techniques, Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT), are compared based on the time-domain channel impulse response (CIR) energy characteristics and they have less complexity and efficient performance than Linear Minimum Mean Square Error (LMMSE) and Least Square Error (LSE). The effect of power limitation in terms of SNR and the number of multipaths for a wireless channel is determined to compare these transform approaches. Two well known performance criteria: Mean Square Error (MSE) and Symbol Error Rate (SER) are used for comparison by using Monte Carlo Simulations for Quadrature Phase Shift Keying (QPSK) modulation.

*Keywords*: *DFT, DCT, LSE, LMMSE, OFDM, Most Significant Taps (MST), CFR, CIR*

## 1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM), which allows the overlapping of the subcarriers but keeps them orthogonal to avoid inter-carrier interference (ICI) and inter-symbol interference (ISI) [1], has been adopted for mobile communication standards due to its ability to combat with the frequency-selective multipath fading channel effects and has high spectral efficiency.

For next generation wireless systems which are specially designed for audio and video processing, the fundamental demand is of the high throughput while maintaining the reliable communication, which can be made possible by the integration of the error-correcting codes, space-time coding (STC) and transmit diversity techniques [2]. For all these operations channel state information (CSI) is required, for which we have two options: Blind estimation or non-blind estimation. In practical wireless systems non-blind channel estimation is preferred due to its dependence on the transmitted data and the previous channel states. For estimation of all sub-channels, training sequences can be added in two modes: Block mode and Comb mode. Later one is preferred due to the presence of Doppler Spread Effect [2].

In frequency domain, channel can be estimated by either Least Square Error (LSE) approach or Linear Minimum Mean Square Error (LMMSE) approach. LSE has less complexity because it does not require channel statistics but the performance is degraded which can be improved by using LMMSE, having more complexity as it utilizes the autocorrelation matrix and the noise variance of the channel. This high computation required by LMMSE can be reduced by many approaches as discussed in [3]. The performance of the low complex LSE can be improved by using a channel filter of more CIR samples or by increasing the multi-path channel taps, as proposed in [3].

Instead of frequency domain, channel can be estimated in time-domain by DFT-based approach, whose performance is better and complexity is less than LSE and LMMSE. The performance can be improved further by making a suitable selection of CIR samples and channel taps by using the Most Significant Taps (MST) method. In this method, the estimated channel in frequency-domain is converted to time-domain by using IDFT. Then this estimated CIR is passed through MST to suppress the noise by discarding certain CIR. The remaining significant CIR is transformed back to frequency domain by DFT, thus improving the performance than LMMSE and reduced complexity due to the presence of fast algorithms FFT and IFFT.

The performance of DFT-based approach degrades in case of non-integer spaced multipath delays due to the presence of the dispersed CIR. To avoid this

problem, new approach has been proposed, named as DCT/EIDCT. In this method, DCT is applied to get the channel response in transform domain, instead of DFT.

The rest of the paper is organized such that in section 2, OFDM system model is described, in section 3 channel estimation algorithms are given along with their simulations in section 4. Conclusions are drawn in last section of the paper.

## 2. System Model

Suppose an OFDM system is transmitting data over N subcarriers, where only $N_D + 1$ subcarriers are carrying useful information while others are used for guard band. On each subcarrier, data $D_{i,n}$ is being transmitted, where $i$ is the OFDM symbol number and $n$ is the subcarrier number. The transmitted signal can be represented as [4]

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{n=-^{N_D}/_2}^{^{N_D}/_2} D_{i,n}\psi_{i,n}(t) \otimes g(t) \qquad (1)$$

Where $g(t)$ is channel impulse response and $\psi_{i,n}(t)$ is the pulse shaping filter used for subcarriers, described by [4]

$$\psi_{i,n}(t) = \begin{cases} e^{j2\pi\left(\frac{n}{T_u}\right)(t-\Delta-iT_s)} & iT_s \leq t < (i+1)T_s \\ 0 & otherwise \end{cases} \qquad (2)$$

$\Delta$ is length of guard interval (GI) and $1/T_u$ is the subcarrier's spacing so $T_s = T_u + \Delta$ is the OFDM symbol duration.
OFDM data is passed through a wireless channel described by the following impulse response [5]

$$g(t,\tau) = \sum_{i=1}^{L} \gamma_i(t) c(\tau - \tau_i) \qquad (3)$$

Where $\gamma_i(t)$ are multipath complex gains, which are wide-sense stationary (WSS) complex Gaussian Processes, limited to Doppler Frequency $f_D$ and $\tau_i$ are multipath delays, which are uncorrelated to each other and $L$ is the number of multipaths.
In wireless channel $c(t)$, the pulse shaping, is normally described by having a square-root raised cosine filter's spectrum. After passing through the channel, the received signal in time domain will be

$$y(n) = x(n) \otimes g(n) + n(n) \qquad (4)$$

Suppose the quasi-stationary channel and perfect synchronization then the received signal at the $n^{th}$ subcarrier of the $i^{th}$ OFDM symbol is given by

$$Y_{i,n} = H_{i,n} . X_{i,n} . G_T(n) G_R(n) + n_{i,n} \qquad (5)$$

Where $G_T(n)$ and $G_R(n)$ are the frequency responses of the transmitter and the receiver's pulse shaping filters, respectively, which are generally assumed to be one within a flat fading channel.

## 3. Channel Estimation

First in this section two state of the art channel estimation algorithms, LMMSE and LSE, are described and then DFT-based and DCT-based channel estimation techniques are explained.

### 3.1 LMMSE Channel Estimation

LMMSE estimation of the channel vector $g$ is given by [6]

$$\hat{g} = \mathbf{\Gamma}_{gy}\mathbf{\Gamma}_{yy}^{-1}y \qquad (6)$$

Where

$$\mathbf{\Gamma}_{gy} = \mathbf{\Gamma}_{gg}F^H X^H \qquad (7)$$

$$\mathbf{\Gamma}_{yy} = XF\mathbf{\Gamma}_{gg}F^H X^H + \sigma_n^2 I_N \qquad (8)$$

where $\mathbf{\Gamma}_{yy}$ is the auto-covariance matrix of the received data $y$ and $\mathbf{\Gamma}_{gy}$ is the cross co-variance matrix between channel vector $g$ and the received signal $y$. $\sigma_n^2$ denotes variance of noise.
The estimated channel frequency response (CFR), $\hat{h}_{mmse}$, is described as

$$\hat{h}_{mmse} = F\hat{g} = FQF^H X^H y \qquad (9)$$

Where DFT-matrix $F$ is used to convert the time-domain estimated channel vector i.e. CIR, to the frequency domain i.e. CFR. The matrix $Q$ is given by [6]

$$Q = \mathbf{\Gamma}_{gg}\left[(F^H X^H XF)^{-1}\sigma_n^2 + \mathbf{\Gamma}_{gg}\right]^{-1}(F^H X^H XF)^{-1} \qquad (10)$$

### 3.2 LSE Channel Estimation

In real-time processing, it is not possible to have prior channel statistics information, which is the fundamental requirement of LMMSE estimation. The

only available information is about the transmitted data [7]. In LSE estimation, no probabilistic statistics

of the channel are required and we have to only make use of the transmitting signal model.
LSE estimation of channel is given by

$$\hat{h}_{ls} = FQ_{ls}F^H X^H y \qquad (11)$$

where
$$Q_{ls} = (F^H X^H X F)^{-1} \qquad (12)$$

$\hat{h}_{ls}$ can also be written as [6]

$$\hat{h}_{ls} = X^{-1}y \qquad (13)$$

The performance and complexity comparison of LMMSE and LSE with their different variants, based on CIR samples and channel taps, is described in [3] and [8].

## 3.3 DFT-based Channel Estimation

Since the energy of the channel is concentrated in time-domain, so DFT-based method is used to suppress the noise in time-domain to achieve good performance at low SNR [9]. The advantage of this method is that it is less complex than LSE since the complexity of N-point DFT operation is *O(NlogN)*. If number of pilot subcarriers is larger than the number of channel taps and all pilot sub-carriers are equi-distanced, then the performance of DFT-based estimation is also good than LSE estimation [10]. For DFT-based channel estimation, first we perform the LSE channel estimation that is given by

$$\hat{h}_{ls} = X^{-1}y$$

By using the N-point inverse-DFT we can obtain the channel impulse response (CIR) from this channel frequency response (CFR), $\hat{h}_{ls}$.

$$\widehat{H}_{ls} = IDFT[\hat{h}_{ls}] \qquad (14)$$

In multipath wireless channels, many samples of CIR have little energy so we take only first L samples having relatively more energy than noise [3], so we get

$$\widehat{H}_{ls} = \begin{cases} IDFT[\hat{h}_{ls}] & 0 \le n \le L-1 \\ 0 & otherwise \end{cases} \qquad (15)$$

Windowing functions can also be applied for this frequency leakage compensation [11]. After IDFT operation we increase samples by padding zeros

$$\widehat{H}_{P,ls} = \begin{cases} \widehat{H}_{ls} & 0 \le n \le L-1 \\ 0 & otherswise \\ \widehat{H}_{ls} & N-L \le n \le N-1 \end{cases} \qquad (16)$$

So CIR samples beyond L samples will contain only noise that is why this part will be discarded. We will consider only first L samples for DFT-based channel estimation.

$$\hat{h}_{ls} = \{DFT[\widehat{H}_{P,ls}] \quad 0 \le n \le N-1 \qquad (17)$$

This method can be used to improve the channel estimation accuracy without increasing the complexity because the IDFT/DFT operations can be implemented with the fast algorithms IFFT/FFT. DFT-CE can be used to improve the performance of LMMSE channel estimation as proposed in [12], because from this method both the channel autocorrelation matrix and noise variance can be estimated.

## 3.4 DCT-based Channel Estimation

When the multipath delays are not integer multiples, then DFT-CE is not suitable due to frequency leakage which causes aliasing. Under this condition the performance can be improved by employing a window-based DFT method [11], but at the cost of more bandwidth utilization. The real time signal has smaller high-frequency components but the DFT approach results in high frequency component. This high frequency component can be reduced by DCT, which is extensively used for voice and picture processing, because DCT employs mirror extension of N-point data sequence to 2N-point data sequence, which removes the discontinuous edge.
First, the channel frequency response on the pilot subcarriers is obtained by using LSE estimation. After that we perform the DCT operation as [13]

$$\widehat{H}_{ls} = DCT[\hat{h}_{ls}] \qquad (18)$$

$$= w_k \sum_{m=0}^{M-1} \hat{h}_{ls} \cos \frac{\pi(2m+1)k}{2M} \quad k = 0,\dots,M-1$$

Where
$$w_k = \frac{1}{\sqrt{M}} , \ k = 0 \ ; \ w_k = \sqrt{\frac{2}{M}} , \ k = 0$$

In next step zeros are inserted in the DCT domain. But different from DFT-based, zeros must be inserted at the end of $\hat{H}_{ls}$.

$$\hat{H}_{P,ls} = \begin{cases} \hat{H}_{ls} & k \leq M-1 \\ 0 & otherwise \end{cases} \quad (19)$$

IDCT can't be directly applied to get CFR because DCT cause a shift in time-domain data. To remove this shift effect extendible IDCT is employed, that is given by [14]

$$\hat{h}_{ls} = \sum_{k=0}^{M-1} w_k \hat{H}_{P,ls} \cos\left(\left(\frac{n}{N} + \frac{1}{2M}\right)\pi k\right) , \quad (20)$$
$$n = 0, \dots, N-1$$

By exchanging the DCT and IDCT processes, the time-shift problem can be avoided but the performance degradation will occur at the spectrum edge [14]. Performance can be further improved by using adaptive filters as proposed in [15].

## 4. Simulation Results

In this section, the performance comparison of DFT-based and DCT-based approaches with LMMSE and LSE channel estimation approaches is evaluated by using MATLAB Monte-Carlo Simulations in terms of Mean Square Error (MSE) and Symbol Error Rate (SER). A Rayleigh fading channel having 64 multi-path channel taps, employing Jake's models, is simulated on an OFDM system using QPSK modulation technique and 64-point FFT.
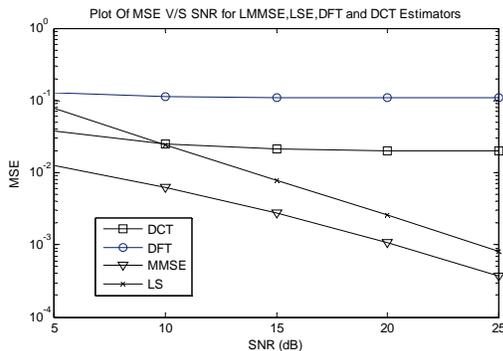


Fig. 1 MSE v/s SNR for Channel Estimators

### 4.1 MSE Comparison

Figure.1 shows the performance comparison of DCT-CE and DFT-CE approach with LMMSE and LSE

methods. It is clear from Figure.1 that LMMSE demonstrates better performance than DFT-CE and DCT-CE but this approach results in more computational time. The complexity can be reduced by using DFT-CE and DCT-CE methods and the performance degradation is not so prominent.

Figure.1 also shows that DCT approach outperforms DFT approach at all SNR values.

In DFT-based CE method, the effect of discarding certain CIR samples by using MST processor is demonstrated in Figure.2. It is clear from Figure.2 that as we go on increasing the number of discarded CIR samples, the performance also degrades which is not prominent at low SNR but at high SNR values, the performance degradation is severe.
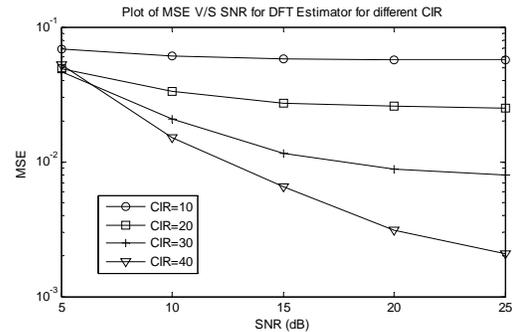


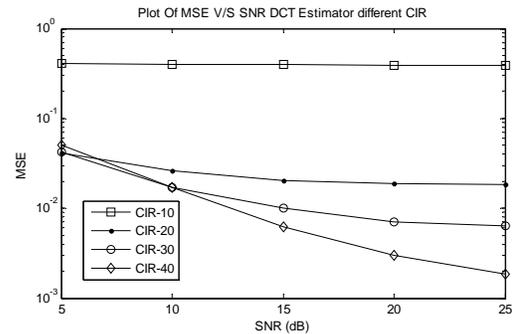Fig.2 MSE v/s SNR for DFT-CE for different CIR Samples



Fig.3 MSE v/s SNR for DCT-CE for different CIR Samples

The same performance behavior is also observed for DCT-CE approach, as shown in Figure.3. When CIR samples are reduced from 20 to 10, the performance degrades significantly. Under low SNR operating conditions, less CIR samples can be considered for less complexity but for high SNR we have to take more CIR samples having significant energy, otherwise the performance will degrade.

There are two options for DCT-CE, either apply DCT first and then IDCT or exchange these operations. The comparison between these two approaches is

shown in Figure.4. The performance of DCT/IDCT is better than IDCT/DCT, especially for high SNR values. But both these methods outperform the DFT-CE. As we go on increasing the SNR, the performance of DCT/IDCT also improves than DFT and IDCT/DCT.

The comparison between DFT and DCT for different number of CIR samples is shown in Figure.5. For DCT, the CIR samples greater than 10 have no effect on performance and only complexity increases. But for DFT, after 20 CIR samples, the performance behavior remains constant. So for DFT we have to consider more CIR samples than DCT approach, to have same performance.
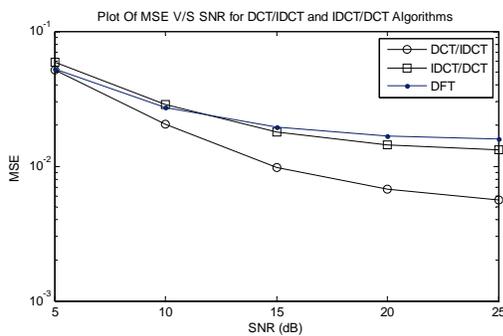


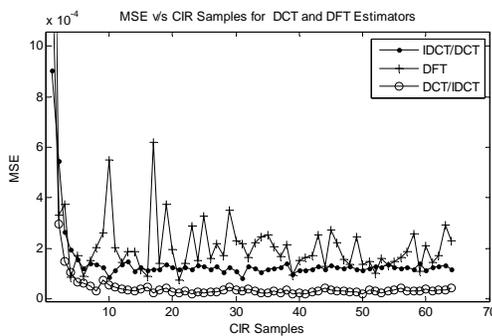Figure.4 MSE v/s SNR for DCT/IDCT and IDCT/DCT



Fig.5 MSE v/s CIR Samples for DCT-CE and DFT-CE

The effect of number of multipaths channel taps on the performance of DFT and DCT is shown in Figure.6. In Figure.6, it is demonstrated that for channel taps more than 10, the performance also remains same and further improvement can be achieved by increasing multipaths channel taps to a value greater than 60. So for less complexity and better performance, approximate 10 to 15 multipaths can be taken, while more multipaths will result only in high complexity.

4.2 SER Comparison

Comparison between DFT and DCT in terms of Symbol Error Rate (SER) is shown in Figure.7. Here again the performance of DCT is better than DFT. By increasing SNR, the performance of DCT improves while that of DFT remains constant, so there is no advantage of increasing SNR while using DFT-CE.

The effect of CIR samples on SER for DFT-CE is shown in Figure.8. For large values of CIR samples, performance improves for high SNR values, while for less CIR samples SNR value has no significant effect on performance. The same behavior is observed for DCT case as shown in Figure.9.
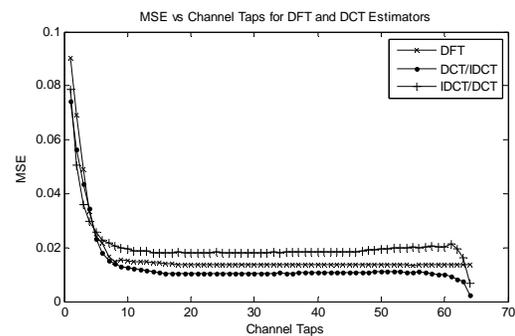


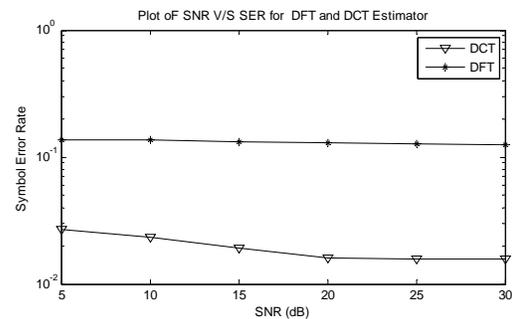Fig.6 MSE v/s Channel Taps for DFT-CE and DCT-CE
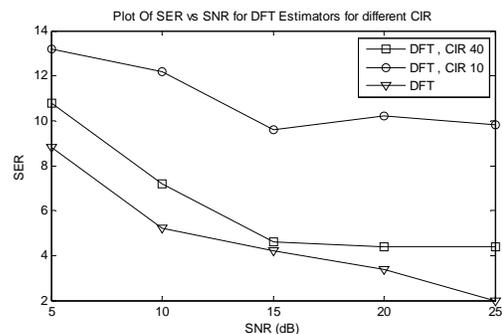


Fig.7 Comparison of SER of DFT-CE and DCT-CE



Fig.8 SER vs SNR of DFT-CE for different CIR Samples

## 5. Conclusion

In this paper, the comparison of DFT-CE and DCT-CE is drawn on the basis of CIR samples and number of multipaths channel taps. These proposed methods show better performance and less complexity because they rely on LSE which does not require any channel statistics. DCT is preferred over DFT, to reduce the

high frequency component, when the spacing between the multipaths delays is non-integer value. For power-limited communication systems, less number of CIR samples are preferred for both DCT and DFT but under high power operating conditions, less CIR samples are discarded for better performance, which results in high complexity. For low SNR values, all DCT and DFT methods have same performance, but by increasing SNR, DCT/IDCT approach results in better performance than IDCT/DCT and DFT. In wireless communication, a system employing approximately 10 multipaths is preferred because more multipaths results only in more multipath delays and more complexity, not performance.
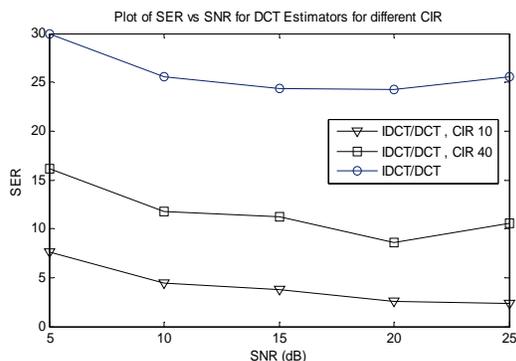


Fig.9 SER v/s SNR of DCT-CE for different CIR Samples

## References

[1] Y. Li, L. J. Cimini, Jr., and N. R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," *IEEE Trans. Comm.*, vol. 46, no. 7, pp. 902-915, July 1998.

[2] V. Srivastava, C. K. Ho, P. H. W. Fung, and S. Sun, "Robust mmse channel estimation in ofdm systems with practical timing synchronization," in *Wireless Communications and Networking Conference, 2004. WCNC.2004 IEEE*, vol. 2, pp.711–716 Vol.2, 2004

[3] Saqib Saleem, Qamar-ul-Islam, "Optimization of LSE and LMMSE Channel Estimation Algorithms based on CIR Samples and Channel Taps", IJCSI International Journal of Computer Science Issues, Vol.8, Issue.1, January 2011

[4] Baoguo Yang, Zhigang Cao, Khalid Ben Latif,"Analysis of Low complexity windowed DFT-based MMSE Channel estimation", IEEE Transactions on Communications, Vol.49, No.11, November 2001.

[5] Ye,Li ,"Pilot symbol aided channel estimation for OFDM in wireless systems", IEEE Transactions on Vehicular Technology, Vol.49, No.4, July 2009.

[6] J.J. van der Beek, O. Edfors, M. Sandell, S.K. Wilson, and P.O.Borgesson, "On channel estimation in OFDM systems," *Proc. VTC'95*, pp. 815-819.

[7] Dimitris G. Manolakis, Vinay K. Ingle. Statistical and Adaptive Signal Processing,Spectral Estimation, Signal Modeling, Adaptive Filtering and Array Processing,Artech House, Boston London

[8] Saqib Saleem, Qamar-ul-Islam, "*Performance and Complexity Comparison of Channel Estimation Algorithms for OFDM System*", IJECS International Journal of Electrical and Computer Sciences, Vol.11, No.02, April 2011

[9] Jun-Hee Jang, Se-Bin Im, Jeong-soon Park, Hyung-Jin Choi," DFT-based decision directed channel estimation for OFDM systems in very large delay spread channels", 14th Asia-Pacific Conference on Communications, 2008. APCC 2008.

[10] Kwak, K, Lee, S, Hong, D, Jihyung Kim," A new DFT-based Channel estimation approach for OFDM with virtual subcarriers by leakage estimation", IEEE Transactions on Wireless Communications,Volume: 7, Issue:6, pp.2004-2008,June 2008

[11] Y. Baoguo, K. B. Letaief, R. S. Cheng, and C. Zhigang, Windowed DFT based pilot-symbol-aided channel estimation for OFDM systems in multipath fading channels," in *Vehicular Technology Conference Proceedings, 2000. VTC 2000-Spring Tokyo. 2000 IEEE 51st*, 2000, pp.1480-1484 vol.2

[12] Jie Ma, Hua Yu, Shouyin Liu," The MMSE Channel Estimation Based on DFT for OFDM System", 5th International Conference on Wireless Communications, Networking and Mobile Computing,2009. WiCom '09.2009

[13] Y. H. Yeh and S. G. Chen, "DCT-based channel estimation for OFDM systems, "*Communications,2004 IEEE International Conference*, vol. 4,pp.2442-2446, June 2004.

[14] Y. H. Yeh and S. G. Chen, "Efficient channel estimation based on discrete cosine transform," *IEEE ICASSP'03*, vol. 4, pp.IV_676-IV_679.

[15] Saqib Saleem, Qamar-ul-Islam, "LMS and RLS Channel Estimation Algorithms for LTE-Advanced", Journal of Computing, Vol.3, Issue.4, pp.155-163, April 2011.