Nonlinear Approach of Actuator Fault Accommodation

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Abstract

In this paper, a method of actuator fault accommodation for a class of nonlinear systems is proposed. It concerns the problem of progressive accommodation to actuator failure. This strategy is based on the optimal nonlinear controller, and its objective to maintain the system closed loop stability. The authors show the interest of the proposed method even for a local analysis when a linear approximation is used. This work focuses on a solution to ensure stability while accommodating to actuator failure. An example is given to illustrate this approach.

Keywords: *Fault tolerant control, progressive accommodation, optimal control, actuator fault, nonlinear system.*

1. Introduction

Modern technological systems rely on sophisticated control functions to meet increased performance requirements. For such systems, Fault Tolerant Control (FTC) needs to be developed. A FTC is a control that can accommodate system component faults and it able to maintain stability and acceptable degree of performance with respect to nominal system operation [1] and accept some graceful performance degradation [2] not only when the system is fault-free but also when there are component malfunctions.

FTC can be classified into passive and active. A Passive FTC (PFTC) can tolerate a predefined set of faults while accomplishing its mission satisfactory without the need for control reconfiguration. Active FTC (AFTC), on the other hand, relies on a Fault Detection and Identification (FDI) process to monitor system performance, and to detect and isolate faults in the system. Accordingly, the control law is reconfigured on-line [3] [4].

Indeed, in the literature, a conventional strategy to solve a nonlinear reconfigurable control problem consists in designing a linear approximation of the model around operating points. Recent papers such as multiple model [5] [6] and sliding modes [7] have been presented. In order to handle nonlinear systems beyond using a linearized approximation, reconfigurable control methods have been proposed using backstepping [8] and nonlinear regulator [9].

Moreover, few papers concern the delays associated with computation times [10] [11] [12]. The former introduced the concept of progressive accommodation whose the objective is to minimize the effect of the accommodation delay. To this end, the reconfigurable control design method is based on a linear quadratic approach.

The objective of this work is to study the validity of the linear approximation approach when the fault holds. More precisely, this paper proposes an analysis of the accommodation delay and its effects on the closed loop stability. This work considers a linear system as an approximation of a nonlinear one around an equilibrium point. The limitation of the linear approach is emphasized when the actuator fault occurs near the boundary of the validity domain of the linearization. In this case, an appropriate nonlinear approach which is valid on the whole physical domain can be helpful.

The present paper is organized as follows: in section 2, the class of affine nonlinear systems is introduced and a necessary background is provided on the main idea of the actuator fault accommodation and optimal regulation problem. Section 3 presents the analysis of the closed loop system stabilization during the fault occurrence with the use of the domain of attraction and the linear approximation validity domain. Section 4 presents the proposed approach of nonlinear progressive accommodation. In section 5, simulation studies have been conducted in an example to illustrate the proposed approach.

2. Preliminaries and Motivation

In the present work, affine nonlinear continuous-time dynamic systems are considered with a state-space representation:

$$\dot{x} = f(x) + Bu \quad (1)$$

$$y = h(x) \quad (2)$$

where $x \in \Re^n$ the vector of state variables is, $u \in \Re^m$ is the control vector and $y \in \Re^l$ is the output vector. f and h are smooth functions with $f(0)=0$. B is a constant matrix of dimension $(n \times m)$. The infinite-time horizon nonlinear regulation problem is defined with the following quadratic performance index in u :

$$V(x) = \min_{u(t)} \int_0^\infty (x^T Q(x) x + u^T R(x) u) dt \qquad (3)$$

in which $Q(x) \geq 0$ and $R(x) > 0$ for all x. Moreover, it is assumed that Q and R are sufficiently smooth so that the value function $V(x)$ is continuously differentiable. In this case, the Hamilton-Jacobi Equation (HJE) is quadratic in $\frac{\partial V}{\partial x}(x)$ such that:

$$\frac{\partial V}{\partial x} f(x) - \frac{1}{4} \frac{\partial V}{\partial x} B R^{-1} B^T \frac{\partial V}{\partial x}^T + x^T Q\, x = 0 \qquad (4)$$

and the optimal feedback control can be designed from:

$$u_n = -\frac{1}{2} R^{-1}(x) B^T \frac{\partial V}{\partial x}^T (x) \qquad (5)$$

In this paper, we consider as defined in [11] a linear representation, that one (or several) actuator fault(s) occur at time t_f. The system can be described by:

$$\dot{x} = f(x) + B_\theta(u) \qquad (6)$$

where:

$$B_\theta(u) = \begin{cases} Bu, & t \in [0, t_f[\\ \beta_f(u, \theta), & t \in [t_f, +\infty[\end{cases} \qquad (7)$$

The function $\beta_f(u,\theta)$ and the parameter θ represent the contribution of the faulty actuator. The complex structure of the system (1) introduces difficulties in solving the optimal control problem.

The calculation of an optimal nonlinear state feedback for nonlinear systems requires the development of numerical algorithms [13] [14], because the optimization problem needs a resolution of the Hamilton-Jacobi equation. Otherwise, the control problem makes mandatory an approximation by system with a simpler structure. Notice that in a local area, the linear system is given by:

$$\dot{x} = Ax + Bu \qquad (8)$$

where $A = \frac{\partial f}{\partial x}\big|_{x=0}$ is the Jacobian matrix of f at point $x=0$. Therefore, the optimal regulation problem is characterized by an Algebraic Riccati Equation (ARE).

As mentioned in [11], in the FTC problem, one has to consider four time periods in order to analyze the system behavior under actuator fault.

1. $t \in [0, t_f[$: nominal system and control u_n.
2. $t \in [t_f, t_{fdi}[$: faulty system under the nominal control u_n and FDI algorithm in process for fault detection, isolation and estimation.
3. $t \in [t_{fdi}, t_{ftc}[$: faulty system under the nominal control u_n and the fault is detected, isolated and estimated.
4. $t \in [t_{ftc}, +\infty[$: faulty system under the accommodated control u_f.

These four time periods are presented in Fig.1.

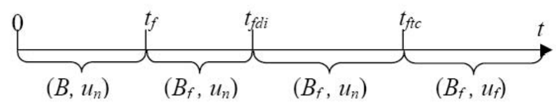

Fig. 1. Description of the fault tolerant control strategy

In practical applications, even if the diagnosis is perfect that is not realistic, the system control is inappropriate on the interval $[t_f, t_{ftc}[$ since the faulty system is controlled by u_n. The progressive accommodation presented in [10] [11], aims at minimizing the interval $[t_{fdi}, t_{ftc}[$. Therefore, thanks to an online control computation, the authors propose an improvement of the closed loop behavior of the fault system in a linear context.

The present paper exposes the limitations of the linear approach and develops an extension of the actuator fault accommodation to a class of affine nonlinear system with unstable free dynamics.

3. Closed loop stability and accommodation to the actuator fault

Number of methods for determining the stability region of nonlinear systems has been proposed in the literature, for example Zubov's method [15]. It computes the entire stability region via a Lyapunov function. Regardless an eventual actuator fault occurrence, the solution of the

Zubov's partial differential equation is used to estimate the closed-loop stability region.
Let the evolution of the nonlinear system be described by the equation (1). At any given point in time t , assume that it is always possible to integrate the dynamic equation (1) for all admissible input control $u(t)$. An optimal control design is designed thanks to the optimization of the performance index (3). The problem of local output regulation involves the design of a feedback controller which ensures that the closed loop system is locally asymptotically stable at the origin, and the regulated output $y(t)$ asymptotically decays to 0 as $t \to +\infty$.

In order to accomplish the above task, the problem of nonlinear control may be solved in a local area using a linear approximation of the system.

In this paper, the authors introduce the notion of the validity domain ν of the linear approximation which allows to synthesizing an optimal controller by an Algebraic Riccati Equation. This study stands for an extension to the class of nonlinear system (1) of the linear approach proposed in [10] [11].

As shown on Fig.2, starting from the initial condition $x_0 \in \nu$, the stability of the nonlinear system in closed loop is ensured in the domain of attraction $\beta(B, u_n)$ using the linear optimal controller. The system converges to the equilibrium point x_{eq}. D_φ stands for the physical operating domain of the system.

When an actuator fault occurs, the nominal model is changed at time t_f and the quadratic performance index (3) is modified. During the time interval $[t_f, t_{ftc}[$, the domain of attraction of the closed loop becomes $\beta(B_f, u_n)$.

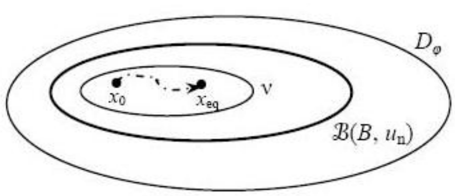

Fig. 2. Description of the validity domain of the linear approximation

If the system is tolerant to the fault, Staroswiecki et al. [10] proposed in a linear approach, an optimal way to progressively accommodate the fault such that the closed loop system is stable. The algorithm is based on the Newton-Raphson algorithm developed in a linear context in [16]. It is considered here that the diagnostic algorithm is computed with no delay, no error that is not realistic. Consequently, as shown on Figure 1, the diagnostic strategy is characterized by the time delay $(t_{fdi} - t_f)$.

Therefore, depending on the nonlinearity of the system, the linear approach to the progressive accommodation may not be able to stabilize the closed loop. The system may leave the validity domain of the linear approximation and the domain of attraction $\beta(B_f, u_n)$ as shown on Fig.3.

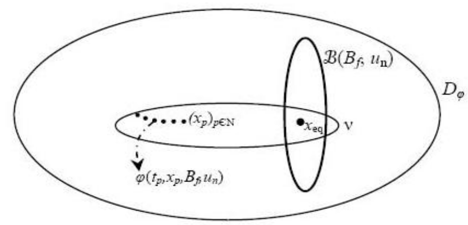

Fig. 3. Evolution in the interval $[t_f, t_{ftc}[$ of the closed loop system under an actuator fault

From Figure 3, x_p stands for the initial condition, t_p is the time delay necessary to cross the boundary of ν and $\varphi(t_p, x_p, B_f, u_n)$ is the solution of the system (1).

From now, if the state corresponding to the solution $\varphi(t_p, x_p, B_f, u_n)$ belongs to the domain of attraction $\beta(B_f, u_f)$, the control is fault tolerant and the solution converges to the equilibrium point as shown on Fig.4.

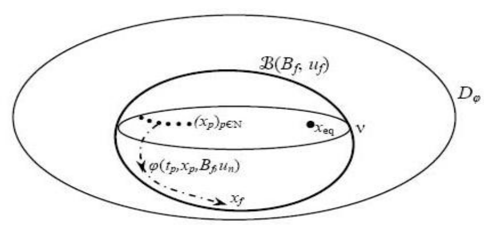

Fig. 4. Evolution in the interval $[t_{ftc}, t[$ of the closed loop system with a fault tolerant control

Nevertheless, if the state corresponding to the solution $\varphi(t_p, x_p, B_f, u_n)$ doesn't belong to the domain of attraction $\beta(B_f, u_f)$, the closed loop system is unstable as presented on Fig.5 and the actuator fault is not accommodated.

In the following, the authors, argue the presented problem, propose an illustration and a solution for the actuator fault accommodation.

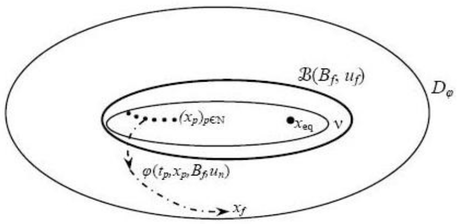

Fig. 5. Evolution in the interval $[t_{ftc}, t[$ of the closed loop system for a non accommodated fault

Consider the class of nonlinear system defined in (1). Let $\beta(B,u)$ be the domain of attraction of the closed loop system defined from (8) with (B,u). The validity domain ν is included in the domain of attraction $\beta(B,u_n)$ where (B,u_n) describes the nominal operating conditions and control. In other words, $\nu \subset \beta(B,u_n)$. This means that the domain of attraction is at least equal to ν since the validity domain of the linear model is ensured on the whole ν.

However, the estimation of the domain of attraction $\beta(B,u_n)$, when it is possible must be done using the nonlinear model (1) as proposed in [15].

In the present paper, the authors consider the case of unstable free dynamics.

In the interval $[t_f, t_{ftc}[$ for the closed loop system defined from (1) with $\beta(B_f,u_n)$, we have $\nu \not\subset \beta(B_f,u_n)$. Otherwise, this means that the actuator fault doesn't affect the performances of the closed loop.

For now, let define the following notations. $\partial\nu$ designates the boundary of ν. $d\left(x_p,\partial\nu\right)$ denotes the distance from x_p to $\partial\nu$. t_p is the time delay necessary to cross the boundary $\partial\nu$. $\varphi\left(t_p,x_p,B,u\right)$ stands for the solution of the closed loop system (1) defined by the pair (B,u) for t_p with the initial condition x_p.

Consequently, there exist two sequences $\left(x_p\right)_{p\in\aleph} \in \nu$ and $\left(t_p\right)_{p\in\square} \in \Re^+$ with $t_p < \frac{1}{P}$ such that $d\left(x_p,\partial\nu\right) < \frac{1}{p}$ and $\varphi\left(t_p,x_p,B_f,u_n\right) \notin \nu$.

Finally, for any t_{ftc}, there exists an initial condition $x^{ftc} \in \nu$ such that $\varphi\left(t_p,x^{ftc},B_f,u_n\right) \notin \nu$, and it is not ensured that $\lim_{t\to\infty}\varphi\left(t,\varphi\left(t_{ftc},x^{ftc},B_f,u_n\right),B_f,\overline{u}\right)$ for any used $\overline{u}$ which is valid in the domain ν in the interval $[t_{ftc},+\infty[$.

4. Proposed approach: Nonlinear Progressive Accommodation (NPA)

In practice, an actuator fault in a co ntrolled system generates changes in inputs/outputs signals and in the parameters of the differential system which describes the dynamics.

The design of a passive fault tolerant controller is sufficient to ensure degraded dynamic performances when the changes in the parameters and signals are small. When the effects of the fault are significant, the global stability of the system may not be ensured, therefore the stabilization of the dynamic system with a f ixed controller may be impossible.

In this paper, the authors consider an actuator fault occurrence under the constraint that the faulty actuator can’t be switched-off and replaced. This last strategy is usually called system reconfiguration. In this section, the authors focus their attention on the fault accommodation in a nonlinear context. They first refer to a fault tolerant control designed beforehand when failure is identified and secondly to an on-line accommodation scheme.

4.1 Nonlinear Progressive Accommodation (NPA)

In the nonlinear case, the infinite-time horizon nonlinear optimal control problem (1), (3), is characterized in terms of Hamilton-Jacobi Equation (4). The difficulty with finding the optimal control is that the HJB equation represents a nonlinear partial differential equation, which is difficult, and often impossible, to solve analytically. The complexity of the HJE prevents any solution excepted in some very simple systems. In order to make real-time implementation possible, one has to avoid solving any partial differential equation. With application to online progressive accommodation and in order to design a suboptimal control design, an alternative is to investigate the Successive Galerkin Approximation (SGA) algorithm to the Hamilton-Jacobi-Bellman Equation (HJBE) [13]. This approach approximates the solution to equation (4), if a stabilizing control to system (1) is known a priori, with the following iterative scheme:

$$u_i(x)=\begin{cases} u_0(x), & i=0 \\ u_n=-\frac{1}{2}R^{-1}B^T\frac{\partial V_{i-1}^T}{\partial x}(x), & i>0 \end{cases} \qquad (9)$$

where V_{i-1} is the performance index of u_{i-1} as calculated from the solution of the Generalized-Hamilton-Jacobi-Bellman (GHJB) equation:

$$\frac{\partial V_{i-1}}{\partial x}(f(x)+Bu_{i-1})+u_{i-1}TR^{-1}u_{i-1}+x^TQx=0 \quad (10)$$

if the u_0 is stabilizing control, then u_i will be stabilizing for all $i\geq 0$, $V_i\rightarrow V$ and $u_i\rightarrow u$ as $i\rightarrow\infty$.

5. Illustrative example

Consider an affine nonlinear continuous-time dynamic system modeled by:

$$\dot{x}=x+x^2+2u \qquad (11)$$
$$y=x \qquad (12)$$

The following problem is first to define an optimal control u_n with respect to a quadratic performance index (3), in nominal conditions given an initial value of the state x_0. Secondly, a fault tolerant control u_f must be synthesized given an acceptable actuator fault.

5.1 Optimal control in nominal conditions

If the problem is local, a linear approximation of the system around the operating point $x=0$ is computed. The optimal control u_n is synthesized thanks to the LQ problem.

Find the optimal control u_n, such that the cost:

$$J(u,x_0)=\int_0^{\infty}(x^TQ(x)x+u^TR(x)u)dt \qquad (13)$$

is minimal.

The optimal solution is known to be:

$$u_n=-R^{-1}B^TPx=-F_nx \qquad (14)$$

where $B=2$, Q and R are symmetric positive definite matrices and P is the solution to the Algebraic Riccati Equation (ARE).

$$PA+A^TP+Q-PBR^{-1}B^TP=0 \qquad (15)$$

With the choice of $Q=R=1$, one finds $u_n=-1.618x$.

5.2 Linear Progressive Accommodation (LPA) to actuator fault

As defined in (7), an actuator fault occurs at $t_f>0$. The state-space representation of the nonlinear faulty system becomes:

$$\dot{x}=x+x^2+B_fu \qquad (16)$$
$$y=x \qquad (17)$$

where $B_f=0.8$. Staroswiecki et al. in [10] proposed a linear approach to the progressive fault accommodation. Given the local linearization of the faulty system (16) around the nominal operating point $x=0$, if the loss of efficiency due the fault occurrence can be admitted, the linear accommodation problem has an admissible solution. Consequently, the linear feedback control law $u_i=-F_ix$ (starting at the time t_{fdi}) is applied on the interval $[t_i,t_{i+1}[$. The description of the linear progressive accommodation strategy is given on Fig.6.

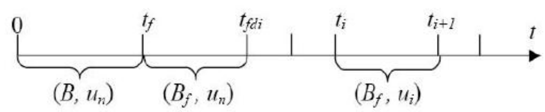

Fig. 6. Description of the progressive accommodation strategy

Based on the linear approximation of the faulty system (A_f, B_f), the feedback control u_i is computed thanks to the Newton-Raphson algorithm presented in [16]. P_i is the unique solution of the Lyapunov equation:

$$P_i\left(A_f-B_fF_{i-1}\right)+\left(A_f-B_fF_{i-1}\right)^TP_i=-Q-F_{i-1}^TRF_{i-1} \qquad (18)$$

The initial F_0 is given and for all $i=1,...,n$, $F_i=R^{-1}B_f^TP_i$. Moreover, the optimal linear fault tolerant controller is defined by P_f.

P_f is the unique positive definite solution of the Algebraic Riccati Equation (ARE):

$$P_f A_f + A_f^T P_f + Q - P_f B_f R^{-1} B_f^T P_f = 0 \qquad (19)$$

and $\lim_{i \to +\infty} P_i = P_f$, where P_i is the solution of (18). The optimal control of the faulty system gives $u_f = -2.8508x$. As an illustration, one can choose the initial condition $x_0 = 0.1$. An actuator fault occurs at the time $t_f = 0.2s$. According to the definition (7), the system is described by:

$$\dot{x} = \begin{cases} x + x^2 + 2u, & t \in [0, t_f] \\ x + x^2 + 0.8u, & t \in [t_f, +\infty] \end{cases} \qquad (20)$$

Let consider the sample computation time $t_e = 1.7s$ and one supposes that the time delay for the fault diagnosis $(t_{fdi} - t_f)$ is equal to t_e. Each iteration takes t_e. The time delay for the FTC computation $(t_{ftc} - t_{fdi})$ is equal to t_e.

Fig.7 presents an illustration of the linear progressive accommodation to the actuator failure. In the interval $[t_f, t_f + t_e]$, the nonlinear faulty system is driven by the linear optimal nominal feedback control u_n. At the time $(t_f + t_e)$, the closed-loop is stabilized using the iterative control u_i in green color line. The first fault tolerant control u_f is applied at the instant $(t_{ftc} = t_{fdi} + t_e)$. The corresponding state $x(t)$ is plotted in red color line.
As expected, the state $x(t)$ decreases to zero a little bit faster with the progressive accommodation than with the FTC.

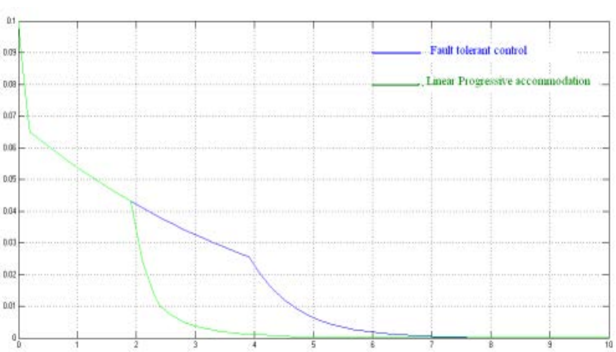

Fig. 7. Illustration of the linear progressive accommodation on the example (11)

Moreover, Table 1 shows the evolution of F_i when linear fault accommodation is applied. The convergence of the Newton-Raphson algorithm on the linear optimal fault tolerant control takes 5 iterations.

Table 1: Evolution of the iterative state feedback gain

Iteration i	$\boldsymbol{F_i}$
0	1.6180
1	4.9154
2	3.4322
3	2.9282
4	2.8526
5	2.8508

By now, for the same actuator fault, there exists and initial condition $x(0)$ such that the nominal closed-loop system stays inside the domain of attraction and the state $x(t)$ doesn't belong to the validity domain of the linear approximation. Therefore, from the instant of the fault occurrence, the closed-loop system leaves the domain of attraction and diverges despite the linear progressive accommodation.

Fig.8 illustrates the divergence of the state $x(t)$ with the linear progressive approach for the given initial condition $x(0) = 0.85$.

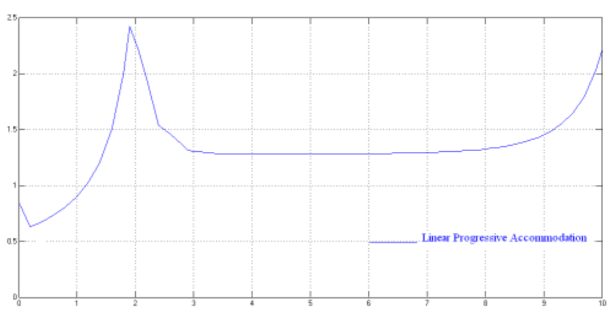

Fig. 8. Illustration of the linear progressive accommodation on the example (11)

5.3 Proposed Nonlinear Progressive Accommodation (NLPA) to actuator fault

The point of departure in the present study is an improvement of the linear approach to the progressive accommodation for the class of nonlinear systems defined in (1).

The time delay needed to begin the accommodation to the actuator fault is equal to $\left(t_{ftc} - t_f\right)$ for a classical fault tolerant control strategy as resumed in Fig.1 and $\left(t_0 - t_f\right)$ for the linear progressive one whose the description is given in Fig.6. t_0 stands for the first instant of the correction with respect to the actuator fault. One can remark that $t_0 = t_{ftc}$ for the classical FTC.

Let define t_c, the first instant of correction for the active or passive fault tolerant control. Whatever t_c, there exists an initial condition $x(0)$ such that the nonlinear faulty system in closed-loop leaves the domain of attraction and becomes unstable.

An alternative to the linear approach of the progressive accommodation issue consists in computing a nonlinear optimal control which is able to accommodate the actuator fault through the minimization of the quadratic performance index (13). Consequently, the optimal control problem in the presence of actuator fault is to find a state feedback control u_{fnl} which minimizes the cost (13) for all possible initial conditions $x(0)$.

To this end, the Hamilton-Jacobi Equation (4) must be solved. An analytic solution of such a problem is not accessible in general that's why a numerical approximation is computed in order to produce a suboptimal control. In the literature, on can find methods to obtain a usable feedback control in [13][14][17].

In the example (20), given the initial condition $x(0) = 0.85$ one can find the nonlinear optimal control which accommodate the actuator fault. The feedback nonlinear control u_i is computed thanks to Successive Galerkin Approximation (SGA) algorithm:

$$u_i(x) = \begin{cases} u_0(x), & i = 0 \\ u_n = -\frac{1}{2} R^{-1} B^T \frac{\partial V_{i-1}^T}{\partial x}(x), & i > 0 \end{cases} \quad (21)$$

V_{i-1} is the solution of the Hamilton-Jacobi-Bellman Equation (HJBE):

$$\frac{\partial V_{i-1}}{\partial x}(f(x) + Bu_{i-1}) + u_{i-1}TR^{-1}u_{i-1} + x^T Qx = 0 \quad (22)$$

The convergence of the Successive Galerkin Approximation (SGA) algorithm to the nonlinear optimal fault tolerant control takes 3 iterations.

The nonlinear optimal fault tolerant control is:

$$u_{fnl}(x) = -1.25x(x + 1 + \sqrt{x^2 + 2x + 1.64}) \quad (23)$$

Fig.9 presents the nonlinear progressive accommodation in green color line.

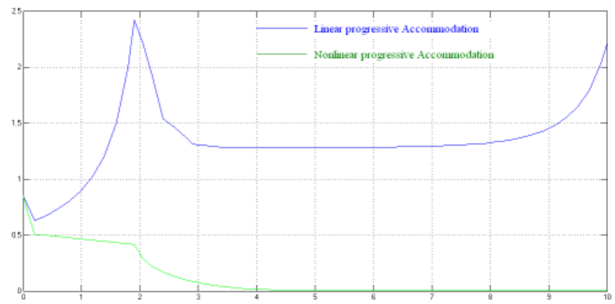

Fig. 9. Illustration of the nonlinear progressive accommodation on the example (11)

The plot shows the improvement of the nonlinear approach for the accommodation. In a sense of stability, the used nonlinear control at the instant of correction t_c ensures the decrease of the state $x(t)$ to zero of the damaged system.

6. Conclusions

This paper underlines the importance of the analysis of the closed-loop system stabilization with the use of the domain of attraction and the linear approximation validity domain in the context of actuator fault accommodation.

This work particulary considers the limitation of the linear progressive accommodation approach when the fault occurs next to the boundary of the validity domain of the linearized model. An example aims at illustrating the argued idea which is developed in the article.

Finally, the nonlinear progressive accommodation is proved to be efficient thanks to an algorithm taking to account the nonlinearity in the active fault tolerant control synthesis.

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