New Approximated Method of Auxiliary Sources for large-size scatterer

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Abstract

In this paper, an efficient approximated method based upon the method of auxiliary sources (MAS) is proposed to solve the two-dimensional scattering problem of large, infinite and perfectly conducting cylinder (PEC). To reduce the size of the total computational cost, the formulation of the MAS is modified by minimizing the number of auxiliary sources considered to implement the solution. It is shown that the standard formulation of the method of auxiliary sources, based on placing a finite number of auxiliary sources in an interior cylinder, can be replaced by a finite number of strips placed on the same interior cylinder. These strips, containing auxiliary sources, are separated by a constant angle. Thus, compared with the standard MAS, the number of auxiliary sources of the new approximated method is reduced; also the proposed method can greatly reduce the computational complexity and the memory requirement. The numerical results obtained in this paper reveal the validity of the proposed approximated method.

Keywords: *Numerical method, method of auxiliary sources (MAS), electromagnetic scattering, approximation of the MAS, radar cross section, boundary condition.*

1. Introduction

The investigation of scattering problem of an incident field by large scatterers is a subject of great interest in the study of the electromagnetic phenomena. Thus, large object scatterer has been an active research topic for many years, due to the complexity of its computation, which needs high computer performance and expensive computational cost.

The complexity of such problem leads to the use of efficient numerical methods for computation of the scattered field. Different methods have been used to solve this problem like the finite element method (FEM) [1] in which the geometry is partitioned into smaller sections, the domain decomposition method (DDM) [2] based on the decomposition of structure into many domains, the localized iterative generalized multiple technique (LIGMT) [3] or method of moment (MoM) [4]. However, the solutions proposed by these methods still having significant computation time and memory cost because it includes the principal of meshing the structure. Thus, these methods require more memory space and a l ong time for computation.

Another numerical method can be used to solve such problem, the method of auxiliary sources MAS which has many advantages; being meshless, not needing a complicated discretisation of the domain, being simple to implement and broadly being used to model scattering problems (photonics, metamaterials, arrays, etc.) [5], [6].

In the present paper, an approximated method based upon the method of auxiliary sources (MAS) [7], [8] is presented. The MAS is a numerical method that is applied to solve radiating and two-dimensional (2D) and three-dimensional (3D) scattering problem. The main idea of this method consists in the elimination of the singularity, which exists in the surface integral equations, by adjusting the position of the auxiliary surface away from the position where the boundary conditions must be enforced.

The problem of scattering by small objects is solved by the method of MAS in [9]. This work has demonstrated the efficiency of this method on reducing the time-consuming and the complexity of such problem, using a finite number of auxiliary sources chosen to be placed regularly on the auxiliary surface, either filaments for 2-D problems or pairs of elementary dipoles for 3-D problems.

In the later case, the scattering problem is solved and many good results are obtained. But, the complexity of the scattering problem becomes more important when the scatterers have large dimensions. Thus, the numbers of auxiliary sources will increase in order to describe all the scattered filed, which implies the increase of the matrix size describing the structure.

The computation resources are limited, so it is impossible to calculate the scattering by these large objects. In addition, the time required to calculate these structures becomes very important, which can be more than a day or more.

In this paper, the MAS formulation of the scattering problem is modified for the treatment of large-size object scatterer. So, the surface of the scatterer is replaced by a finite number of strips placed regularly spaced by a constant angle. On the new constructed structure, the MAS is applied to each strip separately and for each strip an auxiliary surface is associated. Then, a finite number of auxiliary sources is distributed on this elementary auxiliary surface. The solution is constructed by the union of the elementary solutions by taking into account the coupling of all the strips. In this way, the scattered field of the object in a given collocation point is a combination of all the fields generated by auxiliary sources on each strip.

The verification of this approximated method of MAS and the validation of the computer written code is made by solving the problem of scattering by an infinite perfectly conducting cylinder with a plane-wave excitation.

The cylindrical structures, more than other structures, offer suitable and efficient model for the study of many practical objects such as trees, human body, antennas and the big building, which can be the case of our present paper.
The validity of the present approximation is made by comparison to published results.

2. Formulation and equations

Assume an infinite z-axis circular perfectly conducting cylinder (PEC), placed in free space. The structure is illuminated by a transverse magnetic (TMz) plane wave with respect to the z-axis. We denote E^{inc} and H^{inc} respectively the electric and the magnetic field, the components of the electromagnetic fields. A Cartesian coordinate system x, y, z is introduced.

Under these assumptions, the incident electromagnetic fields are given by the following expression [10]:

$$\hat{E}^{inc} = E_0 exp\{j(K_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\}\hat{z} \quad (1)$$

$$\hat{H}^{inc} = -\frac{E_0}{H_0}(\hat{x}\sin\varphi_{inc} - \hat{y}\cos\varphi_{inc}) \; exp\{j(K_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\} \quad (2)$$

Where k_0 is the wave number in the free space, $\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vector respectively in the x, y and z-direction.

The incident field will be scattered by the cylinder giving a scattered field. Because the first field is z-directed, then the scattered field is z-directed too. Therefore the scattering problem will be reduced to bidimensional one.

The purpose of the numerical method is to find the scattered field which satisfies the Maxwell equation, the radiating condition and the boundary conditions of the studied scatterer. The MAS is also based on these boundary conditions, which are defined as the continuity of the tangential component of the total electric and magnetic fields must be enforced in the boundary of the structure in order to construct the solution of the scattering problem. Since the structure is PEC, we consider the continuity of electric field component which is expressed as follows:

$$\hat{n} \times \left(\hat{E}^{inc} + \hat{E}^{scat}\right) = 0 \quad (3)$$

Where E^{scat} is the total electric scattered field from the cylinder.

According to the MAS fundamental concept [9], a set of N auxiliary sources are located inside the scatterer, residing on a fictitious auxiliary surface (circle of radius a), and surround with circular surface containing M collocation points (CPs).
The two surfaces are separated by a distance d_{aux} named auxiliary distance which will be adjusted in order to find the convergent solution of scattering problem.
In this case, we suppose that M=N.

The condition in Eq.3 can be developed for the N auxiliary sources. So, the scattered field at the M collocation points can be written as:
For CP= 1

$$\sum_{n=1}^{N} a_n E_n^{m=1} = -E_{m=1}^{inc} \quad (4)$$

For CP= 2

$$\sum_{n=1}^{N} a_n E_n^{m=2} = -E_{m=2}^{inc} \quad (5)$$

For CP= i

$$\sum_{n=1}^{N} a_n E_n^{m=i} = -E_{m=i}^{inc} \quad (6)$$

For CP= M

$$\sum_{n=1}^{N} a_n E_n^{m=M} = -E_{m=M}^{inc} \quad (7)$$

a_n is the unknown complex current.

E_m^{inc} is the incident field acting on the mth collocation point as written in the Eq. (1).
$E_n^{m=i}$ is the radiated filed by the nth auxiliary source acting on the mth collocation point.

Since we are treating a 2-D problem, each auxiliary source radiates an elementary electric field proportional to the two dimensional Green's function, with a single component parallel to the cylinder axis.
This radiated field can be written as follows [12]:

$$E_n^m = \frac{K_0 \eta}{4} H_0^{(2)} \left[K_0 |r_{CPm} - r_{ASn}|\right] \tag{8}$$

Where $H_0^{(2)}$ is the Hankel function of the second kind of zero order, K_0 the wave number in the free space, r_{CPm} the space vector of the collocation point, r_{ASn} the space vector of the auxiliary source n.

Therefore, we obtain a linear system having N equation with N unknown. The number of unknowns increases with the increase of the number of ASs. It is demonstrated in [11] that the convergence of the MAS solution depends on N. The solution of the scattering problem is given by the solution of the following linear system obtained from the Eq. (4):

$$\begin{bmatrix} E_1^1 & \cdots & E_N^1 \\ E_1^2 & \cdots & E_N^2 \\ \vdots & \ddots & \vdots \\ E_1^{M-1} & \cdots & E_N^{M-1} \\ E_1^M & \cdots & E_N^M \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ \vdots \\ a_N \end{bmatrix} = - \begin{bmatrix} E_1^{inc} \\ \vdots \\ \vdots \\ E_M^{inc} \end{bmatrix}$$

Besides, the number of unknowns increases with the increase of the size of the scatterer. So, if the size of the structure is very important compared with the medium wavelength λ, then the convergence of the MAS needs more ASs than the small objects to converge. As a consequence, the linear system will have more unknowns.

However, computing large-size objects needs significant computing resources, computation time and memory cost. Therefore, it is necessary to find a method to solve the scattering problem for large size scatterer with a minimum of computational cost.

For this, many solutions have been elaborated in order to give a solution for such problem. But, the constraint of computational resources has not been reached. The MAS has shown a good performance and accuracy in the resolution of the scattering problem for the small size of scatterer compared with the other numerical method.

In this paper, in order to reduce the computational cost for the treatment of large-size scatterer, a new approximated method based upon the MAS is presented. The principle of this approximated method is to reduce the size of the matrix describing the scatterer. The main idea here is to replace the structure in Fig.1 by the structure in Fig.2. Indeed, for large objects, this method removes regularly parts of the surface of the structure. So, we obtain a new structure consisted of a series of strips spaced by the angle θ.

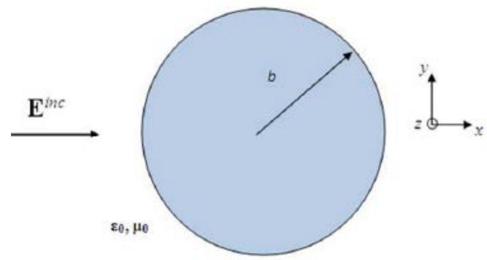

Fig. 1 Geometry of the problem.

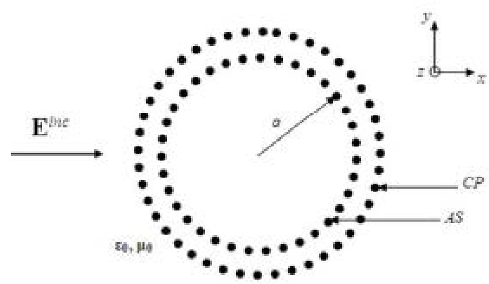

Fig. 2 Base of auxiliary sources repartition with standard MAS.

The Fig.3 shows that the surface of the homogeneous structure in Fig.1 is subdivided in finite, homogeneous strips. If we consider that the number of the strips is p, then, in the approximated structure, a circle arc having a size of $p \times \theta$ is removed and not be considered in the formulation.

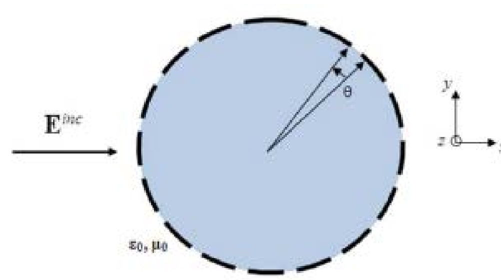

Fig. 3 Geometry of the approximated problem.

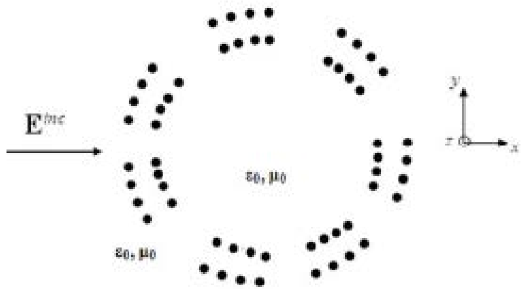

Fig. 4 Base of auxiliary sources repartition for the approximated MAS.

The scattering problem is, now, replaced by p elementary problems.

The formulation of the approximated MAS is based on the discretisation of the p elementary strips into L collocation points. We associate a conformal strip on the auxiliary surface with each strip in the physical surface of the cylinder, which contains also L auxiliary sources distributed on p auxiliary strips (Fig.4).

As the bases of the approximated MAS are constructed by $n \times L$ ASs and $p \times L$ CPs, we consider the boundary conditions, which are expressed by the continuity of the tangential component of the total electric and magnetic fields for each strip. Then, these boundary conditions can be developed as:

For the first strip:
For CP=1:

$$\sum_{n=1}^{L} J_{1n} E_n^{sc1} = -E_{11}^{inc} \tag{9}$$

For CP=i:

$$\sum_{n=1}^{L} J_{1n} E_n^{sci} = -E_{1i}^{inc} \tag{10}$$

For CP=L:

$$\sum_{n=1}^{L} J_{1n} E_n^{scL} = -E_{1L}^{inc} \tag{11}$$

For the pth strip:
For CP=1:

$$\sum_{n=1}^{L} J_{pn} E_n^{sc1} = -E_{p1}^{inc} \tag{12}$$

For CP=i:

$$\sum_{n=1}^{L} J_{pn} E_n^{sci} = -E_{pi}^{inc} \tag{13}$$

For CP=L:

$$\sum_{n=1}^{L} J_{pn} E_n^{scL} = -E_{pL}^{inc} \tag{14}$$

The field generated by the auxiliary sources and distributed on the ith auxiliary strip where the collocation point on the ith strip can be represented by the following linear system:

$$A_{ii} J_i = -E_i^{inc} \tag{15}$$

Where A_{ii} is the elementary matrix describing the ith strip, J_i is the vector of the complex unknown current in the ith strip and E_i is the electric incident vector on the ith strip.

The coupling of the strips is obtained from the calculation of the field on the collocation point on the ith strip generated by the auxiliary sources distributed on the jth auxiliary strip:

$$A_{ij} J_j = -E_i^{inc} \tag{16}$$

Where A_{ij} is the elementary matrix describing the interaction between the jth auxiliary strip and the ith collocation strip.

The equations in Eq.10 and Eq.11 represent all the structure, so we can formulate a n ew linear equation which is written as:

$$AJ = E \tag{17}$$

Where A is the square matrix representing physically the structure, J is the complex unknown current and E the total electric incident field.

The auxiliary sources, on each auxiliary strip, radiate and act on the physical strip. We can construct a square elementary matrix A_{ij} (i=1,2,3,...,p; j=1,2,3,...,p) representing the interaction of jth auxiliary strip with the ith strip of collocation points.

$$\mathcal{A}_{ij} = \begin{pmatrix} E^{sc,m=1}_{n=1} & E^{sc,m=1}_{n=2} & \cdots & E^{sc,m=1}_{n=L} \\ E^{sc,m=2}_{n=1} & E^{sc,m=2}_{n=2} & \cdots & E^{sc,m=2}_{n=L} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ E^{sc,m=L}_{n=1} & E^{sc,m=L}_{n=2} & \cdots & E^{sc,m=L}_{n=L} \end{pmatrix}_{(ij)} \quad (18)$$

Where $E^{sc,m=1}_{n=1}$ is the electric field radiated by the auxiliary source number n=1 on the collocation point number m=1.

As the structure consists of p strips, we obtain $p \times p$ square elementary matrix. The linear system describing the problem can be now written as:

$$\begin{bmatrix} |A_{11}| & \cdots & |A_{1p}| \\ |A_{21}| & \cdots & |A_{2p}| \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ |A_{(p-1)1}| & \cdots & |A_{(p-1)p}| \\ |A_{p1}| & \cdots & |A_{pp}| \end{bmatrix} \begin{bmatrix} J_1 \begin{bmatrix} J_{11} \\ \vdots \\ J_{1L} \end{bmatrix} \\ \vdots \\ J_p \begin{bmatrix} J_{p1} \\ \vdots \\ J_{pL} \end{bmatrix} \end{bmatrix} = - \begin{bmatrix} E_1^{inc} \begin{bmatrix} E_{11}^{inc} \\ \vdots \\ E_{1L}^{inc} \end{bmatrix} \\ \vdots \\ E_p^{inc} \begin{bmatrix} E_{p1}^{inc} \\ \vdots \\ E_{pL}^{inc} \end{bmatrix} \end{bmatrix}$$

J_i (i= 1, 2, 3,..., p) is the complex current vector describing the current generated by the auxiliary sources in the auxiliary strip number i.

E_i^{inc} (i= 1, 2, 3,..., p) is the incident electric vector acting on the L collocation points of the strip number i.

In the following section, an application of the approximated MAS will be presented and discussed in order to validate this method.

3. Numerical results

In order to illustrate the computational efficiency and accuracy and to demonstrate the advantage of the approximated method in solving electrically large problems, we consider a p erfectly conducting cylinder with large-size of radius. To validate the analysis presented in section 2, several numerical results are extracted and compared to reference data, whenever available. In this section, many examples will be presented and discussed.

Therefore, the comparison of the results is based on values of the scattered power from the object which is characterized by the cross section, defined in [12], for 2D problem, as:

$$SW = \lim_{\rho \to +\infty} \left[2\pi\rho \frac{|E^{sca}|^2}{|E^{inc}|^2} \right] \quad (19)$$

The convergence of the approximated solution is based on the value of the boundary condition error for the boundary which is defined as the ratio of the absolute difference between the tangential electric fields intensity around the considered boundary to the maximum magnitude of the corresponding incident field:

$$\Delta E_{bc} = \frac{\left\| E^{sc} + E^{inc} \right\|}{max \left\| E^{inc} \right\|} \quad (20)$$

The convergence rate and the accuracy of the method are only dependent on the number of auxiliary sources N and the distance d_{aux} between auxiliary surface and the boundary. According to MAS, the approximate solution of the boundary problem will tend to exact solution as $N \to \infty$. Therefore, the convergence is ensured [13]. The incident wave frequency is set to 300 MHz.

The Fig.5 shows the RCS of the PEC circular cylinder having b=9.6λ illuminated by a transverse magnetic (TM) with p=43 strips with L=3 auxiliary sources for each one and an auxiliary distance d_{aux}=0.145b and angle $\theta = \pi/p$. A good agreement is obtained between the approximated MAS and the Finite Volume Time Domain (FVTD) on Fig.6, presented in [14].

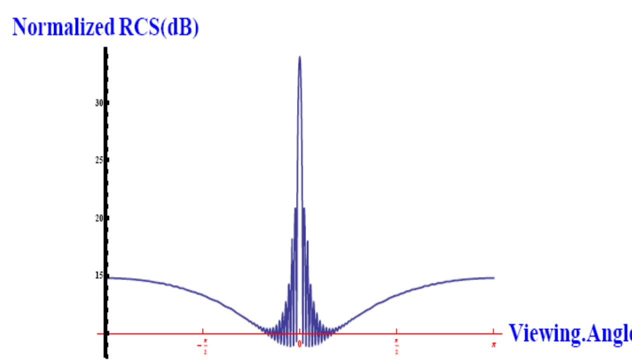

Fig. 5 The normalized scattering cross-section of cylinder with b=9.6λ of radius simulated with approximated MAS.

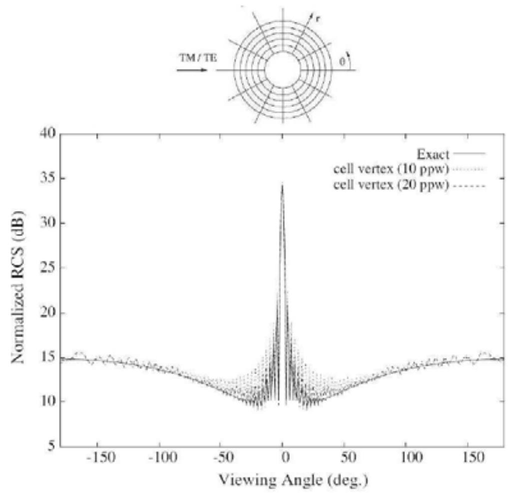

Fig. 6 The normalized scattering cross-section of cylinder with b=9.6λ of radius in [3].

In the second experiment, we consider a cylinder having a radius of size b=40λ. The simulation of the structure allows us to obtain the Fig.7 that is shows a good agreement with the reference for the same structure simulated using the method named Localized Iterative Generalized Multipole Technique (LIGMT) presented in [3]. In this case the convergence of the approximated MAS is obtained for p=180 and L=3, thus a number of auxiliary sources equals N=540 auxiliary sources and an auxiliary distance d_{aux}=0.09b.

In these examples, the number of auxiliary sources required to achieve convergence of the approximated MAS is reduced compared with the results obtained for the standard MAS. Indeed, the number of auxiliary sources that leads to the convergence of the MAS when the radius is b=9.6 λ is $N = p \times L = 129\ AS$, which was equal to N=185 using the standard MAS. For the radius b=40λ, the number of auxiliary sources is $N = 540$ ASs, which was 600 ASs for the standard MAS.

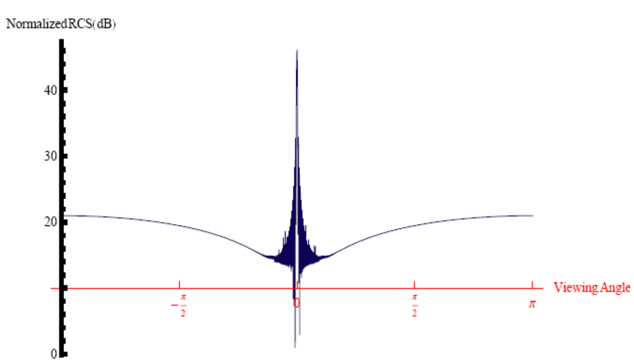

Fig. 7 The normalized scattering cross-section of cylinder with b=40λ of radius simulated with approximated MAS.

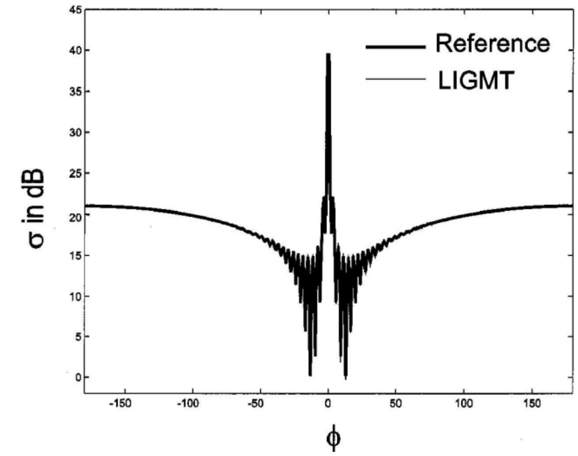

Fig. 8 The normalized scattering cross-section of cylinder with b=40λ of radius in [14].

The maximum boundary condition error along the boundary predicted by the approximated MAS (Fig.9) is 0.145% for cylinder with radius of b=9.6λ and the error given by standard MAS is the same (Fig.10). In addition, the error is minimized to 0.24% in the case of a P EC cylinder with radius of the is b=40λ which was reduced compared with LIGMT in [3] where the error on boundary reaches 1%. Therefore, we can deduce that approximated MAS solution is more accurate than LIGMT.

The result in Fig.5, where b=9.6λ, is the same obtained using the standard MAS, but the number of auxiliary sources for the approximated one is reduced. Indeed, this number is equal to N=L × p=129 AS for the approximated MAS but is N=158 AS giving the same error 0.145%.
For the cylinder having a radius b=40λ, the same error 0.24% is obtained using 540 AS for the approximated MAS and 600 AS.

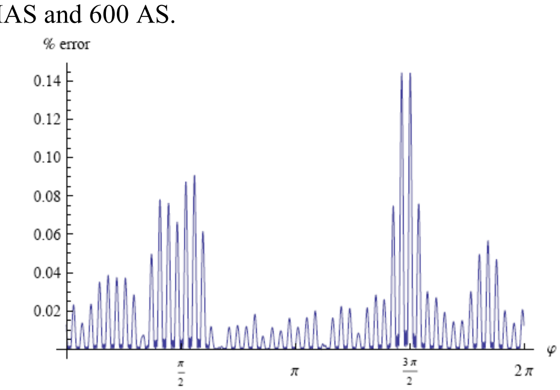

Fig. 9 Boundary condition error for large circular cylinder with radius b=9.6 λ using the approximated MAS.

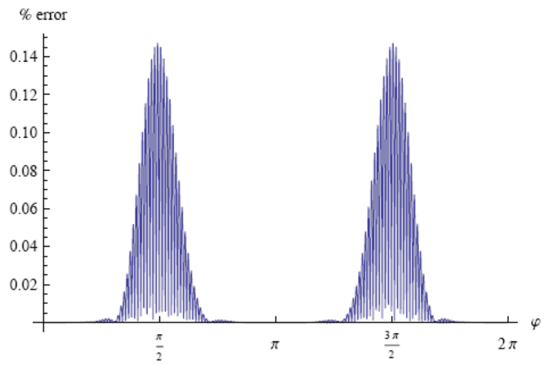

Fig. 10 Boundary condition error for large circular cylinder with radius b=9.6 λ using the standard MAS.

The approximated MAS offers a reduction of the number of auxiliary sources, which implies a reduction of the size of the matrix A. In addition, a comparison between the computational time taken to reach the convergence for the standard MAS and the approximated MAS, we deduce that the time required is minimized.
The perimeter of the physical part of the cylinder PEC considered in the study is reduced by $p \times \theta$. Thus, the complexity of the problem is also reduced.

4. Conclusion

In this paper, we have developed numerical approximated MAS to the scattering problems by large-size objects. Firstly, the structure is cut into a finite number of strips, separated by a constant polar angle, which is also discretized into L collocation points. Secondly, the standard MAS is applied for each separately, thus, the problem complexity is reduced by minimizing the matrix size of the linear system describing the problem.

The approximated MAS reduces the computational cost and memory size needed to simulate large-size objects gives a simplified formulation of the standard MAS.

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