# On Tandem Communication Network Model with DBA and Modified Phase Type Transmission having NHP Arrivals for First Node and Poisson process arrivals for Second Node

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#### Abstract

Communication network models play a predominant role in performance evaluation of many communication systems. The packet arrival processes for data networks are not matching with the Poisson processes due to the nature of bursty and time dependent arrivals. In this paper, a three node communication network model with non homogeneous Poisson arrivals having dynamic bandwidth allocation under modified phase type transmission is introduced for performance evaluation and monitoring of several tele and satellite communications. The system performance measures of the network are derived explicitly. The sensitivity analysis reveals the dynamic bandwidth allocation strategy and non homogeneous Poisson arrivals can reduce the burstyness in buffers and delay in transmission. This model also includes some of the earlier models as particular cases for specific and limiting of parameters

**Keywords:** Tandem Communication Network, Dynamic Bandwidth Allocation, non homogeneous Poisson process, Performance evaluation.

# 1. Introduction

William Cock and Charles Wheatstone (1839) have pioneered the mathematical theory of communication. Thereafter in 1948 a tremendous revolution in communication network modelling is brought by Barden and Barttain. Later AT&T Bell labs, USA network group at MIT, IEEE communication society and other reputed organizations have put considerable efforts for invoking and designing efficient communication systems. (IEEE Communication Society, 2002). Conducting laboratory experiments under variable load conditions is highly complex and time consuming. Hence for efficient design and evaluation of communication networks, the network models are developed and analyzed with various assumptions on constituent processes of the model like arrival process, service process, flow control mechanisms, allocations, routing, etc,. To improve Quality of Service, packet switching gives better utilization over circuit or message switching and yields relatively shorter delay in statistical multiplexing in communication network can reduce the delay in packet switching. Many of communication networks which support tele processing applications are mixed with dynamic engineering skills and statistical multiplexing (Gaujal and Hyon (2002), Parthasarathy et al(2001), Srinivasa Rao et al (2000))

To reduce the congestion in buffers the dynamic bandwidth allocation is evolved as an alternative and efficient control strategy over bit dropping or flow control strategies (Sriram et al (1993), Suresh Varma et al (2007), Padmavathi et al (2009), Nageswara Rao et al (2010)). In DBA, a large portion of the unutilized bandwidth is utilized by changing the transmission rate of packets depending on the content of the buffer connected to it. Much work has been reported recently regarding communication network model with dynamic bandwidth ( Nageswara Rao et al (2011)). In all these papers, the authors assumed that the arrival rate of packets is constant and follows a Poisson process.

IJČSI www.IJCSI.org But in many communication systems the assumption regarding Poisson process are seldom satisfied due to the time dependent nature of arrivals. Therefore, many phenomenon encountered in communication systems which reveal time dependent behaviour of arrival process due to the factors like work load fluctuations, initiating, failures, congestion and flow control, overload peaks, reconfigurations, adaptive routing and others. So to have an efficient performance evaluation entities problem of communication systems like adaptive isolated routing and load balancing, evaluating alternative buffer of changed sharing schemes and effects and to study the effects of flow and control strategies it is needed to develop communication network models with time dependent arrivals and time dependent analysis. In addition to this, Rakesh Singhai et al(2007) have showed that the packet arrival process of heavy tail distribution inter arrivals form a non homogeneous Poisson process and the mean packet arrival rate is not constant but it is time varying. This is also supported by the studies Feldmann (2000), who demonstrated the TCP connection arrival process is bursty and time dependent. The time dependent arrival process can be well characterized by NHP which follows Poisson process.

Very little work has been reported in literature regarding communication network models with direct arrivals to first two nodes having modified phase type transmission and dynamic bandwidth allocation with non homogeneous Poisson arrivals. Hence, in this paper a communication network model is developed and analyzed for these sort of situations. Here it is assumed that the messages arrive to the first and second buffers directly with time dependent arrival rate. After getting transmitted from first buffer the packets may join the second buffer connected to the second node in tandem with first node or get terminated with certain probability. In the second buffer the packets are from first node and directly from outside the network. After getting transmitted from the second node the packets may be routed to the third buffer connected to the third node or get transmitted with certain probability.

The direct arrival to the second node will have tremendous influence on congestion control of the communication system and reduce burstness in the first buffer. Here also it is assumed that transmission completion of each node follows Poisson process with different transmission rates. The dynamic bandwidth allocation strategy is adopted to utilize the ideal bandwidth at nodes and to improve transmission capabilities.

Using difference differential equations the joint probability generating function of the number of packets in each buffer is derived. The performance of the communication network is studied by deriving explicitly the performance measures of the network like the average number of packets in each buffer, the throughput of the node, the mean delay in buffers, the utilization of the transmitters, the variability of the buffer content etc,. The sensitivity of the model with respect to the parameters is also carried. A comparative study of the proposed model with that of Poisson arrivals is also presented. This model is much useful for evaluating several communication systems where the arrivals are time dependent.

# 2. Tandem Communication Network Model with DBA and Modified Phase Type Transmission having NHP Arrivals for First Node and Poisson process arrivals for Second Node:

In this section, a communication network model having three nodes in tandem is studied. The arrivals to the buffer connected at node one is assumed to follow a nonhomogeneous Poisson process with mean arrival rate as a function of time t. It is of the form  $\lambda(t) = \lambda + \alpha t$ . The transmission process from node one to node two follows a Poisson process with parameter  $\mu_1$  It is also assumed that the packets arrive to the second buffer directly from out side of the network in a Poisson process with mean arrival rate  $\epsilon$ . After getting transmitted from node one the packets are forwarded to the second buffer for transmission with probability  $\theta$  or get terminated with probability (1- $\theta$ ) i.e., the packets arrived at second buffer contains the packets received from first node and directly from outside. After getting transmitted from second node the packet are forwarded to the third buffer for transmission with probability  $\pi$  or get terminated with probability (1- $\pi$ ). The transmission process of node two and three also follow Poisson process with parameters  $\mu_2$  and  $\mu_3$  respectively. The transmission rate of each packet is adjusted just before transmission depending on the content of the buffer connected to the transmitter. The packets are transmitted through the transmitters by the first in first out discipline. The schematic diagram representing the communication network model is shown in Figure 1



Figure 1: Schematic diagram of the Communication Network Model

Let  $P_{n_1,n_2,n_3}(t)$  denote the probability that there are  $n_1$  packets in the first buffer and  $n_2$  packets in the second buffer and  $n_3$  packets in the third buffer at time t The difference-differential equations of the network are

$$\begin{split} \frac{\partial P_{n_i,n_2,n_3}(t)}{\partial t} &= - \left( \dot{\lambda}(t) + \varepsilon + n_1 \mu_1 + n_2 \mu_2 + n_3 \mu_3 \right) P_{n_i,n_2,n_3}(t) + \dot{\lambda}(t) P_{n_i-l,n_2,n_3}(t) + (n_1 + 1) \mu_1 \theta P_{n_i+l,n_2-l,n_3}(t) \\ &+ (n_1 + 1) \mu_1 (1 - \theta) P_{n_i+l,n_2,n_3}(t) + \varepsilon P_{n_i,n_2-l,n_3}(t) + (n_2 + 1) \mu_2 \pi P_{n_i,n_2+l,n_3-l}(t) \\ &+ (n_2 + 1) \mu_2 (1 - \pi) P_{n_i,n_3+l,n_3}(t) + (n_3 + 1) \mu_3 P_{n_i,n_2,n_3+l}(t), \qquad n_1 > 0, n_2 > 0, n_3 > 0 \end{split}$$

$$\begin{split} \frac{\partial P_{n_i,n_2,0}(t)}{\partial t} &= -\left(\lambda(t) + \varepsilon + n_1\mu_1 + n_2\mu_2\right) P_{n_i,n_2,0}(t) + \lambda(t)P_{n_i-1,n_2,0}(t) + (n_1+1)\mu_i \theta P_{n_i+1,n_2-1,0}(t) \\ &+ (n_1+1)\mu_i (1-\theta)P_{n_i+1,n_2,0}(t) + \varepsilon P_{n_i,n_2-1,0}(t) + (n_2+1)\mu_2 (1-\pi)P_{n_i,n_2+1,0}(t) \\ &+ \mu_3 P_{n_i,n_2,1}(t), \qquad n_1 > 0, n_2 > 0 \end{split}$$

$$\begin{aligned} \frac{\partial t_{0,n_2,n_3}(t)}{\partial t} &= -\left(\lambda(t) + \varepsilon + n_2\mu_2 + n_3\mu_3\right)P_{0,n_2,n_3}(t) + \mu_1 \theta P_{1,n_2-1,n_3}(t) + \mu_1(1-\theta)P_{1,n_2,n_3}(t) \\ &+ \varepsilon P_{0,n_2-1,n_3}(t) + (n_2+1)\mu_2\pi P_{0,n_2+1,n_3-1}(t) + (n_2+1)\mu_2(1-\pi)P_{0,n_2+1,n_3}(t) \\ &+ (n_3+1)\mu_3P_{0,n_2,n_3+1}(t) , \qquad n_2 > 0, n_3 > 0 \end{aligned}$$

$$\frac{\partial \xi_{n_1,0,n_2}(\mathbf{r})}{\partial t} = -\left(\lambda(t) + \varepsilon + n_1\mu_1 + n_3\mu_3\right)P_{n_1,0,n_3}(t) + \lambda(t)P_{n_1-1,0,n_3}(t) + (n_1+1)\mu_1(1-\theta)P_{n_1+1,0,n_3}(t) + \mu_2\pi P_{n_1,1,n_3-1}(t) + \mu_2(1-\pi)P_{n_1,1,n_3}(t) + (n_3+1)\mu_3P_{n_1,0,n_3+1}(t), \qquad n_1 > 0, n_3 > 0$$

$$\begin{aligned} \frac{\partial P_{n_i,0,0}(t)}{\partial t} &= -\left(\lambda(t) + \varepsilon + n_1\mu_1\right)P_{n_i,0,0}(t) + \lambda(t)P_{n_i-1,0,0}(t) + (n_1+1)\mu_1(1-\theta)P_{n_i+1,0,0}(t) \\ &+ \mu_2(1-\pi)P_{n_i,1,0}(t) + \mu_3P_{n_i,0,1}(t), \qquad n_1 > 0 \end{aligned}$$

$$\begin{split} \frac{\partial P_{0,n_2,0}(t)}{\partial t} &= -\left(\lambda(t) + \varepsilon + n_2\mu_2\right) P_{0,n_2,0}(t) + \mu_1 \left(1 - \theta\right) P_{1,n_2,0}(t) + \mu_1 \theta P_{1,n_2-1,0}(t) \\ &+ \varepsilon P_{0,n_2-1,0}(t) + (n_2 + 1)\mu_2 \left(1 - \pi\right) P_{0,n_2+1,0}(t) + \mu_3 P_{0,n_2,1}(t), \qquad n_2 > 0 \end{split}$$

$$\frac{\partial P_{0,0,n_3}(t)}{\partial t} = -\left(\lambda(t) + \varepsilon + n_3\mu_3\right)P_{0,0,n_3}(t) + \mu_1\left(1 - \theta\right)P_{1,0,n_3}(t) + \mu_2\pi P_{0,1,n_3-1}(t) + \mu_2\left(1 - \pi\right)P_{0,1,n_3}(t) + (n_3 + 1)\mu_3P_{0,0,n_3+1}(t), \qquad n_3 > 0$$

$$\frac{\partial P_{0,0,0}(t)}{\partial t} = -(\lambda(t) + \varepsilon)P_{0,0,0}(t) + \mu_1(1-\theta)P_{1,0,0}(t) + \mu_2(1-\pi)P_{0,1,0}(t) + \mu_3P_{0,0,1}(t)$$
(2.1)

Let 
$$P(s_1, s_2, s_3; t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} P_{n_1, n_2, n_3}(t) s_1^{n_1} s_2^{n_2} s_3^{n_3}$$
 be the joint

probability generating function of  $P_{n_1,n_2,n_3}(t)$ . (2.2)

Multiplying the equation (5.2.1) with  $S_1^{n_1}S_2^{n_2}S_3^{n_3}$  and summing over all  $n_1$ ,  $n_2$  and  $n_3$  we get

$$\begin{aligned} \frac{dP}{dt} &= -\lambda(t) P(s_1, s_2, s_3; t) + \lambda(t) s_1 P(s_1, s_2, s_3; t) - \varepsilon P(s_1, s_2, s_3; t) \\ &+ \varepsilon s_2 P(s_1, s_2, s_3; t) - \mu_1 s_1 \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_1} + \mu_1 s_2 \theta \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_1} \\ &+ \mu_1 (1 - \theta) \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_1} - \mu_2 s_2 \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_2} + \mu_2 s_3 \pi \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_2} \\ &+ \mu_2 (1 - \pi) \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_2} - \mu_3 s_3 \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_3} + \mu_3 \frac{\partial P(s_1, s_2, s_3; t)}{\partial s_3} \end{aligned}$$
(2.3)

After simplifying, we get

$$\frac{dP}{dt} = \lambda(t)(s_1 - 1)P(s_1, s_2, s_3; t) + \varepsilon(s_2 - 1)P(s_1, s_2, s_3; t) - \mu_1(s_1 - 1 - \theta(s_2 - 1)))\frac{\partial P(s_1, s_2, s_3; t)}{\partial s_1} - \mu_2(s_2 - 1 - \pi(s_3 - 1)))\frac{\partial P(s_1, s_2, s_3; t)}{\partial s_2} - \mu_3(s_3 - 1)\frac{\partial P(s_1, s_2, s_3; t)}{\partial s_3}$$

$$(2.4)$$

Solving the equation (5.2.4) by Lagrangian's method, the auxiliary equations are

$$\frac{dt}{1} = \frac{ds_1}{\mu_1(s_1 - 1 + \theta(1 - s_2))} = \frac{ds_2}{\mu_2(s_2 - 1 + \pi(1 - s_3))} = \frac{ds_3}{\mu_3(s_3 - 1)}$$
$$= \frac{dP}{\lambda(t)P(s_1, s_2, s_3; t)(s_1 - 1) + \varepsilon P(s_1, s_2, s_3; t)(s_2 - 1)}$$
(2.5)

To solve the equations in (2.5) the functional form of  $\lambda(t)$  is required. Let the mean arrival rate of packets is  $\lambda(t) = \lambda + \alpha t$ , where  $\lambda > 0$ ,  $\alpha > 0$  are constants.

Solving the first and fourth terms in equation (2.5), we get

$$a = (s_3 - 1)e^{-\mu_3 t} \tag{2.6a}$$

Solving the first and third terms in equation (2.5), we get

$$b = (s_2 - 1)e^{-\mu_2 t} + \frac{\pi\mu_2}{\mu_3 - \mu_2}(s_3 - 1)e^{-\mu_2 t}$$
(2.6b)

Solving the first and second terms in equation (2.5), we get

$$c = (s_1 - 1)e^{-\mu_1 t} + \frac{\theta \mu_1}{\mu_2 - \mu_1} (s_2 - 1)e^{-\mu_1 t} + \frac{\pi \theta \mu_1 \mu_2}{(\mu_3 - \mu_1)(\mu_2 - \mu_1)} (s_3 - 1)e^{-\mu_1}$$
(2.6c)

Solving the first and fifth terms in equation (2.5), we get  

$$d = P(s_1, s_2, s_3; t) \exp\left\{-\left[\frac{(s_1-1)}{\mu_1}\left(\lambda + \alpha t - \frac{\alpha}{\mu_1}\right) + \frac{(s_2-1)}{\mu_2}\left(\varepsilon + \theta\left(\lambda + \alpha t - \frac{\alpha(\mu_1 + \mu_2)}{\mu_1\mu_2}\right)\right)\right] + \frac{(s_3-1)\pi}{\mu_3}\left(\varepsilon + \theta\left(\lambda + \alpha t - \frac{\alpha(\mu_1\mu_2 + \mu_2\mu_3 + \mu_1\mu_3)}{\mu_1\mu_2\mu_3}\right)\right)\right]\right\}$$
(2.6d)

where, a, b, c and d are arbitrary constants.

Using the initial conditions  $P_{000}(0)=1$ ,  $P_{000}(t)=0 \quad \forall t>0$ . The general solution of (2.5) gives the probability generating function of the number of packets in the first, second and third buffers at time t, as

$$P(s_{1}, s_{2}, s_{1}; t) = \exp\left\{\frac{(s_{1}-1)}{\mu_{1}}\left(1-e^{-\mu_{1}}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right) + \frac{(s_{1}-1)\alpha t}{\mu_{1}} + \left(\frac{(s_{2}-1)}{\mu_{2}}\right)\left(1-e^{-\mu_{2}}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)\right) + \left(\frac{(s_{2}-1)\theta}{\mu_{2}}\right)\alpha t + \left(\frac{(s_{2}-1)\theta}{\mu_{2}-\mu_{1}}\right)\left(e^{-\mu_{2}}-e^{-\mu_{2}}\left(\lambda-\frac{\alpha}{\mu_{1}}\right) + \left(\frac{(s_{3}-1)\pi}{\mu_{3}}\right)\left(1-e^{-\mu_{2}}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{3}}+\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)\right) + \left(\frac{(s_{3}-1)\pi \theta \alpha t}{\mu_{3}}\right) + \left(\frac{(s_{3}-1)\pi \theta}{\mu_{3}-\mu_{2}}\right)\left(e^{-\mu_{2}}-e^{-\mu_{2}}\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}-\frac{1}{\mu_{1}}\right)\right)\right) + \left(s_{2}-1\right)\pi \theta \mu_{2}\left(\frac{e^{-\mu_{2}}}{(\mu_{2}-\mu_{3})(\mu_{3}-\mu_{1})} + \frac{e^{-\mu_{2}}}{(\mu_{3}-\mu_{2})(\mu_{2}-\mu_{1})} + \frac{e^{-\mu_{2}}}{(\mu_{1}-\mu_{3})(\mu_{2}-\mu_{1})}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right)\right)$$

$$for \lambda < min \{\mu_{1}, \mu_{2}, \mu_{3}\}$$
(2.7)

# **3 PERFORMANCE MEASURE OF THE NETWORK:**

In this section, we derive and analyze the performance measures of the communication network under transient conditions. Expanding  $P(s_1,s_2, s_3;t)$  given in equation (2.7) and collecting the constant terms, we get the probability that the network is empty as

$$\begin{split} P_{000}(t) &= \exp\left\{-\left[\frac{1}{\mu_{1}}\left(1-e^{-\mu_{1}t}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right)+\frac{\alpha t}{\mu_{1}}+\frac{1}{\mu_{2}}\left(1-e^{-\mu_{2}t}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)\right.\\ &+\frac{\theta\alpha t}{\mu_{2}}+\frac{\theta}{\mu_{2}-\mu_{1}}\left(e^{-\mu_{2}t}-e^{-\mu_{1}t}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right)+\frac{\pi}{\mu_{3}}\left(1-e^{-\mu_{2}t}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{3}}+\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)\right.\\ &+\frac{\pi\theta\alpha t}{\mu_{3}}+\frac{\pi\theta}{(\mu_{3}-\mu_{2})}\left(e^{-\mu_{2}t}-e^{-\mu_{2}t}\right)\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\\ &+\pi\theta\mu_{2}\left(\frac{e^{-\mu_{2}t}}{(\mu_{2}-\mu_{3})(\mu_{3}-\mu_{1})}+\frac{e^{-\mu_{2}t}}{(\mu_{3}-\mu_{2})(\mu_{2}-\mu_{1})}+\frac{e^{-\mu_{2}t}}{(\mu_{1}-\mu_{3})(\mu_{2}-\mu_{1})}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right)\right]\right\} \end{split}$$

$$(3.1)$$

Taking  $s_2 = 1$ ,  $s_3 = 1$  in equation (2.7), we get the probability generating function of the first buffer size as

$$P(s_1, t) = \exp\left\{\frac{(s_1 - 1)}{\mu_1} \left(\lambda - \frac{\alpha}{\mu_1}\right) \left(1 - e^{-\mu_1 t}\right) + \frac{(s_1 - 1)}{\mu_1} \alpha t\right\}$$
for  $\lambda < \mu_1$  (3.2)

Expanding P( $s_1$ ,t) and collecting the constant terms, we get the probability that the first buffer is empty as

$$P_{0.}(t) = \exp\left\{-\left[\frac{1}{\mu_{1}}\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\left(1 - e^{-\mu_{1}t}\right) + \frac{1}{\mu_{1}}\alpha t\right]\right\} \quad (3.3)$$

Taking  $s_1 = 1$ ,  $s_2 = 1$  in equation (2.7), we get the probability generating function of the second buffer size as

$$P(s_{2},t) = \exp\left\{\frac{(s_{2}-1)}{\mu_{2}}\left(1 - e^{-\mu_{2}t}\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{(s_{2}-1)\theta}{\mu_{2}}\alpha t + \frac{(s_{2}-1)\theta}{\mu_{2} - \mu_{1}}\left(e^{-\mu_{2}t} - e^{-\mu_{1}t}\right)\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right)\right\}$$
  
for  $\lambda < \min\left\{\mu_{1}, \mu_{2}\right\}$  (3.4)

Expanding  $P(s_2,t)$  and collecting the constant terms, we get the probability that the second buffer is empty as

$$P_{\underline{\alpha}}(t) = \exp\left[-\left[\frac{1}{\mu_{2}}\left(1 - e^{-\mu_{2}t}\right)\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\theta}{\mu_{2}}\alpha t + \frac{\theta}{\mu_{2} - \mu_{1}}\left(e^{-\mu_{2}t} - e^{-\mu_{1}t}\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right)\right]$$
(3.5)

Taking  $s_1 = 1$ ,  $s_2 = 1$  in equation (2.7), we get the probability generating function of the third buffer size as

$$P(s_{3},t) = \exp\left\{\frac{(s_{1}-1)\pi}{\mu_{3}}\left(1-e^{-\mu t}\right)\left\{\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{3}}+\frac{1}{\mu_{4}}+\frac{1}{\mu_{4}}\right)\right)\right) + \frac{(s_{1}-1)\pi\theta \,\alpha t}{\mu_{3}} + \frac{(s_{1}-1)\pi\theta}{\mu_{3}-\mu_{2}}\left(e^{-\mu t} - e^{-\mu t}\right)\left(\lambda - \alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{4}}\right)\right)\right) + (s_{1}-1)\mu_{2}\pi\theta\left(\frac{e^{-\mu t}}{(\mu_{2}-\mu_{1})(\mu_{3}-\mu_{4})} + \frac{e^{-\mu t}}{(\mu_{3}-\mu_{2})(\mu_{2}-\mu_{4})} + \frac{e^{-\mu t}}{(\mu_{4}-\mu_{4})(\mu_{2}-\mu_{4})}\right)\left(\lambda - \frac{\alpha}{\mu_{4}}\right)\right]$$
for  $\lambda < \min\{\mu_{1}, \mu_{2}, \mu_{3}\}$  (3.6)

Expanding P( $s_3$ ,t) and collecting the constant terms, we get the probability that the third buffer is empty as

$$P_{,0}(t) = \exp\left\{-\left|\frac{\pi}{\mu_{3}}\left(1 - e^{-\mu_{3}}\right)\left(\mathcal{E} + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{3}} + \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\pi\theta\alpha_{1}}{\mu_{3}} + \frac{\pi\theta}{\mu_{3} - \mu_{2}}\left(e^{-\mu_{3}} - e^{-\mu_{3}}\right)\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right) + \frac{\mu_{2}\pi\theta\left(\frac{e^{-\mu_{3}}}{(\mu_{2} - \mu_{3})(\mu_{3} - \mu_{1})} + \frac{e^{-\mu_{3}}}{(\mu_{3} - \mu_{2})(\mu_{2} - \mu_{1})} + \frac{e^{-\mu_{3}}}{(\mu_{1} - \mu_{3})(\mu_{2} - \mu_{1})}\right)\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right]\right\}$$

$$(3.7)$$

The mean number of packets in the first buffer is

$$L_{1}(t) = \frac{\partial P(s_{1}, t)}{\partial s_{1}}\Big|_{s_{1}=1} = \frac{1}{\mu_{1}} \left(\lambda - \frac{\alpha}{\mu_{1}}\right) \left(1 - e^{-\mu_{1}t}\right) + \frac{1}{\mu_{1}} \alpha t$$
(3.8)

The utilization of the first transmitter is

$$U_{1}(t) = 1 - P_{0..}(t) = 1 - \exp\left\{-\left[\frac{1}{\mu_{1}}\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\left(1 - e^{-\mu_{1}t}\right) + \frac{1}{\mu_{1}}\alpha t\right]\right\}$$
(3.9)

The mean number of the packets in second buffer is

$$L_{2}(t) = \frac{\partial P(s_{2}, t)}{\partial s_{2}}\Big|_{s_{2}=1} = \frac{1}{\mu_{2}} \Big(1 - e^{-\mu_{2}t} \left(\varepsilon + \theta \left(\lambda - \alpha \left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{1}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{2}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{2}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{2}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\alpha}{\mu_{2}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} \left(\lambda - \frac{\theta}{\mu_{2}}\right)\right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2}} \left(e^{-\mu_{2}t} - e^{-\mu_{2}t} + e^{-\mu_{2}t} \left(\mu_{2} - \frac{\theta}{\mu_{2}}\right)\right)$$

The utilization of the second transmitter is

$$U_{2}(t) = 1 - P_{.0}(t) = 1 - \exp\left\{-\left[\frac{1}{\mu_{2}}\left(1 - e^{-\mu_{2}t}\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\theta}{\mu_{2}}\alpha t + \frac{\theta}{\mu_{2} - \mu_{1}}\left(e^{-\mu_{2}t} - e^{-\mu_{1}t}\right)\right)\right\} + \frac{\theta}{\mu_{2}}\alpha t + \frac{\theta}{\mu_{2} - \mu_{1}}\left(\varepsilon^{-\mu_{2}t} - e^{-\mu_{1}t}\right)\right\}$$
(3.11)

The mean number of the packets in  $L_3(t) = \frac{\partial P(s_3, t)}{\partial s_3}\Big|_{s_3=1}$ 

$$=\frac{\pi}{\mu_{3}}\left(1-e^{-\mu_{4}}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{3}}+\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)+\frac{\pi\theta\alpha\,t}{\mu_{3}}+\frac{\pi\theta}{\mu_{3}-\mu_{2}}\left(e^{-\mu_{4}}-e^{-\mu_{4}}\right)\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)+\pi\theta\mu_{2}\left(\frac{e^{-\mu_{4}}}{(\mu_{2}-\mu_{3})(\mu_{3}-\mu_{1})}+\frac{e^{-\mu_{4}}}{(\mu_{3}-\mu_{2})(\mu_{2}-\mu_{1})}+\frac{e^{-\mu_{4}}}{(\mu_{1}-\mu_{3})(\mu_{2}-\mu_{1})}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right)$$
(3.12)

The utilization of the third transmitter is

$$U_{3}(t) = 1 - P_{,0}(t) = 1 - \exp\left\{-\left[\frac{\pi}{\mu_{3}}\left(1 - e^{-\mu_{3}}\right)\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{3}} + \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\pi\theta\alpha t}{\mu_{3}} + \frac{\pi\theta}{\mu_{3} - \mu_{2}}\left(e^{-\mu_{3}} - e^{-\mu_{3}}\right)\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right) + \pi\theta\mu_{2}\left(\frac{e^{-\mu_{3}}}{(\mu_{2} - \mu_{3})(\mu_{3} - \mu_{1})} + \frac{e^{-\mu_{3}}}{(\mu_{3} - \mu_{2})(\mu_{2} - \mu_{1})} + \frac{e^{-\mu_{3}}}{(\mu_{1} - \mu_{3})(\mu_{2} - \mu_{1})}\right)\left(\lambda - \alpha\left(\frac{1}{\mu_{1}} + \frac{1}{\mu_{1}}\right)\right)\right]\right\}$$
  
(3.13)

The variance of the number of packets in the first buffer is

$$\mathbf{V}_{1}(t) = \frac{1}{\mu_{1}} \left( \lambda - \frac{\alpha}{\mu_{1}} \right) \left( 1 - e^{-\mu_{1}t} \right) + \frac{1}{\mu_{1}} \alpha t$$
(3.14)

The variance of the number of packets in the second buffer is

$$V_{2}(t) = \frac{1}{\mu_{2}} \left( 1 - e^{-\mu_{2}t} \left( \mathcal{E} + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left( e^{-\mu_{2}t} - e^{-\mu_{1}t} \left( \lambda - \frac{\alpha}{\mu_{1}} \right) \right) \right)$$
(3.15)

The variance of the number of packets in the third buffer is

$$\begin{aligned} V_{3}(t) &= \frac{\pi}{\mu_{3}} \left( 1 - e^{-\mu t} \right) \left( \varepsilon + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{3}} + \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) + \frac{\pi \theta \alpha t}{\mu_{3}} + \frac{\pi \theta}{\mu_{3} - \mu_{2}} \left( e^{-\mu t} - e^{-\mu t} \right) \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \\ &+ \pi \theta \mu_{2} \left[ \frac{e^{-\mu t}}{(\mu_{2} - \mu_{3})(\mu_{3} - \mu_{1})} + \frac{e^{-\mu t}}{(\mu_{3} - \mu_{2})(\mu_{2} - \mu_{1})} + \frac{e^{-\mu t}}{(\mu_{1} - \mu_{3})(\mu_{2} - \mu_{1})} \right] \left( \lambda - \frac{\alpha}{\mu_{1}} \right) \end{aligned}$$

$$(3.16)$$

The throughput of the first transmitter is

$$\left(1-P_{0..}(t)\right) = \mu_{1}\left[1+\exp\left\{\frac{1}{\mu_{1}}\left(\lambda-\frac{\alpha}{\mu_{1}}\right)\left(1-e^{-\mu_{1}t}\right)+\frac{1}{\mu_{1}}\alpha t\right\}\right]$$
(3.17)

 $\sum$ 

The mean delay in the first buffer is

 $\mu_1$ 

$$W_{1}(t) = \frac{L_{1}(t)}{\mu_{1}\left(1 - P_{0..}(t)\right)} = \frac{\frac{1}{\mu_{1}} \left(\lambda - \frac{\alpha}{\mu_{1}}\right) \left(1 - e^{-\mu_{1}t}\right) + \frac{1}{\mu_{1}} \alpha t}{\mu_{1} \left[1 + \exp\left\{\frac{1}{\mu_{1}} \left(\lambda - \frac{\alpha}{\mu_{1}}\right) \left(1 - e^{-\mu_{1}t}\right) + \frac{1}{\mu_{1}} \alpha t\right\}\right]}$$
(3.18)

1 (

The throughput of the second transmitter is  

$$\mu_{2} \left( 1 - P_{.o.}(t) \right) = \mu_{2} \left[ 1 + \exp \left[ \frac{1}{\mu_{2}} \left( 1 - e^{-\mu_{1}t} \left( \varepsilon + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) + \frac{\theta}{\mu_{2}} \alpha t + \frac{\theta}{\mu_{2} - \mu_{1}} \left( e^{-\mu_{1}t} - e^{-\mu_{1}t} \left( \lambda - \frac{\alpha}{\mu_{1}} \right) \right) \right] \right]$$
(3.19)

The mean delay in the second buffer is

$$W_{2}(t) = \frac{L_{2}(t)}{\mu_{2}(1-P_{n}(t))}$$

$$= \frac{\frac{1}{\mu_{2}}\left(1-e^{-\mu_{2}}\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\theta}{\mu_{2}}\alpha t + \frac{\theta}{\mu_{2} - \mu_{1}}\left(e^{-\mu_{2}} - e^{-\mu_{1}}\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right)\right)}{\mu_{2}\left[1 + \exp\left[\frac{1}{\mu_{2}}\left(1-e^{-\mu_{2}}\right)\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\theta}{\mu_{2}}\alpha t + \frac{\theta}{\mu_{2} - \mu_{1}}\left(e^{-\mu_{2}} - e^{-\mu_{1}}\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right)\right)\right]}\right]}$$

$$(3.20)$$

The throughput of the third transmitter is

$$\mu_{3}(1-P_{,0}(t)) = \mu_{3} \left\{ 1 + \exp\left[\frac{\pi}{\mu_{3}}(1-e^{-\mu_{0}t})\left(\varepsilon + \theta\left(\lambda - \alpha\left(\frac{1}{\mu_{3}} + \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right)\right) + \frac{\pi\theta\,\alpha\,t}{\mu_{3}} + \frac{\pi\theta}{\mu_{3} - \mu_{2}}\left(e^{-\mu_{0}t} - e^{-\mu_{0}t}\left(\lambda - \alpha\left(\frac{1}{\mu_{2}} + \frac{1}{\mu_{1}}\right)\right) + \pi\theta\mu_{2}\left(\frac{e^{-\mu_{0}t}}{(\mu_{2} - \mu_{3})(\mu_{3} - \mu_{1})} + \frac{e^{-\mu_{0}t}}{(\mu_{3} - \mu_{2})(\mu_{2} - \mu_{1})} + \frac{e^{-\mu_{0}t}}{(\mu_{1} - \mu_{3})(\mu_{2} - \mu_{1})}\right)\left(\lambda - \frac{\alpha}{\mu_{1}}\right)\right] \right\}$$

$$(3.21)$$

The mean delay in the third buffer is

W<sub>3</sub>(t) = 
$$\frac{L_3(t)}{\mu_3 (1 - P_{..0}(t))}$$

$$\frac{\pi}{\mu_{3}}\left(1-e^{-\mu_{f}}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{3}}+\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)+\frac{\pi\theta\alpha t}{\mu_{3}}+\frac{\pi\theta}{\mu_{3}-\mu_{2}}\left(e^{-\mu_{f}}-e^{-\mu_{f}}\right)\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)\\ =\frac{1}{\mu_{3}\left(1-e^{-\mu_{f}}\left(\frac{e^{-\mu_{f}}}{\mu_{3}}\left(1-e^{-\mu_{f}}\right)\left(\varepsilon+\theta\left(\lambda-\alpha\left(\frac{1}{\mu_{3}}+\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)\right)+\frac{\pi\theta\alpha t}{\mu_{3}}+\frac{\pi\theta}{\mu_{3}-\mu_{2}}\left(e^{-\mu_{f}}-e^{-\mu_{f}}\right)\left(\lambda-\alpha\left(\frac{1}{\mu_{2}}+\frac{1}{\mu_{1}}\right)\right)\right)}{\pi\theta\mu_{2}\left(\frac{e^{-\mu_{f}}}{(\mu_{2}-\mu_{3})(\mu_{3}-\mu_{1})}+\frac{e^{-\mu_{f}}}{(\mu_{3}-\mu_{2})(\mu_{2}-\mu_{1})}+\frac{e^{-\mu_{f}}}{(\mu_{3}-\mu_{2})(\mu_{2}-\mu_{1})}\right)\left(\lambda-\frac{\alpha}{\mu_{1}}\right)\right]\right\}}$$
(3.22)

The mean number of packets in the entire network at time t is

$$\begin{split} L(t) &= \frac{1}{\mu_{1}} \left( 1 - e^{-\mu_{1}t} \left( \lambda - \frac{\alpha}{\mu_{1}} \right) + \frac{1}{\mu_{1}} \alpha t + \frac{1}{\mu_{2}} \left( 1 - e^{-\mu_{2}t} \left( \varepsilon + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) + \frac{\theta}{\mu_{2}} \alpha t \right) \\ &+ \frac{\theta}{\mu_{2} - \mu_{1}} \left( e^{-\mu_{2}t} - e^{-\mu_{1}t} \left( \lambda - \frac{\alpha}{\mu_{1}} \right) + \frac{\pi}{\mu_{3}} \left( 1 - e^{-\mu_{3}t} \right) \left( \varepsilon + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{3}} + \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) \right) \\ &+ \frac{\pi \theta \alpha t}{\mu_{3}} + \frac{\pi \theta}{\mu_{3} - \mu_{2}} \left( e^{-\mu_{3}t} - e^{-\mu_{1}t} \right) \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \\ &+ \pi \theta \mu_{2} \left( \frac{e^{-\mu_{3}t}}{(\mu_{2} - \mu_{3})(\mu_{3} - \mu_{1})} + \frac{e^{-\mu_{2}t}}{(\mu_{3} - \mu_{2})(\mu_{2} - \mu_{1})} + \frac{e^{-\mu_{1}t}}{(\mu_{1} - \mu_{3})(\mu_{2} - \mu_{1})} \right) \left( \lambda - \frac{\alpha}{\mu_{1}} \right) \end{split}$$

$$(3.23)$$

The variability of the number of packets in the network is

$$\operatorname{var}(N) = \frac{1}{\mu_{1}} \left( 1 - e^{-\mu_{1}} \left( \lambda - \frac{\alpha}{\mu_{1}} \right) + \frac{1}{\mu_{1}} \alpha t + \frac{\theta}{\mu_{2}} \left( 1 - e^{-\mu_{2}} \right) \left( \varepsilon + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) + \frac{\theta}{\mu_{2}} \alpha t \\ + \frac{\theta}{\mu_{2} - \mu_{1}} \left( e^{-\mu_{2}t} - e^{-\mu_{1}t} \right) \left( \lambda - \frac{\alpha}{\mu_{1}} \right) + \frac{\pi}{\mu_{3}} \left( 1 - e^{-\mu_{3}t} \right) \left( \varepsilon + \theta \left( \lambda - \alpha \left( \frac{1}{\mu_{3}} + \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \right) \right) \\ + \frac{\pi \theta \alpha t}{\mu_{3}} + \frac{\pi \theta}{\mu_{3} - \mu_{2}} \left( e^{-\mu_{3}t} - e^{-\mu_{3}t} \right) \left( \lambda - \alpha \left( \frac{1}{\mu_{2}} + \frac{1}{\mu_{1}} \right) \right) \\ + \pi \theta \mu_{2} \left( \frac{e^{-\mu_{3}t}}{(\mu_{2} - \mu_{3})(\mu_{3} - \mu_{1})} + \frac{e^{-\mu_{3}t}}{(\mu_{3} - \mu_{2})(\mu_{2} - \mu_{1})} + \frac{e^{-\mu_{1}t}}{(\mu_{1} - \mu_{3})(\mu_{2} - \mu_{1})} \right) \left( \lambda - \frac{\alpha}{\mu_{1}} \right)$$

(3.24)

# 4 PERFORMANCE EVALUATION OF THE NETWORK

In this section, the performance of the communication network proposed in this section is discussed through numerical illustration. Different values of the parameters are considered for bandwidth allocation and arrival of packets. After interacting with the technical staff at internet providing station, it is observed that the packet parameter  $\lambda$ varies from  $2x10^4$  packets/sec to  $7x10^4$  packets/sec,  $\alpha$ varies from -0.5 to 1.5,  $\theta$  varies from 0.1 to 0.9,  $\varepsilon$  varies from 0.2 to 0.6  $\pi$  varies from 0.1 to 0.9, with an average packet size of 53 bytes. After transmitting from node one, the forward transmission rate  $\mu_1$ varies from  $5x10^4$  packets/sec to  $9x10^4$ packets/sec. The rate of transmission from node two  $\mu_2$  varies from 15x10<sup>4</sup> packets/sec to 19x10<sup>4</sup> packets/sec. The rate of transmission from node three  $\mu_3$  varies from  $25 \times 10^4$  packets/sec to  $29 \times 10^4$  packets/sec. In all the nodes, the dynamic bandwidth allocation strategy is considered i.e., the transmission rate of each packet depends on the number of packets in the buffer connected to it at that instant.

From equations (3.9), (3.11), (3.13), (3.17), (3.19) and (3.21), the utilization of the transmitters and throughput of

three nodes are computed for different values of the parameters t,  $\lambda$ ,  $\alpha$ ,  $\theta$ ,  $\varepsilon$ ,  $\pi$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  are presented in table 1. The relationship between parameters and utilization of transmitters and throughput of three nodes are shown in Figure 1.

As the time (t) and parameter ( $\lambda$ ) increases, the utilization of transmitters are increases for fixed values of the other parameters. It is also observed that as the parameter  $(\alpha)$ increases, the utilization of transmitters at three nodes are increases for fixed values of the other parameters. The parameter ( $\theta$ ) increases, the utilization of transmitters at first node is fixed and second and third nodes are increasing for fixed values of the other parameters. The parameter ( $\varepsilon$ ) increases, the utilization of first node is constant and second and third nodes are increasing for fixed values of the other parameters. The parameter  $(\pi)$ increases, the utilization of transmitters at first and second nodes are fixed and the third node it is increases for fixed values of the other parameters. As the transmission rate  $(\mu_1)$  increases, the utilization of the first node decreases and the other two nodes increases when the other parameters remain fixed. As the transmission rate  $(\mu_2)$ increases, the utilization of first node is constant and the utilization of the second node decreases and third node increases when the other parameters remain fixed. Similarly, as the transmission rate  $(\mu_3)$  increases the utilization of the first and second nodes are fixed and third node is decreases when other parameters remain fixed.

It is observed as time value (t) increases, the throughput of first, second and third nodes are increasing for fixed values of the other parameters. As the parameter ( $\lambda$ ) varies from  $3x10^4$  packets/sec to  $7x10^4$  packets/sec, the throughput of the first, second and third nodes are increasing when other parameters remain fixed. When the arrival parameter ( $\alpha$ ) varies from -0.5 to 1.5, the throughput of the first, second and third nodes are increasing when other parameters remain fixed. When the parameter ( $\theta$ ) varies from 0.1 to 0.9, the throughput of the first node is constant, second and third nodes are increasing when other parameters remain fixed. When the parameter ( $\epsilon$ ) varies from 0.2 to 0.6, the throughput of the first node is constant, second and third nodes are increasing when other parameters remain fixed. When the parameter ( $\pi$ ) varies from 0.1 to 0.9, the throughput of the first node and second nodes are constant and third node is increases when other parameters remain fixed. When the transmission rate  $(\mu_1)$  varies from  $5 \times 10^4$  packets/sec to 9x10<sup>4</sup> packets/sec, the throughput of the first, second and the third nodes are increasing when other parameters remain fixed. The transmission rate  $(\mu_2)$  varies from  $15 \times 10^4$  packets/sec to  $19 \times 10^4$  packets/sec, the throughput of the first node remains constant, the second and the third nodes are increasing when other parameters remain fixed. Similarly the transmission rate  $(\mu_3)$  varies from  $25 \times 10^4$ 

packets/sec to  $29 \times 10^4$  packets/sec, the throughput of the first and second nodes remain constant and for the third node is increasing when other parameters remain fixed.

#### Table 1

Values of Emptiness probabilities and Utilization of the communication Network with DBA and NHP arrivals

| ť   | λ\$ | α   | θ   | £   | Π   | μ <sub>1</sub> \$ | μ <sub>2</sub> \$ | μ <sub>3</sub> \$ | U <sub>1</sub> (t) | $U_2(t)$ | U <sub>3</sub> (t) | Thp <sub>1</sub> | Thp <sub>2</sub> | Thp <sub>3</sub> |
|-----|-----|-----|-----|-----|-----|-------------------|-------------------|-------------------|--------------------|----------|--------------------|------------------|------------------|------------------|
| 0.1 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.14926            | 0.01302  | 0.00092            | 0.74630          | 0.19523          | 0.02300          |
| 0.3 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.28800            | 0.02254  | 0.00143            | 1.43998          | 0.33807          | 0.03564          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.7 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.38684            | 0.02860  | 0.00173            | 1.93420          | 0.42893          | 0.04333          |
| 0.9 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.41492            | 0.03022  | 0.00183            | 2.07458          | 0.45334          | 0.04568          |
| 2.0 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.53233            | 0.03750  | 0.00227            | 2.66163          | 0.56250          | 0.05687          |
| 5.0 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.74334            | 0.05656  | 0.00347            | 3.71670          | 0.84840          | 0.08678          |
| 0.5 | 3   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.45884            | 0.03206  | 0.00201            | 2.29418          | 0.48095          | 0.05033          |
| 0.5 | 4   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.54960            | 0.03771  | 0.00241            | 2.74799          | 0.56560          | 0.06029          |
| 0.5 | 5   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.62514            | 0.04332  | 0.00281            | 3.12569          | 0.64976          | 0.07025          |
| 0.5 | 6   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.68801            | 0.04890  | 0.00321            | 3.44005          | 0.73344          | 0.08021          |
| 0.5 | 7   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.74034            | 0.05444  | 0.00361            | 3.70168          | 0.81662          | 0.09016          |
| 0.5 | 2   | -   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.28504            | 0.02387  | 0.00150            | 1.42518          | 0.35808          | 0.03746          |
| 0.5 | 2   | 0   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.30731            | 0.02471  | 0.00154            | 1.53653          | 0.37066          | 0.03842          |
| 0.5 | 2   | 0.5 | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.32888            | 0.02555  | 0.00158            | 1.64440          | 0.38324          | 0.03939          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1.5 | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.37004            | 0.02722  | 0.00165            | 1.85018          | 0.40835          | 0.04133          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1   | 0.3 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.05216  | 0.00324            | 1.74892          | 0.78242          | 0.08104          |
| 0.5 | 2   | 1   | 0.5 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.07725  | 0.00487            | 1.74892          | 1.15880          | 0.12166          |
| 0.5 | 2   | 1   | 0.7 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.10168  | 0.00649            | 1.74892          | 1.52522          | 0.16221          |
| 0.5 | 2   | 1   | 0.9 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.12546  | 0.00811            | 1.74892          | 1.88194          | 0.20269          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1   | 0.1 | 0.3 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.03285  | 0.00201            | 1.74892          | 0.49278          | 0.05034          |
| 0.5 | 2   | 1   | 0.1 | 0.4 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.03927  | 0.00241            | 1.74892          | 0.58912          | 0.06032          |
| 0.5 | 2   | 1   | 0.1 | 0.5 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.04565  | 0.00281            | 1.74892          | 0.68482          | 0.07029          |
| 0.5 | 2   | 1   | 0.1 | 0.6 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.05199  | 0.00321            | 1.74892          | 0.77989          | 0.08026          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.3 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00484            | 1.74892          | 0.39580          | 0.12088          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.5 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00805            | 1.74892          | 0.39580          | 0.20114          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.7 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.01125            | 1.74892          | 0.39580          | 0.28114          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.9 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.01444            | 1.74892          | 0.39580          | 0.36089          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 6                 | 15                | 25                | 0.31180            | 0.02705  | 0.00165            | 1.87080          | 0.40582          | 0.04114          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 7                 | 15                | 25                | 0.28016            | 0.02752  | 0.00167            | 1.96110          | 0.41273          | 0.04168          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 8                 | 15                | 25                | 0.25367            | 0.02784  | 0.00168            | 2.02933          | 0.41756          | 0.04208          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 9                 | 15                | 25                | 0.23134            | 0.02807  | 0.00169            | 2.08202          | 0.42100          | 0.04236          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 16                | 25                | 0.34978            | 0.02483  | 0.00162            | 1.74892          | 0.39721          | 0.04045          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 17                | 25                | 0.34978            | 0.02344  | 0.00162            | 1.74892          | 0.39842          | 0.04052          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 18                | 25                | 0.34978            | 0.02219  | 0.00162            | 1.74892          | 0.39947          | 0.04059          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 19                | 25                | 0.34978            | 0.02107  | 0.00163            | 1.74892          | 0.40040          | 0.04065          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 25                | 0.34978            | 0.02639  | 0.00161            | 1.74892          | 0.39580          | 0.04036          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 26                | 0.34978            | 0.02639  | 0.00155            | 1.74892          | 0.39580          | 0.04039          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 27                | 0.34978            | 0.02639  | 0.00150            | 1.74892          | 0.39580          | 0.04042          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 28                | 0.34978            | 0.02639  | 0.00144            | 1.74892          | 0.39580          | 0.04045          |
| 0.5 | 2   | 1   | 0.1 | 0.2 | 0.1 | 5                 | 15                | 29                | 0.34978            | 0.02639  | 0.00140            | 1.74892          | 0.39580          | 0.04047          |

\*= Seconds, \$ = Multiplication of 10,000 Packets/sec





Figure 1 : The relationship between Utilization, Throughput and various other parameters

From equations (3.8), (3.10), (3.12), (3.23) and (3.18), (3.20), (3.22), the mean number of packets in the buffers and in the network, mean delay in transmission of three nodes are computed for different values of t,  $\lambda$ ,  $\alpha$ ,  $\theta$ ,  $\pi$ ,  $\mu_1$ ,  $\mu_2$ , and  $\mu_3$  and presented in table 2. The relationship between the parameters and the performance measure are shown in the figure 2.

It is observed that as time (t) varies from 0.1 second to 5 seconds, the mean number of packets in the three buffers and in the network are increasing when other parameters are fixed. When the parameter ( $\lambda$ ) varies from  $3 \times 10^4$ packets/sec to  $7x10^4$  packets/sec, the mean numbers of packets in the first, second and third buffers and in the network are increasing when other parameters remain fixed. When the parameter ( $\alpha$ ) varies from -0.5 to 1.5, the mean number of packets in the first, second and third buffers and in the network are increasing when other parameters remain fixed. When the parameter  $(\theta)$  varies from 0.1 to 0.9, the mean number of packets in the first buffer is constant, second and third buffers and in the network are increasing when other parameters remain fixed. When the parameter ( $\varepsilon$ ) varies from 0.2 to 0.6, the mean number of packets in the first buffer is constant, second and third buffers and in the network are increasing when other parameters remain fixed. When the parameter ( $\pi$ ) varies from 0.1 to 0.9, the mean number of packets in the first and second buffers are fixed and in the third buffer and in the network are increasing when other parameters remain fixed.

#### Table 2

Values of mean number of packets and mean delay of the communication network with DBA and NHP Arrivals

| ť   | λs | α    | Θ   | e   | π   | $\mu_1^{s}$ | $\mu_2^{\$}$ | μ3 <sup>\$</sup> | $L_1(t)$ | $L_2(t)$ | $L_3(t)$ | $W_1(t)$ | $W_2(t)$ | $W_3(t)$ | $L_n(t)$ |
|-----|----|------|-----|-----|-----|-------------|--------------|------------------|----------|----------|----------|----------|----------|----------|----------|
| 0.1 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.16165  | 0.01310  | 0.00092  | 0.21660  | 0.06710  | 0.04002  | 0.17567  |
| 0.3 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.33967  | 0.02280  | 0.00143  | 0.23589  | 0.06743  | 0.04003  | 0.36390  |
| 0.5 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.43045  | 0.02674  | 0.00162  | 0.24612  | 0.06756  | 0.04003  | 0.45881  |
| 0.7 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.48913  | 0.02901  | 0.00173  | 0.25288  | 0.06764  | 0.04003  | 0.51988  |
| 0.9 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.53600  | 0.03069  | 0.00183  | 0.25837  | 0.06769  | 0.04004  | 0.56852  |
| 2.0 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.75998  | 0.03822  | 0.00228  | 0.28553  | 0.06795  | 0.04005  | 0.80048  |
| 5.0 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 1.36000  | 0.05822  | 0.00348  | 0.36592  | 0.06863  | 0.04007  | 1.42170  |
| 0.5 | 3  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.61403  | 0.03259  | 0.00202  | 0.26765  | 0.06776  | 0.04004  | 0.64864  |
| 0.5 | 4  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.79762  | 0.03844  | 0.00241  | 0.29025  | 0.06796  | 0.04005  | 0.83847  |
| 0.5 | 5  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.98120  | 0.04428  | 0.00281  | 0.31391  | 0.06815  | 0.04006  | 1.02830  |
| 0.5 | 6  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 1.16478  | 0.05013  | 0.00321  | 0.33859  | 0.06835  | 0.04006  | 1.21813  |
| 0.5 | 7  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 1.34836  | 0.05598  | 0.00361  | 0.36426  | 0.06855  | 0.04007  | 1.40796  |
| 0.5 | 2  | -0.5 | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.33552  | 0.02416  | 0.00150  | 0.23543  | 0.06748  | 0.04003  | 0.36119  |
| 0.5 | 2  | 0    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.36717  | 0.02502  | 0.00154  | 0.23896  | 0.06750  | 0.04003  | 0.39373  |
| 0.5 | 2  | 0.5  | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.39881  | 0.02588  | 0.00158  | 0.24252  | 0.06753  | 0.04003  | 0.42627  |
| 0.5 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.43045  | 0.02674  | 0.00162  | 0.24612  | 0.06756  | 0.04003  | 0.45881  |
| 0.5 | 2  | 1.5  | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.46209  | 0.02760  | 0.00165  | 0.24976  | 0.06759  | 0.04003  | 0.49135  |
| 0.5 | 2  | 1    | 0.1 | 0.2 | 0.1 | 5           | 15           | 25               | 0.43045  | 0.02674  | 0.00162  | 0.24612  | 0.06756  | 0.04003  | 0.45881  |
| 0.5 | 2  | 1    | 0.3 | 0.2 | 0.1 | 5           | 15           | 25               | 0.43045  | 0.05357  | 0.00325  | 0.24612  | 0.06847  | 0.04006  | 0.48727  |

| 0.5 | 2 | 1 | 0.5 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.08040 | 0.00488 | 0.24612 | 0.06938 | 0.04010 | 0.51573 |
|-----|---|---|-----|-----|-----|---|----|-------------------|---------|---------|---------|---------|---------|---------|
| 0.5 | 2 | 1 | 0.7 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.10723 | 0.00651 | 0.24612 | 0.07030 | 0.04013 | 0.54419 |
| 0.5 | 2 | 1 | 0.9 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.13406 | 0.00814 | 0.24612 | 0.07124 | 0.04016 | 0.57265 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.02674 | 0.00162 | 0.24612 | 0.06756 | 0.04003 | 0.45881 |
| 0.5 | 2 | 1 | 0.1 | 0.3 | 0.1 | 5 | 15 | 250.43045         | 0.03340 | 0.00202 | 0.24612 | 0.06779 | 0.04004 | 0.46587 |
| 0.5 | 2 | 1 | 0.1 | 0.4 | 0.1 | 5 | 15 | 250.43045         | 0.04007 | 0.00242 | 0.24612 | 0.06801 | 0.04005 | 0.47293 |
| 0.5 | 2 | 1 | 0.1 | 0.5 | 0.1 | 5 | 15 | 250.43045         | 0.04673 | 0.00282 | 0.24612 | 0.06824 | 0.04006 | 0.47999 |
| 0.5 | 2 | 1 | 0.1 | 0.6 | 0.1 | 5 | 15 | 250.43045         | 0.05339 | 0.00322 | 0.24612 | 0.06846 | 0.04006 | 0.48706 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.02674 | 0.00162 | 0.24612 | 0.06756 | 0.04003 | 0.45881 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.3 | 5 | 15 | 250.43045         | 0.02674 | 0.00485 | 0.24612 | 0.06756 | 0.04010 | 0.46204 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.5 | 5 | 15 | 250.43045         | 0.02674 | 0.00808 | 0.24612 | 0.06756 | 0.04016 | 0.46527 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.7 | 5 | 15 | 250.43045         | 0.02674 | 0.01131 | 0.24612 | 0.06756 | 0.04023 | 0.46850 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.9 | 5 | 15 | 250.43045         | 0.02674 | 0.01454 | 0.24612 | 0.06756 | 0.04029 | 0.47173 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.02674 | 0.00162 | 0.24612 | 0.06756 | 0.04003 | 0.45881 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 6 | 15 | 250.37368         | 0.02743 | 0.00165 | 0.19974 | 0.06759 | 0.04003 | 0.40275 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 7 | 15 | 250.32872         | 0.02790 | 0.00167 | 0.16762 | 0.06760 | 0.04003 | 0.35829 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 8 | 15 | 250.29258         | 0.02823 | 0.00168 | 0.14418 | 0.06761 | 0.04003 | 0.32250 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 9 | 15 | 250.26310         | 0.02847 | 0.00170 | 0.12637 | 0.06762 | 0.04003 | 0.29326 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | 250.43045         | 0.02674 | 0.00162 | 0.24612 | 0.06756 | 0.04003 | 0.45881 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 16 | 250.43045         | 0.02514 | 0.00162 | 0.24612 | 0.06329 | 0.04003 | 0.45721 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 17 | 250.43045         | 0.02372 | 0.00162 | 0.24612 | 0.05952 | 0.04003 | 0.45579 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 18 | 250.43045         | 0.02244 | 0.00162 | 0.24612 | 0.05618 | 0.04003 | 0.45452 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 19 | 250.43045         | 0.02130 | 0.00163 | 0.24612 | 0.05319 | 0.04003 | 0.45338 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | <b>25</b> 0.43045 | 0.02674 | 0.00162 | 0.24612 | 0.06756 | 0.04003 | 0.45881 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | <b>26</b> 0.43045 | 0.02674 | 0.00155 | 0.24612 | 0.06756 | 0.03849 | 0.45874 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | <b>27</b> 0.43045 | 0.02674 | 0.00150 | 0.24612 | 0.06756 | 0.03706 | 0.45869 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | <b>28</b> 0.43045 | 0.02674 | 0.00145 | 0.24612 | 0.06756 | 0.03574 | 0.45864 |
| 0.5 | 2 | 1 | 0.1 | 0.2 | 0.1 | 5 | 15 | <b>29</b> 0.43045 | 0.02674 | 0.00140 | 0.24612 | 0.06756 | 0.03451 | 0.45859 |

\* = Seconds, \$ = Multiplication of 10,000 Packets/sec



Figure 2 : The relationship between mean no. of packets, mean delay and various parameters

When the transmission rate  $(\mu_1)$  varies from  $5 \times 10^4$ packets/sec to  $9x10^4$  packets/sec, the mean number of packets in the first buffer and in the network are decreasing and in the second and third buffers are increasing when other parameters remain fixed. Similarly the transmission rate  $(\mu_2)$  varies from  $15x10^4$  packets/sec to  $19x10^4$ packets/sec, the mean number of packets in the first buffer remains fixed and the mean number of packets in the second buffer and in the network are decreasing and the mean number of packets in the third buffer is increases, when other parameters remain fixed. Similarly the transmission rate ( $\mu_3$ ) varies from  $25 \times 10^4$  packets/sec to  $29 \times 10^4$  packets/sec, the mean number of packets in the first and second buffers remains constant and the mean number of packets in the third buffer and in the network are decreasing when other parameters remain fixed.

It is observed that as time (t) and the parameter ( $\lambda$ ) are increasing, the mean delay in buffers are increasing for fixed values of the other parameters. It is also observed that as parameter ( $\alpha$ ) varies the mean delay in buffers are increasing for fixed values of the other parameters. The parameter ( $\theta$ ) varies the mean delay in first buffer remains constant and second and third buffers are increasing for fixed values of the other parameters. The parameter  $(\epsilon)$ varies the mean delay in first node is constant and second and third nodes are increasing for fixed values of other parameters. The parameter  $(\pi)$  varies the mean delay in first and second buffers remain fixed and third buffer is increases for fixed values of the other parameters. When the transmission rate  $(\mu_1)$  increases, the mean delay in the first buffer decreases and in the second and third buffers increasing when the other parameters remain fixed. Similarly, the transmission rate  $(\mu_2)$  increases the mean delay in the first buffer remains fixed, second buffer decreases, and third buffer increases when other parameter remains fixed. Similarly, the transmission rate  $(\mu_3)$ increases the mean delay in the first and second buffers are fixed and the third buffer are decreasing, when other parameter remain fixed.

From this analysis it is observed that the dynamic bandwidth allocation strategy has a significant influence on all performance measures of the network. It is further observed that the performance measures are highly sensitive towards smaller values of time. Hence, it is optimal to consider dynamic bandwidth allocation under and non-homogeneous Poisson arrivals and evaluate the performance of the network under transient conditions. It is also to be observed that the congestion in buffers and delays in transmission got reduced to a minimum level by adopting dynamic bandwidth allocation.

## **5 SENSIVITY ANALYSIS:**

Sensitivity analysis of the model is performed to steady the effect of changes in the parameters t,  $\lambda$ ,  $\alpha$ ,  $\theta$ ,  $\epsilon$ ,  $\pi$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  on the mean number of packets, the utilization of transmitters, the mean delay in transmitters and the throughput of the first, second and third nodes.

The following data has been considered for the sensitivity analysis, t = 0.5 sec,  $\lambda = 2x10^4$  packets/sec,  $\alpha=1$ ,  $\theta = 0.1$ ,  $\varepsilon = 0.2$ ,  $\pi = 0.1$ ,  $\mu_1 = 5x \ 10^4$  packets/sec,  $\mu_2 = 15x \ 10^4$  packets/sec, and  $\mu_3 = 25x10^4$  packets/sec.

The mean number of packets, the utilization of nodes, the mean delay, and the throughput of the first, second and third buffers are computed with variation of -15%, -10%, -5%, 0%, +5%, +10% and +15% on the model parameters and presented in table 3.

Table 3: Sensitivity Analysis of the model

| arameter | Performance |         |         | %       | 6 change | in Para | neter   |         |  |
|----------|-------------|---------|---------|---------|----------|---------|---------|---------|--|
|          | Measures    | -15     | -10     | -5      | 0        | 5       | 10      | 15      |  |
|          | $L_1(t)$    | 0.40200 | 0.41206 | 0.42151 | 0.43045  | 0.43892 | 0.44699 | 0.45469 |  |
|          | $L_2(t)$    | 0.01891 | 0.01932 | 0.01971 | 0.02008  | 0.02042 | 0.02074 | 0.02104 |  |
|          | $L_3(t)$    | 0.00231 | 0.00236 | 0.00239 | 0.00243  | 0.00247 | 0.00250 | 0.00253 |  |
|          | $U_1(t)$    | 0.33102 | 0.33771 | 0.34395 | 0.34978  | 0.35527 | 0.36045 | 0.36536 |  |
|          | $U_2(t)$    | 0.01873 | 0.01914 | 0.01952 | 0.01988  | 0.02021 | 0.02052 | 0.02082 |  |

| t=0.5          | $U_3(t)$           | 0.002310.002350.002390.002430.002460.002500.00253        |
|----------------|--------------------|--|
|                | $W_1(t)$           | 0.242890.244030.245100.246120.247090.248020.24890        |
|                | $W_2(t)$           | 0.067300.067310.067330.067340.067350.067360.06737        |
|                | $W_2(t)$           | 0.040050.040050.040050.040050.040050.040050.04005        |
|                | Thp                | 1.6551111.688561.719741.748921.776341.802241.82678       |
|                | Thp <sub>2</sub>   | 0.280920.287090.292820.298170.303170.307870.31230        |
|                | Thp <sub>3</sub>   | 0.057800.058830.059800.060710.061570.062390.06317        |
|                | $L_1(t)$           | 0.375370.393730.412090.430450.448810.467170.48552        |
|                | $L_2(t)$           | 0.018320.018910.019490.020080.020660.021250.02183        |
|                | $L_3(t)$           | 0.002190.002270.002350.002430.002510.002590.00267        |
|                | $U_1(t)$           | 0.312970.325470.337740.349780.361610.373220.38463        |
|                | $U_2(t)$           | 0.018160.018730.019300.019880.020450.021020.02160        |
| λ=2            | $U_3(t)$           | 0.002190.002270.002350.002430.002510.002590.00267        |
|                | $W_1(t)$           | 0.239880.241950.244030.246120.248230.250340.25247        |
|                | $W_2(t)$           | 0.067280.067300.067320.067340.067360.067380.06740        |
|                | W <sub>3</sub> (t) | 0.040040.040050.040050.040050.040050.040050.04005        |
|                | Thp <sub>1</sub>   | 1.564841.627331.688681.748921.808061.866121.92313        |
|                | Thp <sub>2</sub>   | 0.272350.280960.289570.298170.306760.315350.32393        |
|                | Thp <sub>3</sub>   | 0.054730.056720.058720.060710.062700.064690.06668        |
|                | $L_1(t)$           | 0.420960.424120.427290.430450.433610.436780.43994        |
|                | L <sub>2</sub> (t) | 0.019820.019910.019990.020080.020160.020250.02034        |
|                | L <sub>3</sub> (t) | 0.002410.002420.002420.002430.002440.002450.00245        |
|                | $U_1(t)$           | 0.343580.345660.347720.349780.351840.353880.35593        |
|                | $U_2(t)$           | 0.019620.019710.019790.019880.019960.020050.02013        |
| α=1            | U3(t)              | 0.002410.002410.002420.002430.002440.002440.00245        |
|                | $W_1(t)$           | 0.245040.245400.245760.246120.246490.246850.24721        |
|                | $W_2(t)$           | 0.067330.067330.067340.067340.067340.067340.06735        |
|                | W <sub>3</sub> (t) | 0.040050.040050.040050.040050.040050.040050.04005        |
|                | Thp <sub>1</sub>   | 1.717911.728281.738611.748921.759191.769421.77963        |
|                | Thp <sub>2</sub>   | 0.294370.295640.296900.298170.299430.300690.30196        |
|                | Thp <sub>3</sub>   | 0.060130.060320.060510.060710.060900.061090.06129        |
|                | $L_1(t)$           | 0.430450.430450.430450.430450.430450.430450.43045        |
|                | $L_2(t)$           | 0.018070.018740.019410.020080.020750.021420.02209        |
|                | $L_3(t)$           | 0.002190.002270.002350.002430.002510.002590.00268        |
|                | $U_1(t)$           | 0.349780.349780.349780.349780.349780.349780.349780.34978 |
|                | $U_2(t)$           | 0.017900.018560.019220.019880.020530.021190.02185        |
| $\theta = 0.1$ | $U_3(t)$           | 0.002180.002270.002350.002430.002510.002590.00267        |
|                | $W_1(t)$           | 0.246120.246120.246120.246120.246120.246120.246120.24612 |
|                | $W_2(t)$           | 0.067270.067290.067320.067340.067360.067380.06741        |
|                | $W_3(t)$           | 0.040040.040050.040050.040050.040050.040050.04005        |
| 1              | Thp <sub>1</sub>   | 1.748921.748921.748921.748921.748921.748921.748921.74892 |
|                | Thp <sub>2</sub>   | 0.268550.278430.288300.298170.308020.317870.32772        |
|                | Thp <sub>3</sub>   | 0.054600.056640.058670.060710.062740.064780.06681        |

The performance measures are highly affected by the changes in the values of time (t) and parameter ( $\lambda$ ), ( $\alpha$ ), ( $\theta$ ), ( $\epsilon$ ) and ( $\pi$ ). As t increases from -15% to +15% the average number of packets in the three buffers and in the network are increasing along with an increase in the mean delay in buffers, the utilization of transmitters and throughput of the three nodes. As the parameter ( $\lambda$ ) increases from -15% to +15% the average number of packets in the three buffers and in the network are increasing along with an increase in the mean delay, the utilization of transmitters and the throughput of the three nodes. Similarly, for the parameter  $(\alpha)$ , the utilization of transmitters and the throughput of nodes are increasing in the communication network. Overall analysis of the parameters reflects that the dynamic bandwidth allocation strategy for congestion control will tremendously reduce the delay in communication and improve the voice quality by reducing burstness in buffers.

## **6 COMPARATIVE STUDY**

To study the effect of non homogeneous Poisson arrival assumption on the communication network a comparative study between the performance measures of the network models with non homogeneous Poisson arrivals and Poisson arrivals is performed. The performance measures of both models are computed with fixed values of the parameters ( $\lambda$ ,  $\alpha$ ,  $\theta$ ,  $\epsilon$ ,  $\pi$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ ) and different values of t = 0.3, 0.5, 2, 5 seconds are presented in table 4.

| Table 4  |    |
|--|----|
| Comparative study of models with Non-Homogeneous a | nd |
| Homogeneous Poisson arrivals                       |    |

| Time (t) | Parameters         | Model with | Model With |            |             |
|----------|--------------------|------------|------------|------------|-------------|
| Sec      | Measured           | NHP α=1    | HP α=0     | Difference | % Variation |
|          | L1(t)              | 0.3396     | 0.3107     | 0.0289     | 4.45        |
|          | L <sub>2</sub> (t) | 0.0227     | 0.0221     | 0.0006     | 1.48        |
|          | L <sub>3</sub> (t) | 0.0014     | 0.0014     | 0.0000     | 0.01        |
|          | $U_1(t)$           | 0.2879     | 0.2671     | 0.0208     | 3.76        |
|          | $U_2(t)$           | 0.0225     | 0.0218     | 0.0006     | 1.47        |
| t=0.5    | U <sub>3</sub> (t) | 0.0014     | 0.0014     | 0.0001     | 0.01        |
|          | $W_1(t)$           | 0.2358     | 0.2326     | 0.0032     | 0.68        |
|          | $W_2(t)$           | 0.0674     | 0.0674     | 0.0001     | 0.02        |
|          | W <sub>3</sub> (t) | 0.0400     | 0.0399     | 0.0001     | 0.03        |
|          | Thp <sub>1</sub>   | 1.4399     | 1.3355     | 0.1044     | 3.76        |
|          | Thp <sub>2</sub>   | 0.3380     | 0.3283     | 0.0097     | 1.47        |
|          | Thp <sub>3</sub>   | 0.0356     | 0.0356     | 0.0001     | 0.01        |
|          | L1(t)              | 1.3600     | 0.4000     | 0.9600     | 54.55       |
|          | L <sub>2</sub> (t) | 0.0516     | 0.0200     | 0.0316     | 44.10       |
|          | L <sub>3</sub> (t) | 0.0062     | 0.0024     | 0.0038     | 43.89       |
|          | U <sub>1</sub> (t) | 0.7433     | 0.3297     | 0.4137     | 38.55       |
| - 5      | U <sub>2</sub> (t) | 0.0502     | 0.0198     | 0.0304     | 43.47       |
| 1=3      | U <sub>3</sub> (t) | 0.0061     | 0.0024     | 0.0037     | 43.81       |
|          | $W_1(t)$           | 0.3659     | 0.2427     | 0.1233     | 20.25       |
|          | $W_2(t)$           | 0.0684     | 0.0673     | 0.0011     | 0.78        |
|          | $W_3(t)$           | 0.0401     | 0.0400     | 0.0001     | 0.09        |
|          | Thp <sub>1</sub>   | 3.7167     | 1.6484     | 2.0683     | 38.55       |
|          | Thp <sub>2</sub>   | 0.7537     | 0.2970     | 0.4567     | 43.47       |
|          | Thp <sub>3</sub>   | 0.1534     | 0.0599     | 0.0935     | 43.81       |

As t increases the percentage variation of performance measures between the two models is increasing. For the model with non-homogeneous Poisson arrivals with dynamic bandwidth allocation has more utilization compared to that of the model with Poisson arrivals with dynamic bandwidth allocation. From this analysis it is observed that the assumption of non-homogeneous Poisson arrivals have a significant influence on all the performance measures of the network. This model also includes the two node tandem communication network model when  $\mu_3$  is zero.

## 7. CONCLUSIONS:

In this paper a novel and new communication model which is much useful for analyzing the communication systems more effectively and efficiently is developed and analyzed. The work presented in this paper focus on the improvement of three node tandem communication network with Dynamic Bandwidth Allocation and modified phase type transmission having NHP arrivals for first node and Poisson arrivals for second node. The variable traffic conditions (bursty traffic/ time dependent traffic) is characterized by non homogeneous Poisson process with time dependent arrival rates. It is shown that the dynamic bandwidth allocation can reduce congestion in buffers and delay in transmission by utilizing iid bandwidth. The developed network model much useful for the evaluating the performance of several networks like LAN, WAN, MAN and Computer Communication Systems under variable traffic conditions by predicting performance measures more close to the reality. This mode

also includes some of the earlier models as particular cases for specific values of the parameters.

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