Fuzzy Kernel and Fuzzy Subsemiautomata with Thresholds

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Abstract

In 1967, Wee introduced the concept of fuzzy automata, using Zadeh's concept of fuzzy sets. A group semiautomaton has been extensively studied by Fong and Clay .This group semiautomaton was fuzzified by Das and he introduced fuzzy semiautomaton, fuzzy kernel and fuzzy subsemiautomaton over finite group. Fuzzy subgroup with thresholds was defined by Yuan et. al. In this paper, we introduce the idea of fuzzy kernel and fuzzy subsemiautomaton with thresholds. Further, we discuss some results concerning them.

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1. Introduction

Lofti A. Zadeh introduced fuzzy sets in 1965. Rosenfeld defined fuzzy subgroups in 1971. Anthony and Sherwood replaced "min" in Rosenfeld axiom by t-norm and introduced T-Fuzzy subgroup. Bhakat and Das introduced ($\in$, $\in\vee q$)-fuzzy normal, quasinormal and maximal subgroup in 1992.Also in 1997 they introduced fuzzy kernel and fuzzy subsemiautomaton of a fuzzy semiautomaton over a finite group using the notions of a fuzzy normal subgroup and a fuzzy subgroup of a group. In the year 1999 Sung-jin Cho et al. introduced the notion of T-fuzzy semiautomata, T-fuzzy kernel, and T-fuzzy subsemiautomata over a finite group. In 2003, Yuan et. al defined fuzzy subgroup with thresholds which is a generalization of Rosenfeld's fuzzy subgroup and Bhakat and Das's fuzzy subgroup. This paper defines fuzzy kernel with thresholds, fuzzy subsemiautomaton with thresholds and discusses some results concerning them.

2. Preliminaries

In this section we summarize some preliminary definitions and results which are required for developing main results.

Let $(G,*)$ denote a group. We sometimes write G for $(G,*)$ when the operation $*$ is understood.

2.1 Definition [7]

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A: X \to [0,1]$ and $A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

2.2 Definition [4]

A fuzzy subset λ of a group G is a fuzzy subgroup of G if for all $x, y \in G$

(i) $\lambda(x*y) \geq \lambda(x) \wedge \lambda(y)$

(ii) $\lambda(x^{-1}) \geq \lambda(x)$

2.3 Definition [4]

A fuzzy subgroup λ of G is called a fuzzy normal subgroup of G if $\lambda(x*y*x^{-1}) \geq \lambda(y)$ for all $x, y \in G$.

2.4 Definition [6]

Let $\lambda, \mu \in [0,1]$ and $\lambda < \mu$ .Let A be a fuzzy subset of a group G . Then A is called a fuzzy subgroup with thresholds of G if for all $x, y \in G$

(i) $A(x*y) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu$

(ii) $A(x^{-1}) \vee \lambda \geq A(x) \wedge \mu$

2.5 Definition [2]

Let $\alpha, \beta \in [0,1]$ and $\alpha < \beta$. Let μ be a fuzzy subset of a group G . Then μ is called a fuzzy normal subgroup with thresholds of G if $\mu(y^{-1}*x*y) \vee \alpha > \mu(x) \wedge \beta, \forall x, y \in G.$

2.6 Definition [4]

A fuzzy semiautomaton over a finite group $(Q,*)$ is a triple (Q, X, μ) where X is a finite set and μ is a fuzzy subset of $Q \times X \times Q$.

2.7 Definition [4]

Let $S = (Q, X, \mu)$ be a fuzzy semiautomaton over a finite group G . A fuzzy subset λ of Q is called fuzzy kernel of S if the following conditions hold. For all $p, q, r, k \in Q, x \in X$

(i) λ is a fuzzy normal subgroup of Q

(ii) $\lambda(p*r^{-1}) \geq \mu(q*k, x, p) \wedge \mu(q, x, r) \wedge \lambda(k)$

2.8 Definition [4]

Let $S = (Q, X, \mu)$ be a fuzzy semiautomaton over a finite group G. A fuzzy subset λ of Q is called fuzzy subsemiautomaton of S if the following conditions hold:

(i) λ is a fuzzy subgroup of Q

(ii) $\lambda(p) \geq \mu(q, x, p) \wedge \lambda(q)$ *for all* $p, q \in Q, x \in X$.

2.9 Definition [4]

Let λ *and* μ be fuzzy subsets of G . The product $\lambda*\mu$ of λ and μ is defined by $(\lambda*\mu)(x) = \vee\{\lambda(y) \wedge \mu(z) / y, z \in G \text{ such that } x = y*z\}$

3. Main Results

3.1 Definition

Let $S = (Q, X, \mu)$ be a fuzzy semiautomaton over a finite Group. A fuzzy subset λ of Q is called a fuzzy kernel of S with thresholds if

(i) λ is a fuzzy normal subgroup of Q with thresholds

(ii) $(\lambda(p*r^{-1}) \vee \alpha) \geq \mu(q*k, x, p) \wedge \mu(q, x, r) \wedge \lambda(k) \wedge \beta$ for all $p, q, r, k \in Q$

3.2 Definition

Let $S = (Q, X, \mu)$ be a fuzzy semiautomaton over a finite Group. A fuzzy subset λ of Q is called a fuzzy subsemiautomaton of S with thresholds if the following conditions hold

(i) λ is a fuzzy subgroup of Q with thresholds

(ii) $\left(\lambda(p)\vee\alpha\right)\geq\mu(q,x,p)\wedge\lambda(q)\wedge\beta$ for all $p,q\in Q\, and\ x\in X$.

3.3 Definition

Let λ and μ be fuzzy subsets of G with thresholds α,β . The product $\lambda*\mu$ is defined by

$$\left((\lambda*\mu)(x)\vee\alpha\right)=\vee\begin{Bmatrix}\lambda(y)\wedge\mu(z)\wedge\beta\,/\\ y,z\in G\, such\, that\ x=y*z\end{Bmatrix}$$

Note

Let $S=(Q,X,\mu)$ be a fuzzy semiautomaton over a finite group in the remaining of the results. The element $'e'$ be the identity of $(Q,*)$.

3.4 Proposition

Let $\alpha,\beta\in[0,1]$ and $\alpha<\beta$. Let λ be a fuzzy kernel of $S=(Q,X,\mu)$ with thresholds α,β . Then λ is a fuzzy subsemiautomaton of S with thresholds α,β if and only if $\left(\lambda(p)\vee\alpha\right)\geq\mu(e,x,p)\wedge\lambda(e)\wedge\beta$ for all $p\in Q, x\in X$

Proof: We have $\left(\lambda(p)\vee\alpha\right)=\lambda\left(p*r^{-1}*r\right)\vee\alpha$

$\geq\lambda\left(p*r^{-1}\right)\wedge\lambda(r)\wedge\beta$ (By definition 2.2)

$$\geq\left(\lambda\left(p*r^{-1}\right)\wedge\lambda(r)\wedge\beta\right)\vee\alpha$$

$$=\left(\mu\left(p*r^{-1}\right)\vee\alpha\right)\wedge\lambda(r)\wedge\beta$$

$$\geq\left(\mu(q,x,p),\wedge\mu(e,x,r)\wedge\lambda(q)\wedge\beta\right)\wedge\lambda(r)\wedge\beta$$

(By definition 3.1)

$$\geq\left(\mu(q,x,p)\wedge\mu(e,x,r)\wedge\lambda(e)\wedge\lambda(q)\wedge\beta\right)$$

Since $\lambda(r)\geq\mu(e,x,r)\wedge\lambda(e)$ by given condition

$$\geq\mu(q,x,p)\wedge\mu(e,x,r)\wedge\lambda(q)\wedge\beta$$

Since $\lambda(e)\geq\lambda(q)$

$$\geq\mu(q,x,p)\wedge\lambda(q)\wedge\beta$$

3.5 Proposition

Let $\alpha,\beta\in[0,1]$ and $\alpha<\beta$. Let λ be a fuzzy kernel of $S=(Q,X,\mu)$ with thresholds α,β and ν be a fuzzy subsemiautomaton of S with thresholds α,β . Then $\lambda*\nu$ is a fuzzy subsemiautomaton of S with thresholds α,β .

Proof: Since λ is a fuzzy normal sub group with thresholds and ν is a fuzzy sub group with thresholds of Q , it follows that $\lambda*\nu$ is a fuzzy sub group of Q with thresholds α,β

$\left((\lambda*\nu)(p)\vee\alpha\right)=\left((\lambda*\nu)\left(p*r*r^{-1}\right)\right)\vee\alpha$, By definition 3.3

$$\geq\left(\lambda\left(p*r^{-1}\right)\wedge\nu(r)\right)\wedge\beta$$

$$\geq\left(\left(\lambda\left(p*r^{-1}\right)\wedge\nu\left(r\right)\right)\vee\alpha\right)\wedge\beta$$

Using Lemma 2.1 in [2]

$$=\left(\lambda\left(p*r^{-1}\right)\vee\alpha\right)\wedge\left(\nu\left(r\right)\vee\alpha\right)\wedge\beta,$$ since $\vee$ is distributive.

$$\geq\left(\mu\left(a*b,x,p\right)\wedge\mu\left(a,x,r\right)\wedge\lambda\left(b\right)\wedge\beta\right)\wedge$$
$$\left(\mu\left(a,x,r\right)\wedge\nu\left(a\right)\wedge\beta\right)\wedge\beta$$

$$=\mu\left(a*b,x,p\right)\wedge\lambda\left(b\right)\wedge\nu\left(a\right)\wedge\beta$$

$$\left(\left(\lambda*\nu\right)\left(p\right)\vee\alpha\right)\geq\vee\begin{Bmatrix}\mu\left(a*b,x,p\right)\wedge\lambda\left(b\right)\wedge\nu\left(a\right)\wedge\beta/\\ \forall a,b\in Q,a*b=q\end{Bmatrix}$$

$$=\mu\left(q,x,p\right)\wedge\left(\vee\begin{Bmatrix}\lambda\left(b\right)\wedge\nu\left(a\right)\wedge\beta/\\ a*b=q,\forall a,b\in Q\end{Bmatrix}\right)$$

$$=\mu\left(q,x,p\right)\wedge\left(\lambda*\nu\right)\left(q\right)\vee\alpha$$

$$>\mu\left(q,x,p\right)\wedge\left(\lambda*\nu\right)\left(q\right)\wedge\beta$$

$$\geq\mu\left(q,x,p\right)\wedge\left(\lambda*\nu\right)\left(q\right)\wedge\beta$$

3.6 Proposition

Let $\alpha,\beta\in\left[0,1\right]$ and $\alpha<\beta$ .If λ and ν are fuzzy kernels of S with thresholds α,β then $\lambda*\nu$ is a fuzzy kernel of S with thresholds α,β .

Proof: Since λ and ν are fuzzy normal subgroups of Q with thresholds then $\lambda*\nu$ is also fuzzy normal sub group of Q with thresholds

$$\left(\left(\lambda*\nu\right)\left(p*r^{-1}\right)\vee\alpha\right)\geq\left(\lambda*\nu\right)\left(p*q^{-1}*q*r^{-1}\right)\vee\alpha$$

$$\geq\lambda\left(p*q^{-1}\right)\wedge\nu\left(q*r^{-1}\right)\wedge\beta$$ (By normal subgroup definition)

$$=\left(\lambda\left(p*q^{-1}\right)\vee\alpha\right)\wedge\left(\nu\left(q*r^{-1}\right)\vee\alpha\right)\wedge\beta$$

Using Lemma 2.1 in[2]

$$\geq\left(\mu\left(a*b*c,x,p\right)\wedge\mu\left(a*b,x,q\right)\wedge\lambda\left(c\right)\wedge\beta\right)\wedge$$
$$\left(\mu\left(a*b,x,q\right)\wedge\mu\left(a,x,r\right)\wedge\nu\left(b\right)\wedge\beta\right)\wedge\beta$$
(By definition 3.1)

$$=\mu\left(a*b*c,x,p\right)\wedge\lambda\left(c\right)\wedge\nu\left(b\right)\wedge\beta$$

$$\left(\text{Since }\mu\left(a*b*c,x,p\right)\leq\mu\left(a*b*,x,p\right)\right)$$

$$\left(\lambda*\nu\right)\left(p*r^{-1}\right)\vee\alpha\geq\vee\begin{Bmatrix}\mu\left(q*b*c,x,p\right)\wedge\mu\left(q,x,r\right)\\ \wedge\lambda\left(c\right)\wedge\nu\left(b\right)\wedge\beta/\\ b,c\in Q,b*c=k\end{Bmatrix}$$

$$=\mu\left(q*k,x,p\right)\wedge\mu\left(q,x,r\right)\wedge\left(\vee\begin{Bmatrix}\lambda\left(c\right)\wedge\nu\left(b\right)\wedge\beta/\\ b*c=k,b,c\in Q\end{Bmatrix}\right)$$

$$= \mu(q*k,x,p) \wedge \mu(q,x,r) \wedge (\lambda*\mu)(k) \vee \alpha$$

$$> \mu(q*k,x,p) \wedge \mu(q,x,r) \wedge (\lambda*\mu)(k) \wedge \beta$$

References

[1] Anthony J. M. and Sherwood, H., A characterization of fuzzy groups, Fuzzy Sets and System, 7, (1982) pp 297-305.

[2] Bao Qing Hu, Fuzzy groups and T- fuzzy Groups with Thresholds, Advances in fuzzy mathematics Vol. 5, No.1, (2010) pp. 17-29.

[3] Bhakat S. K., (∈, ∈Vq) -fuzzy normal, quasinormal and maximal Subgroups, Fuzzy Sets and Systems, 112, (2000) pp. 299-312.

[4] P. Das, On some properties of fuzzy semiautomaton over a finite group, Information Sciences 101, (1997) pp.71-84

[5] Rosenfeld, A., Fuzzy groups, Journal of Mathematical Analysis and Applications, 35, (1971) pp. 512-517.

[6] Yuan, X., Zhang, C., and Ren, Y., Generalized fuzzy groups and many valued implications, Fuzzy Sets and Systems, 138, (2003) pp. 205-211.

[7] Zadeh, L. A., Fuzzy sets, Information and Control,8,(1965)pp.338-353

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