Image Restoration Using Thresholding Techniques on Wavelet Coefficients

Rubeena Vohra[1], Akash Tayal[2]

[1] Assistant Professor, BVCOE, New Delhi

[2]Assistant Professor, IGIT, New Delhi

Abstract

Image restoration from corrupted image is a classical problem in the field of image processing. Additive random noise can easily be removed using simple threshold methods with linear and non-linear filtering techniques. De-noising of natural images corrupted by Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet de-noising scheme thresholds the wavelet coefficients arising from the standard discrete wavelet transform. In this paper, it is proposed to investigate the suitability of different wavelet bases and the decomposition levels on the performance of image de-noising algorithms in terms of peak signal -to- noise ratio.

***Keywords**: Image, De-noising, Wavelet Transform*

1. Introduction

Image restoration is the removal or reduction of degradations that are incurred while the image is being obtained. Degradation comes from blurring as well as noise due to electronic and photometric sources. In addition to blurring the image is often corrupted by noise during its acquisition and transmission. For example, in the image acquisition, the performance of imaging sensors is affected by a variety of factors, such as, environmental conditions and by the quality of the sensing elements themselves. For instance, in acquiring images with a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image. Images are also corrupted during transmission, due to interference in the channel used for transmission. The main objective of de-noising techniques for random noise removal is to suppress the noise while preserving the original image details. Statistical filters like Average filter [5], [6], Median filter [7] can be used for removing such noises but the wavelet based de-noising techniques proved better results than these filters. In general, image de-noising imposes a compromise between noise reduction and preserving significant image details. To achieve a good performance in this respect, a de-noising algorithm has to adapt to image discontinuities. It compresses essential information in a signal into relatively few, large coefficients, which represent image details at different resolution scales. In recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal and image de-noising [2] because wavelet provides an appropriate basis for separating noisy signal from image signal. Many wavelet based thresholding techniques like VisuShrink, SureShrink have proved better efficiency in image denoising. We describe here an efficient thresholding technique for denoising by analyzing the statistical parameters of the wavelet coefficients. The threshold is estimated and the coefficients are killed or remain unchanged or shrinked, depending on the type of thresholding (i.e. hard or soft). The first method estimates the threshold level by a median estimator, which implements the noise standard deviation from the coefficients of the diagonal subband of the first level (i.e. HH) and is called global. The second method refers to a median estimator, which is applied on all the detail coefficients of each level, so is level dependent. Eventually, the third approach employs a median estimator which is applied on the horizontal-vertical-diagonal detail coefficients of each subband, so is detail dependent. This paper is organized as follows. Section 2 is a brief review of the discrete wavelet transform. In section 3, the concept of wavelet thresholding is developed. Section 4 explains the proposed method of de-noising based on wavelet decomposition. Experimental evaluation is performed in section 5 and finally conclusions are given in section 6.

2. Discrete Wavelet Transform (DWT)

The mathematical approach to the discrete wavelet transform (DWT) is based on the fact that a function f (t) can be linearly represented as:

$$f(t) = \sum_k a_k \psi_k(t) \quad (1)$$

where a_k are the analysis coefficients and ψ_k the analyzing functions, which are called basis functions, if the above analysis is unique. If the basis functions are orthogonal, that is,

$$\langle \psi_k(t), \psi_l(t) \rangle = \int \psi_k(t)\psi_l(t)dt = 0 \quad \text{for } k \neq l \quad (2)$$

the coefficients can be estimated from the following equation:

$$a_k = \langle f(t), \psi_k(t) \rangle = \int f(t)\psi_k(t)dt \quad (3)$$

where *f(t)* is given from (1). In general, a 2-D signal may be transformed by DWT as:

$$f(t) = \sum_k \sum_j \alpha_{j,k} \psi_{j,k}(t) \quad (4)$$

where $\alpha_{j,k}$ and $\psi_{j,k}$ are the transform coefficients and basis functions respectively. Equation (4) is the inverse transform, given by $\alpha_{j,k}$ and $\psi_{j,k}$. Therefore, a function f(t) may be represented by transform coefficients, which are estimated from the internal product of that function with an orthogonal basis function. Inversely, the desired function may be reconstructed from these coefficients and the basis function. These basis functions are called wavelets [1], [3].

Another consideration of the wavelets is the subband coding theory or multiresolution analysis [4]. The first component to multiresolution analysis is vector spaces. For each vector space, there is another vector space of higher resolution until you get to the final image. The basis of each of these vector spaces is the scale function for the wavelet. We can consider an image a vector space such as V_j would be perfectly normal image and V_{j-1} would be that image at a lower resolution until we get V_o where there is only one pixel in the entire image. For such vector space V_j there is an orthogonal compliment called W_j and the basis function for this vector space is the wavelet. If the function $\phi(x) \in Vo$ such that the set of functions of $\phi(x)$ and its integer translates $\{\phi(x-k)/k \in z\}$ forms a basis for space V_o which is termed as scaling function or father function. The subspace Vj are nested which implies $V_j \in V_{j+1}$. It is possible to decompose V_{j+1} in V_j and W_j.

$$V_j \oplus W_j = V_{j+1} \quad (5)$$

Also, $W_j \in V_j$ (6)

$\Psi(x) \in Wo$ obeys translation property such that $\Psi(x-k) \in Wo$, $k \in z$ [11] . form a basis function for space W_o which is termed as wavelet function or mother function. DWT scaling function for 2-D DWT can be obtained by multiplying two 1-D scaling functions: $\phi(x,y) = \phi(x)\phi(y)$. Wavelet function for 2-D DWT can be obtained by multiplying two wavelet functions. For 2-D case there exists three wavelet functions that scan details in horizontal $\Psi(x, y) = \Psi(x)\phi(y)$, vertical $\Psi(x, y) = \phi(x)\Psi(y)$ and diagonal direction $\Psi(x, y) = \Psi(x)\Psi(y)$. As a result, there are three types of detailed images for such resolution: horizontal, vertical and diagonal.

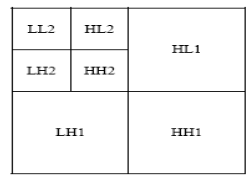

Fig 1: Two- level decomposition

3. Wavelet Thresholding

Let f= {fij, i, j=1, 2 ...M} denotes a M x M matrix of original image to be recovered and M is some integer power of 2. During the transmission, the signal f is corrupted by independent and identically distributed zero mean, white Gaussian noise nij with standard deviation σ i.e. nij ~ N (0, σ^2) and at the receiver end, the noisy observation gij=fij+nij is obtained. The goal is to estimate the signal f from the noisy observations gij such that the Mean Square Error (MSE) is minimum. To achieve this gij is transformed into wavelet domain, which decomposes the gij into many subbands, which separates the signal into so many frequency bands. The small coefficients in the subbands are dominated by noise, while coefficients with large absolute value carry more signal information than noise. Replacing noisy coefficients (small coefficients below certain value) by zero and an inverse wavelet transform may lead to reconstruction that has lesser noise. Normally Hard Thresholding and Soft Thresholding techniques are used for such denoising process. Hard and Soft thresholding [14] with threshold λ are defined as follows. The hard thresholding operator is defined as:

$$D(U, \lambda) = U \text{ for all } |U| < \lambda \quad (7)$$

= 0 otherwise

The soft thresholding operator on the other hand is defined as:

$$D(U, \lambda) = sign(U) * max(0, |U| - \lambda) \quad (8)$$

4. De-noising Algorithm

1. Transform the noisy image into orthogonal domain by discrete 2D wavelet transform.

2. Apply hard or soft thresholding the noisy detail coefficients of the wavelet transform.

3. Perform inverse discrete wavelet transform to obtain the de-noised image.

Here, the threshold plays an important role in the de-noising process. Normally, hard thresholding and soft thresholding techniques are used for such de-noising process. Hard thresholding is a keep or kill rule whereas soft thresholding shrinks the coefficients [13] above the threshold in absolute value. It is a shrink or kill rule. The following are the methods of threshold selection for image de-noising based on wavelet transform:

4.1 Visushrink

It is the de-noising technique introduced by Donoho [12], [8] , it uses the threshold value t that is proportional to standard deviation of noise follows "Hard Thresholding Rule". The universal rule for threshold T can be calculated using the formulae,

$$T=\sigma\sqrt{2\log n} \quad (9)$$

This method performs well under a number of applications because wavelet transform has the compaction property of having only a small number of large coefficients. All the rest wavelet coefficients are very small. This algorithm offers the advantages of smoothness and adaptation. However, it exhibits visual artifacts.

4.2 Sureshrink

A threshold chooser based on Stein's Unbiased Risk Estimator (SURE) was proposed by Donoho and Johnstone and is called as SureShrink. It is a combination of the universal threshold and the SURE threshold. This method specifies a threshold value tj for each resolution level j in the wavelet transform which is referred to as level dependent thresholding. The goal of SureShrink is to minimize the mean squared error, defined as

$$MSE=1/MN\sum_{y=1}^{M}\sum_{x=1}^{N}[I(x,y)-I^{'}(x,y)]^2 \quad (10)$$

where $I^{'}(x,y)$ is the estimate of the signal while I(x,y) is the original signal without noise. The SureShrink suppresses noise by thresholding the empirical wavelet coefficients. The SureShrink threshold t* is defined as:

$$t^*=\min(t,\sigma^2\log n) \quad (11)$$

where t denotes the value that minimizes Stein's Unbiased Risk Estimator, σ is the noise variance computed from Equation (11), and n is the size of the image. The Sureshrink method follows the soft thresholding rule. The thresholding employed here is adaptive, i.e., a threshold level is assigned to each dyadic resolution level by the principle of minimizing the Stein's Unbiased Risk Estimator for threshold estimates. It is smoothness adaptive, which means that if the unknown function contains abrupt changes or boundaries in the image, the reconstructed image also does.

4.3 Bayesshrink

BayesShrink was proposed by Chang, Yu and Vetterli [10]. The goal of this method is to minimize the Bayesian risk, and hence its name, BayesShrink. It uses soft thresholding and is subband-dependent, which means that thresholding is done at each band of resolution in the wavelet decomposition. Like the SureShrink procedure, it is smoothness adaptive. The Bayes threshold, tB, is defined:

$$tB=\sigma^2/\sigma_s \quad (12)$$

where σ^2 is the noise variance and σ is the signal variance without noise. The noise variance σ^2 is estimated from the subband HH1 by the median estimator. From the definition of additive noise we have

$$w(x,y)=s(x,y)+n(x,y). \quad (13)$$

Since the noise and the signal are independent of each other, it can be stated that

$$\sigma_w^2=\sigma_s^2+\sigma^2 \quad (14)$$

σ_W^2 can be computed as shown below:

$$\sigma_w^2=1/n^2\sum_{x,y-1}^{n}w^2(x,y) \quad (15)$$

The variance of the signal σ_S^2 is computed as:

$$\sigma_s=\sqrt{\max(\sigma_w^2-\sigma^2,0)} \quad (16)$$

5. EXPERIMENTAL RESULTS

The above said methods are evaluated using the quality measure Peak Signal to Noise ratio which is calculated using the formulae,

$$PSNR=10*\log_{10}(255)^2/MSE(db) \quad (17)$$

where MSE is the mean squared error between the original image and the reconstructed de-noised image. Quantitatively assessing the performance in practical application is complicated issue because the ideal image is normally unknown at the receiver end. So this paper uses the following method for experiments. One original image is applied with Gaussian noise with variance value 0.001. In this paper, different wavelet bases are used in all methods. For taking the wavelet transform of the image, readily available MATLAB routines are taken. In each sub-band, individual pixels of the image are shrinked based on the threshold selection. A de-noised wavelet transform is created by shrinking pixels. The inverse wavelet transform is the de-noised image. In this paper three images of different sizes are denoised by applying the techniques discussed above. The simulation results are shown below of three images: “imde1.jpg”,”imde2.jpg”, “imde3.jpg” with its original image, noise corrupted image with Gaussian noise at variance 0.001 and its denoised image.

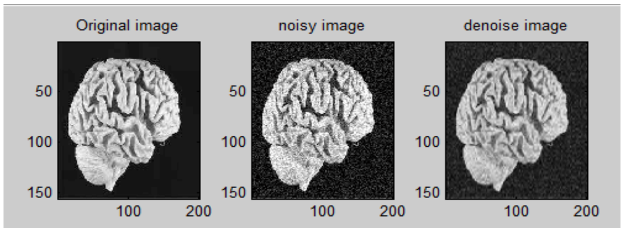

Fig.2:“imde1.jpg”with its original, noisy and denoised pattern

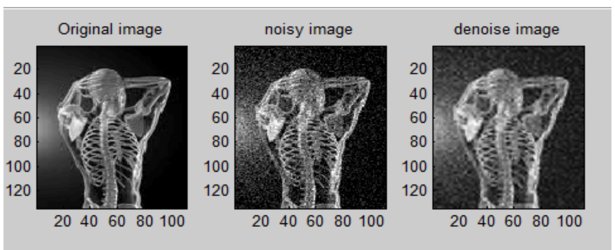

Fig.3:“imde2.jpg”with its original, noisy and denoised pattern

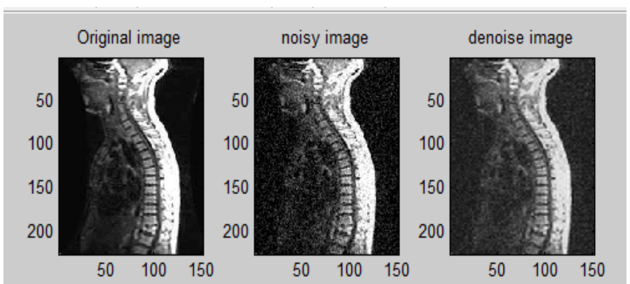

Fig.4:“imde2.jpg”with its original, noisy and denoised pattern

Table 1: signal-to-noise ratios of the thresholding techniques compared to weiner filter

Wavelet	Image	Figure1	Figure2	Figure3
	Median	31.6	35.2	33.5
haar	Visushrink	29.2	32.8	30.8
	Sureshrink	31.8	35.0	32.8
	Bayeshrink	30.0	33.3	31.4
Db16	Visushrink	30.4	33.4	31.4
	Sureshrink	32.8	36.4	33.7
	Bayeshrink	29.0	33.3	30.4
Coif5	Visushrink	30.5	33.8	31.5
	Sureshrink	**33.0**	**36.2**	**34.0**
	Bayeshrink	29.9	33.4	30.4
Sym8	Visushrink	30.5	33.5	31.6
	Sureshrink	32.9	36.0	33.8
	Bayeshrink	30.0	34.0	32.2

Throughout the text, we tried to present numerous original interpretations, pictorial explanations and discussions broadening our viewpoints on this topic. The use of the localized context-dependent hard and soft thresholding operators have resulted in some improvement in the performance of the various standard wavelet thresholding methods studied in this paper.
For the above mentioned three methods, image de-noising is performed using wavelets for the second level decomposition and the results are shown in figure1, figure2 and figure3 along with the table formulated for noise variance 0.001. Along with the comparison to the Weiner Filter “Sureshrink” gave the best possible results.

6. CONCLUSION

In this paper, the image de-noising using discrete wavelet transform is analyzed. The experiments were conducted to study the suitability of different wavelet bases and also different window sizes. Among all discrete wavelet bases, coiflet performs well in image de-noising. For Gaussian noise (0, 0.09) – PSNR improves by the use of Hard Thresholding technique. Experimental results also show that Sureshrink gives better result than Visushrink and Bayesshrink as compared to Weiner filter.

References

[1] M. Vetterli, J. Kovacevic,Wavelets and subband coding, Englewood Cliffs, NJ, Prentice Hall, 1995.
[2] C.S.Burrus, R.A.Gopinath, H.Guo, "Introduction to Wavelets and Wavelet Transforms", Prentice Hall, 1998, pp. 2-18.
[3] G. Strang,T. Nguyen, "Wavelets and Filter Banks", Wellesley, 1997.
[4] S. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation", IEEE Trans. PAMI, vol. 11, no. 7, pp. 674-693.
[5] Maher A. Sid-Ahmed. (1995). Image Processing-Theory algorithm and architecture. McGraw-Hill, pp 78-80.
[6] Rafael C.Gonzalez & Richard E.Wodds. (1993). Digital Image Processing. Addison Wesley publishing Company , pp 41-43.
[7] D.L. Donoho. (1994). Ideal spatial adoption by wavelet shrinkage. Biometrika, volume 81, pp.425-455.
[8] D.L. Donoho and I.M. Johnstone. (1995). Adapting to unknown smoothness via wavelet shrinkage. Journal of American Statistical Association., Vol. 90, no. 432, pp1200-1224.
[9] S. Grace Chang, Bin Yu and M. Vattereli. (2000). Wavelet Thresholding for Multiple Noisy Image Copies. IEEE Transaction. Image Processing, vol. 9, pp.1631- 1635.
[10] S. Grace Chang, Bin Yu and M. Vattereli. (2000). Spatially Adaptive Wavelet Thresholding with Context Modeling for Imaged noising. IEEE Transaction - Image Processing, volume 9, pp. 1522-1530.
[11] M. Vattereli and J. Kovacevic. (1995). Wavelets and Subband Coding. Englewood Cliffs. NJ, Prentice Hall.
[12] Maarten Janse. (2001). Noise Reduction by Wavelet Thresholding.Volume 161, Springer Verlag, United States of America, I edition.
[13] Carl Taswell. (2000). The what, how and why wavelet shrinkage denoising. Computing in science and Engineering, pp.12-19.
[14]D.L. Donoho. (1995). De-noising by soft thresholding. IEEE Transactions on Information Theory, volume 41, pp.613-627