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Analysis of Image Denoising using Wavelet Coefficient and Adaptive Subband Thresholding Technique

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Abstract

Image denoising is a common procedure in digital image processing aiming at the removal of noise which may corrupt an image during its acquisition or transmission while sustaining its quality. A statistical model is proposed depending on the magnitude of wavelet coefficients and noise variance for each coefficient is estimated based on the subband it belongs to using Maximum Likelihood (ML) estimator or a Maximum a Posterior (MAP) estimator. An adaptive thresholding is proposed which is applied to each subband coefficient except the low pass or approximation subband. This is done by fixing the optimum thresholding value depending on the decomposition level. The proposed method describes a new method for suppression of noise in image by fusing the wavelet denoising technique with optimized thresholding function to which a multiplying factor (α) is included to make the threshold value dependent on decomposition level. Due to this, the proposed technique yields significantly superior image quality by preserving the edges, producing a better PSNR value. The efficiency is proved on comparing with Bayes shrink (BS), Modified Bayes Shrink (MBS) and Normal Shrink (NS) for different noise level.

Keywords: *Image denoising, decomposition level, wavelet coefficients, optimum thresholding*

1. Introduction

Estimating an image that is corrupted by additive noise has been of interest to many researchers for practical as well as theoretical reason. The problem is to recover the original image from the noisy data by sustaining the possible important image features. Various statistical wavelet models for images have been proposed. A simple and popular model is independent and identically distributed (iid) Generalized Gaussian Distribution (GGD) model for wavelet coefficients [1, 2]. This model has been successfully used in image denoising and restoration. In recent years, there has been a fair amount of research on filtering using wavelet coefficients thresholding because wavelets provide an appropriate basis for separating noisy signal from the image signal. The problem of wavelet based denoising can be expressed as an estimation of clean coefficients from noisy data with Bayesian estimation techniques. If the Maximum a Posterior (MAP) estimator is used for this problem, the solution requires a prior knowledge about the distribution of wavelet coefficients. Based on the distribution type, the corresponding estimator (shrinkage function) is obtained. The classical soft threshold shrinkage function can also be obtained by a Laplacian probability density function (pdf) [3]. Many researchers have proposed the bivariate pdfs for modeling the interscale dependency [3, 4, 5, 6]. Usually these pdfs improve the denoising results they may lead to complicated algorithms. Intrascale dependency states that pdfs using spatial local parameters are able to capture the statistical properties of wavelets [7, 8]. Mihcak [8] proposes a Gaussian pdf with local variance for denoising and earns impressive results with his simple algorithm. In this paper, we use a Laplacian pdf with local variance to model the heavy-tailed property and interscale and intrascale dependencies of wavelet coefficients. This pdf is univariate we estimate the local variance of each coefficients using its spatial adjacent and its parent's spatial adjacent to incorporate both inter and intrascale dependencies in this estimation. Denoising is commonly done by wavelet shrinkage irrespective to the type of DWT applied.

Wavelet shrinkage is a method of removing noise from images by shrinking the empirical wavelet coefficients in the wavelet domain and it is a non linear image denoising procedure to remove the noise. A common shrinkage approach is thresholding [9, 10] which sets the wavelet coefficients with "small" magnitudes to zero while retains shrinking in magnitude for the remaining ones. Originally, Donoho and Johnstone proposed the use of a universal threshold uniformly throughout the entire wavelet decomposition tree which was found to be more efficient [11, 12, 13, 14, 15]. Although thresholding with a uniform threshold per subband is attractive due to its simplicity, the performance is limited and the denoising quality is often not satisfactory. Thus wavelet shrinkage methods using separate threshold in each subband have been developed over recent years. Some methods of selecting thresholds that are adaptive to different spatial characteristics have been recently proposed and investigated [16, 17, 18]. In general, adaptive approaches have found to be more effective than their global counterparts.

In this paper a new model is proposed based on wavelet coefficients and noise variance is estimated for each coefficient depending on the subband using Maximum Likelihood (ML) estimator. A multiplying factor (α) is included in the optimum threshold formula to make the threshold value dependent on decomposition level. After computing threshold, apply soft thresholding to each noisy coefficient. By inverting the multi scale decomposition, the resultant quality image with less blurring and preserving more detail information is reconstructed.

2. Wavelet Coefficient Model

Considering the advantages and limitations of the statistical model a new model is proposed based on the wavelet coefficients. Wavelet coefficients with large magnitudes are representatives of edges or some textures. While those with small magnitude are associated with smooth regions such as the background.

In this smooth region the signal variance for every sub band are estimated by a ML estimator. This method presents effective results but their spatial adaptivity is not well suited near object edges where the variance field is not smoothly varied. To overcome this the coefficient in each subband except the first fine scale is partitioned into two classes based on the magnitudes of their parents, namely significant class and insignificant class in the corresponding region .The significant class represents high activity regions and insignificant class corresponds to smooth region. The sizes of the two classes are controlled by the significance threshold T. If the magnitude of the parent is larger than T then the coefficient is included in significant class otherwise it is included in insignificant class.

The two classes have different statics. The histogram of the coefficient in insignificant class is highly concentrated around zero while that of significant class is more spread out. Hence the coefficients in significant class are modeled as independent identically distributed (iid) Laplacian with zero mean. For the coefficient in insignificant model which corresponds to homogeneous regions, the usage of intrascale model in Estimation Quantization [EQ] coder is appropriate [19]. It provides a good fit for the first order statics of wavelet coefficients and well models the nonstationary nature of low-activity regions.

2.1 Statistical Model

The observation model is expressed as follows $Y = X + V$, where Y is the wavelet transform of the degraded image, X is the wavelet transform of the original image, V denotes the wavelet transform of the noise components following the Gaussian distribution $N(0,\sigma_V^2)$ Since X and V are mutually independent, the variances of Y, X and V are given by

$$\sigma_Y^2 = \sigma_X^2 + \sigma_V^2 \tag{1}$$

3. Estimation of Noise Variance

The following steps are used to evaluate the variance estimate for each wavelet coefficients depending on the subband.

Step 1: The noise variance σ_V^2 can be accurately estimated from the first decomposition level diagonal subband HH_1 by the robust and accurate median estimator [20].

$$\sigma_V^2 = \left(\frac{median|y(V)|}{0.6745} \right)^2 \tag{2}$$

Where $y(V)$ represents the coefficients HH_1 subband.

Step 2: The coarse subbands are not processed because the coarse subband has very high SNR. These coefficients are considered reliable.

Step 3: For each of the three subbands (horizontal, vertical and diagonal orientations) coefficients within the subband are modeled as identically independently distributed with zero mean and variance $\sigma_{\bar{x},j}^2$ (where j indicates the subband). The variance estimate is computed from the noisy coefficients in subband j as

$$\sigma_{\bar{x},j}^2 = \max\{0, \mathrm{var}\{\overline{y_i}, i \in subbandj\} - \sigma_v^2\} \tag{3}$$

Using MAP, estimation of $\bar{x}$ is obtained by applying a soft threshold λ as given in equation (4) to each noisy coefficient.

$$\lambda = \sqrt{2}\sigma_v^2 / \sigma_{\bar{x},j}^2, \; j \in subband \tag{4}$$

Step 4: In each of the other high sub bands, coefficients are assigned either to significant or insignificant classes depending on the magnitude of their estimated parent relative to the significance threshold T, where

$$T = \sigma\sqrt{2\log N^2} \tag{5}$$

(i) Coefficients in significant class are modeled as iid Laplacian with zero mean and their variance $\sigma^2_{\bar{x},insig}$ is estimated from the noisy coefficients as mentioned in step 3. Again the MAP estimator is a simple soft thresholding scheme where its threshold value is adjusted to the signal variance.

(ii) Coefficients in insignificant class which has small magnitude representing smooth areas, $\sigma^2_{\bar{x},insig}$ is estimated using ML estimator in order to have an estimate for a local neighborhood $\sigma^2_{\bar{x}}$ where variance is assumed to be constant. The estimate of the class coefficient variance is

$$\sigma^2_{\bar{x},insig} = \frac{1}{M}\left(\sum_{V=1}^{M} y^2(V) - \sigma^2_V\right) \tag{6}$$

where M represents the number of wavelet coefficients residing in local neighborhood *N*. Considering the coefficients belonging to a insignificant class inside the window are used by excluding the one which belong to significant class, the MAP estimator is given by

$$\bar{x} = \frac{\sigma^2_{\bar{x},insig}}{\sigma^2_{\bar{x},insig} + \sigma^2_V}\,\overline{y_i} \tag{7}$$

Thus the coefficient of estimates corresponding to the high subband are obtained by repeating the above steps from parent to child subband, starting from the coarse scale and terminating in the highest subband.

4. Optimum Value Threshold and Proposed Technique

Wavelet thresholding [21, 22, 23] is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising. It removes noise by killing coefficients that are insignificant relative to some threshold and turns out to be simple and effective which depends heavily on the choice of a thresholding parameter. The choice of this threshold determines the efficacy of denoising to a great extent.

4.1 Threshold Selection

Finding an optimum value thresholding is not an easy task. A small threshold may yield a result close to the input, but the result may still be noisy. A large threshold on the other hand, produces a signal with a large number of zero coefficients. This leads to a smooth signal. Paying too much attention to smoothness destroys details and it may cause blur and artifacts in image processing.

Soft thresholding method is used to analyze the performance of denoising system for different levels of DWT decomposition, as it results in better denoising performance than other denoising methods. Also it leads to less severe distortion of the object of interest than other thresholding methods [24]. Several approaches have been suggested for setting the threshold for each band of the wavelet decomposition. A common approach is to compute the sample variance σ^2 of the coefficients in each band and set the threshold to any multiple of standard deviation σ for that band [25]. Thus, to implement a soft threshold of the DWT coefficients for a particular wavelet band, the coefficients of that band should be thresholded as shown in Fig. 1(a). The soft thresholding is generally represented by,

$$d_{ik}^{soft} = \begin{matrix} sign(d_{ik})(|d_{ik}| - \lambda^*) if d_{ik} > \lambda^* \\ 0 if d_{ik} \le \lambda^* \end{matrix} \tag{8}$$

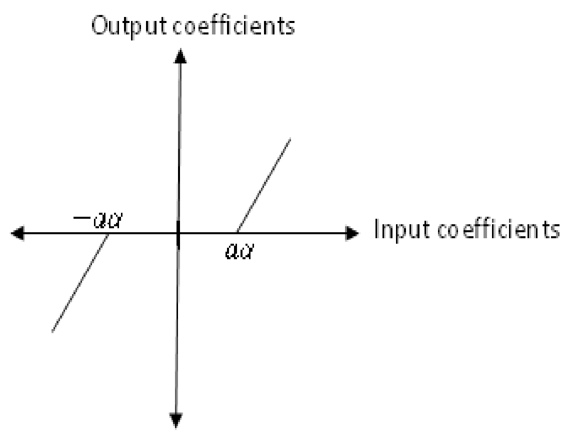

Fig.1 Soft threshold Characteristics with λ = aσ

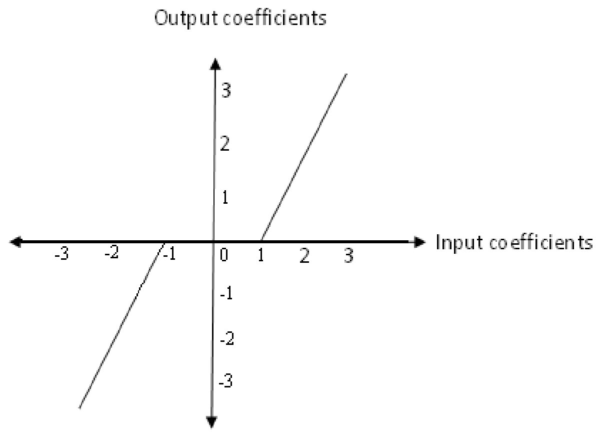

Fig.2 Soft threshold Characteristics with λ = 1

4.2 Optimum Value Threshold

An adaptive thresholding is proposed by fixing the optimum thresholding value depending on the decomposition level. At every decomposition level, four frequency subbands are obtained namely LL, LH, HL, HH. The next level should be applied to the low frequency subband LL only. This process is continued until a prespecified level (level 2) is reached. In wavelet domain, as the level of subbands increases its coefficients becomes smoother. That is, subband HL_2 is smoother than the corresponding subband in the first level (HL_1) and so the threshold value of HL_2 should be smaller than that of HL_1.

In the wavelet decomposition, the magnitude of the coefficient varies depending on the decomposition level. Therefore, if all levels are processed with one threshold value, the processed image may be overly smoothened so that sufficient information preservation is not possible and the image gets blurred. To overcome this problem and to obtain a significantly superior quality image, the multiplying factor α is included in the threshold formula to get better PSNR value by preserving edges where

$$\alpha = 2^{L-K}\sqrt{\log M} \qquad (9)$$

L is the number of wavelet decomposition level, K is the level at which the subband is available, M is the total number of wavelet coefficients. Using this multiplication factor α the optimum threshold formula for the proposed technique is given by

$$\lambda^{*} = \alpha\lambda \qquad (10)$$

Where λ, σ are calculated using equations (4) and (9) respectively.

4.3 Proposed threshold algorithm

The proposed block diagram of wavelet based image denoising system is shown in Fig. 2. Wavelet Based Denoising method relies on the fact that noise commonly manifests itself as fine- grained structure in the image and DWT provides a scale based decomposition. Thus, most of the noise tends to be represented by wavelet coefficient at the finer scales. Discarding these coefficients would result in a natural filtering of the noise on the basis of scale. As the coefficients at such scales also tend to be primary carriers of edge information, the DWT noisy coefficients can be made zero if their values are below its optimum threshold value. On the other hand, the edge relating coefficients are usually above the threshold. The inverse DWT of the thresholded coefficients is the denoised image.

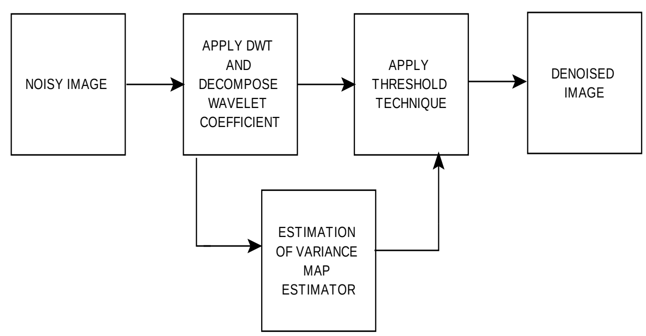

Fig. 3 Block diagram of the proposed method

Algorithm:
The complete algorithm of the proposed wavelet based denoising technique is explained in the following steps:
Input: Noisy image
Output: Denoised image
Step 1: Perform Multiscale decomposition of the image corrupted by Gaussian noise using wavelet transform.
Step 2: Estimate the noise variance σ_V^2 using equation (2) for each scale and compute the scale parameter.
Step 3: For each of the three subbands variance estimate is computed from the noisy coefficient in subband j using equation (3).
Step 4: In each of the other high subbands the estimates of the class coefficient variance are estimated using equation (3) and (6).
Step 5: Calculate threshold value using optimum value threshold formula as given in equation (10) after finding the multiplying factor σ for each subband using the relation given in (9)
After computing threshold for each subband except the low pass or approximation subband, apply soft thresholding to each wavelet coefficient using threshold

given in equation (8), by substituting the threshold value obtained in Step 5.

Step6: Invert the multiple decomposition to reconstruct the denoised image.

5. Experiment and Results

The above algorithm has been applied on several natural gray scale test images like Lena Barbara and pepper at different Gaussian Noise level (standard deviation $\sigma = 0.001, 0.002, 0.003, 0.004$. Here we used Daubechius (Db4), the least asymmetric compactly supported wavelet at two levels of decomposition. Performance of noise reduction algorithm is measured using quantitative performance measure such as Peak Signal to Noise Ratio (PSNR) as given in Table-1 and interms of visual quality of images, as shown in fig-4.To evaluate the performance of the proposed method, it is compared with the Baye's shrink, Modified Baye's shrink and Normal shrink using PSNR which is defined as

$$PSNR = 20\log_{10}(\frac{255^2}{MSE})dB \tag{11}$$

Where MSE denotes the Mean Square Error between the original and denoised image given by

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(X_{ij} - Y_{ij})^2 \tag{12}$$

Where M, N are width and height of image. Y –Noisy image X –original image.

5.1 Statiscal Analysis

Consider the PSNR value obtained for Barbara Image of different noise level using our proposed technique and Modified Bayes Shrink method as two samples X_1 and X_2 of sizes n_1 and n_2 $(n_1 = n_2 = 10)$ respectively. The significant differences between these two samples were tested using Student's t-test: Two samples assuming equal variance.

By using the test statistic $t = \frac{\bar{x_1} - \bar{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ ($\sim t(n1 + n2 - 2)$) d.f (degrees of freedom) for the above sample it is seen that the calculated value of t is greater than the critical value of t at 5% level of significance for 18 d.f. Due to this the null hypothesis $H_0 : \mu_1 = \mu_2$ is rejected and $H_0 : \mu_1 > \mu_2$ (1 tailed test) is accepted. By this significant test the PSNR value obtained for Barbara image of different noise level by our proposed method is significant than by using modified Bayes Shrink method. A similar test is also done by considering the other sample as Bayes Shrink Method, Normal Shrink Method. In these cases also it is observed that the proposed method yields a significant value. Hence Adaptive Subband Threshold Technique out performs the other wavelet method by possessing high PSNR value. This method finds application in denoising images those are corrupted during transmission which is normally random in nature.

Table- 1 Comparison of PSNR of different wavelet shrinkage method for different images corrupted by Gaussian noise.

Image	Noise level	Noisy	Baye's Shrink	Normal shrink	Modified Baye's Shrink	Proposed method
Barbara	0.001	69.08	73.10	73.15	73.05	73.01
	0.002	62.17	66.14	66.53	66.20	67.17
	0.003	58.13	61.80	62.24	61.88	63.06
	0.004	55.28	58.69	59.12	58.79	59.99
Lena	0.001	69.07	75.11	75.63	75.19	76.37
	0.002	62.15	67.35	67.99	67.46	69.27
	0.003	58.10	62.63	63.23	62.73	64.51
	0.004	55.24	59.28	59.84	59.38	61.04
House	0.001	69.01	75.12	75.79	75.29	76.77
	0.002	62.09	67.13	67.86	67.34	69.14
	0.003	58.04	62.37	63.05	62.57	64.45
	0.004	55.18	59.02	59.64	59.21	60.75

From Table -1 it is observed that the proposed thresholding technique out performs the other denoising methods by possessing high PSNR value

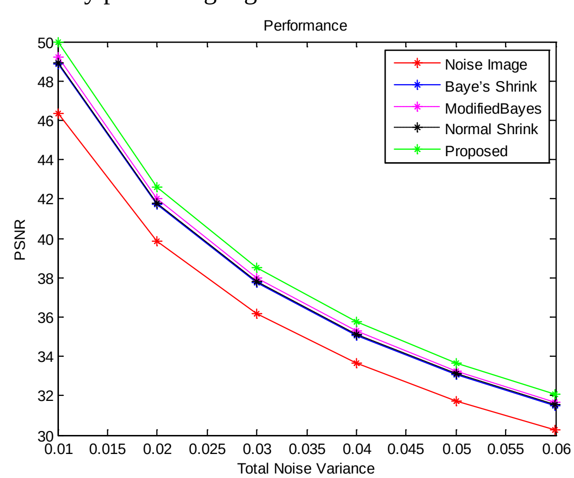

Fig. 3: Comparison of PSNR Value of different denoising methods for Barbara image

From Fig. 3 it is observed that Baye's shrink performs little denoising in high activity sub regions to preserve the sharpness of the edges but completely denoise the flat subparts of the image. Normal shrink preserve edges better than noise removal method using Baye's shrink. Modified Baye's shrink yields a better results for denoising and also adopts thresholding strategy by preserving edges better than Baye's and Normal shrink. The proposed thresholding algorithm gives better performance than other spatial domain filter like Baye's shrink, Normal shrink and Modified Baye's shrink by giving a better PSNR value. Further it out performs the performance of the above mentioned thresholding algorithm by preserving the edges as well as removing the noise, due to the advantages of using the multiplying factor α included in the optimum value threshold formula and subband thresholding.

Fig. 5: Quality performance of different denoising methods for Barbara image (noise standard deviation $\sigma_V = 0.001$) (a) Original image (b) Noisy Image (PSNR=69.07) (c) Adaptive Threshold (PSNR=73.01) (d) Normal shrink (PSNR=73.15) (e) Modified Baye's shrink (PSNR=73.05) (f) Baye's Shrink (PSNR=73.02)

6. Conclusion

The proposed threshold estimation method is based on the analysis of statistical parameters like standard deviation, variance of the sub band coefficients using ML or MAP estimator which is more sub bands adaptive. Since the decomposition level dependent is included as a multiplying factor α in the optimum value threshold formula along with sub band variance estimation, the proposed technique yields significantly superior image quality by preserving edges and a better PSNR value. It is also observed that the images corrupted with less noise densities, single level of decomposition is sufficient. While for images corrupted with higher noise density second level of decomposition is required irrespective of the images.

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