Reduction of Measuring Items- Contemporary Issue in Assessing Internal Consistency

Oluwaseun Gbenga Fadare[1,*], Hezekiah Oluleye Babatunde[2], Gbenga Olayinka Ojo[3], John Niyi Iyanda[4]
Fisayo Caleb Sangogboye[5]

[1,*] Department of Computer Science , Joseph Ayo Babalola Univeristy,
Ikeji-Arakeji, Osun-State, Nigeria.

Corresponding Author

[2]Department of Computer Science , Osun-State Univeristy,
Osogbo, Osun-State, Nigeria.

[3]Department of Mathematics and Statistics , Joseph Ayo Babalola Univeristy,
Ikeji-Arakeji, Osun-State, Nigeria.

[4]Language & ICT Lab., Joseph Ayo Babalola Univeristy,
Ikeji-Arakeji, Osun-State, Nigeria.

[5]Department of Computer Science , Joseph Ayo Babalola Univeristy,
Ikeji-Arakeji, Osun-State, Nigeria.

Abstract

This paper discusses principal component analysis (PCA) as an underlying factor reduction and as a complex, sequential variable reduction procedure in measuring and evaluating internal consistency of measuring instrument. This review paper provides theoretically and practical contemporary-issues on PCA and factor analysis, variable redundancy illustration, concepts of principal component, number of meaningful component to retain, and factor analysis extraction method in multivariate analysis. This paper collects in one review article information for researchers and practitioners in understanding the subject matter in further simplification of steps in multivariate analysis.

Keywords: *Principal Component Analysis (PCA), Factor Analysis, Measuring Instrument, Latent Variables, Eigenvalues, Communality.*

1.0 Introduction

Measurement is at the core of doing research. Measurement is the assignment of number to conceptual event. In almost all research, everything has to be reduced to numbers eventually. A measurement is said to be consistent if the measurement can produce similar results if used again in similar circumstance. Internal reliability refers to the extent to which a measure is consistent within itself. A survey instrument measures practically nothing if its internal consistency is unreliable. Hence, PCA is one of the practical methods used in measuring internal consistency of measuring items. Principal component Analysis is applied and performed specifically to measure an account of observed variables, in order to reduce these variables to smaller number of artificial variables called principal components that will considerably account for most of the variances in the observed variables. Then PCA may be used as criterion variables in subsequent analyses.

PCA is a widely utilized and broadly applied statistical default method of extraction in many SPSS and SAS which likely contributes to its popularity. It became widely known in decades ago when computers were slow and expensive to use; it was a quicker, cheaper alternative to factor analysis [1]. It is computed without regard to any underlying structure caused by latent variables; components are calculated using all of the variance of the manifest variables, and all of that variance appears in the solution [2]. PCA is an integral part of factor analysis, which predicts that any latent variable can cause the manifest variables to covary. Some researchers had argued that PCA and principal factor analysis cannot be separated from each other and they are related and interwoven.

In this paper, efforts are made to clarify the two terms, citing necessary and appropriate statistical reasons from doing so. The goal of PCA is to decompose a data table with correlated measurements into a new set of uncorrelated that is orthogonal variables. These variables are called depending upon the context, principal components, factors, eigenvectors, singular vectors or loadings. Each unit is assigned a set of scores which correspond to its projection on the components. The results of the analysis are sometimes presented with graphs; plotting the projections of the units onto the components, and the loadings of the variable. The importance of each component is expressed by the variance (i.e. eigenvalues) by the proportion of the variance explained.

1.1 PCA versus Factor Analysis

These two techniques are used to analyze groups of correlated variables representing one or more related ideologies, for instance, indicators of socioeconomic status, work satisfaction, health, self – esteem. PCA is used to find optimal ways of combining variables into small number of subsets, while factor analysis is used to identify the structure underlying such variables and to estimate scores to measure latent factors themselves. The main applications of these techniques can be found in the analysis of multiple indicators, measurement and validation of complex constructs index and scale construction, and data reduction. Another difference between the two approaches has to do with the variance that is analyzed. In PCA, all of the observed variances are analyzed, while in factor analysis, it is only the shared variance that is analyzed.

2.0 Illustration of Variable Redundancy Procedures

PCA is a variable reduction procedure; quite practically applicable when there is redundancy, large enough, in the observed variable measured. Redundancy means that some of the variables are correlated with one another, possibly because they are measuring the same construct. Because of this redundancy, researcher reduces the observed variables into a smaller number of principal components (artificial variables) that will account for most of the variables in the observed variables. An assumed example of a research items will be used to illustrate the concepts of variable redundancy. If there exit eight-items measures of work satisfaction as shown below:

Please response to each of the following statements by rating the answer from 1 to 7 in which 1= “strongly disagree and 7= “strongly agree”

My boss treats me with consideration.

My boss consults me concerning important issues/decisions that affect my work.

My boss gives me recognition when I do a good work.

My boss gives me the support I need to do my work well.

My boss likes my work.

My salary is fair.

My salary is commensurate to the amount of my responsibility.

My salary is comparable to the pay earned by other employees whose jobs are similar to mine

Administering this questionnaire to over 350 employees using the above scale, one of the likely problems that one can encounter after the survey is the concept of redundancy. Items 1 to 5 and 6 to 8 treat this problem respectively. Table 1 shows fictitious correlation matrix among the 8 items

Table 1

Varia-bles	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1	*1.0*							
2	*.75*	*1.0*						
3	*.82*	*.83*	*1.0*					
4	*.69*	*.91*	*.87*	*1.0*				
5	*.72*	*.88*	*.81*	*.79*	*1.0*			
6	*.03*	*.02*	*.04*	*.05*	*.09*	*1.0*		
7	*.05*	*.03*	*.05*	*.07*	*.60*	*.60*	*1.0*	
8	*.02*	*.04*	*.03*	*.08*	*.50*	*.72*	*0.7*	*1.00*

The above 8 items show two distinct patterns. Item 1-5 shows strong correlations with one another. This is because items 1-5 are measuring the same construct. In the same way, items 6 to 8 shows weak correlation with one another, a possible indication that they all measure the same construct as well. Having this apparent redundancy; it can be deduced that items 1 to 8 exhibit two different constructs. PCA preaches that these variables (items 1 to 8 correlations) should be reduced into two distinct components; item 1 to 5 as an employees' satisfaction with boss and item 5 to 7 as a single new variable reflecting satisfaction with salary. Researchers can now use these two new artificial variables, rather than the eight original variables as predictor variable in multivariate analysis

2.1 Concepts of Principal components

Principal component can be technically defined as a linear combination of optimally-weighted observed variable. Subject scores on a principal component are computed so that each subject would have scores on two components discussed above. One score on the satisfaction with boss component, and other score on the satisfaction with salary component.

The general form for the formula to compute scores on the first component extracted (created) in a principal component analysis is shown:

C1 = k11(X1) + k12(X 2) + k13(X 3) ... k1p (Xp)

where

C1 = the subject's score on principal component 1 (the first component extracted)

k1p = the regression coefficient for observed variable p, as used in creating principal component 1

Xp = the subject's score on observed variable p.

Assuming that questions 1 to 5 were assigned relatively large regression weights that range from-34 to .51, while items 6 to 8 were assigned very small weight ranging from .02 to .04. Component 1, which is the extracted (satisfaction with boss component1), will be computed and have similar representation with that shown below:

C1 = .51(X1) + .44(X2) + .40(X3) + .48(X4) + .34(X5) + .02(X6) + .01(X7) + .04(X8)

While X_1 to X_8 represents coefficient matrix among the 8 items as shown above

Implication of this is that items 1 to 5 represent or account for maximal of total variance in the observed variable as regards to C1. Similarly, component 2, which is the extracted (satisfaction with salary), will be computed and have similar representation with that shown below:

C2 = .01(X1) + .03(X2) + .04(X3) + .02(X4) + .041(X5) + .52(X6) + .49(X7) + .53(X8)

Items 1-5 were assigned relatively small regression weight that ranges from .01 to .04 while items 6 to 8 were assigned large weight ranging from .49 to .53

Therefore, items 6 to 8 accounts for maximal amount of total variance in the observed variables as regards to C2. Hence scores based on C1 and C2 can be computed and results used in further analysis.

2.2 Characteristic of Principal Components

Characteristic of components C1 and C2 as used above will have the following: The first component extracted in a PCA accounts for a maximal amount of total variance in the observed variances. This means the first component will be correlated with at least, some or most of the observed variance. The second component will account for a maximal amount of variance in the data set that was not accounted for by the first component. This means that the second component will be correlated with some of the observed variances that did not exhibit strong correlations with component 1. Secondly, second component will be uncorrelated with the first component.

2.3 Determining the Number of "Meaningful" components to Retain

Though, our research (fictitious) shows two components extracted but in actual sense the number of components extracted is equal to the number of variables being analyzed; it is necessary and compulsory for researchers to decide how many of these components are truly meaningful and worthy of being retained for rotation and interpretation. Generally, one expects that only the first few components will account for meaningful amounts of variance and that the later components will tend to account for trivial variance, sometimes it is like that. The next step determines how many meaningful components should be retained for interpretation. There are four criteria used in achieving this decision.

(1) Eigenvalue-one criterior
(2) The scree test
(3) The proportional of variance accounted for
(4) The interpretability criterion

One of the most commonly used criteria for solving the number of components problem is the eigenvalue-one criterior, also known as the Kaiser criterion [3]. This method retains and interprets any component with an eigenvalue greater than 1. On the other hand, a component with an eigenvalue less than 1.00 is accounting for less variance than had been contributed by one variable. The purpose of PCA is to reduce a number of observed variables into a relatively smaller number of components; this cannot be achieved if one retains component that accounts

IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 6, No 2, November 2011
ISSN (Online): 1694-0814
www.IJCSI.org

for less variance than had been contributed by individual variables. For this reason, components with eigenvalues less than 1.0 are viewed as trivial and are not retained. This method becomes a problem when a small moderate number of variables are analyzed and the variables communalities are high. Stevens in 1986, [4] reviewed studies that have investigated the accuracy of the eigenvalue-one criterior and recommends its use only when less than 30 variables are being analyzed and communalities are greater than 0.07 or when the analysis us based on over 250 observations and the mean communality greater than or equal to 0.60

2.4 The scree test.

A graphical method is the *scree* test first proposed by Cattell in 1966. Cattell, [5] proposed to find the place where the smooth decrease of eigenvalues appears to level off to the right of the plot. An example of scree test is shown below in figure.

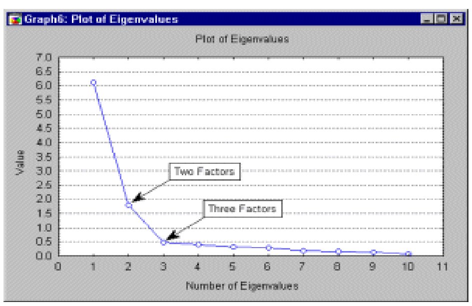

Figure 1

From the graph, we would probably retain 2 or 3 factors. This method involves plotting eigenvalue associated with each component, with a view of detecting a "break" between the components with relatively large eigenvalues and those with small eigenvalues. The components that appear before the break are assumed to be meaningful and are retained for rotation, those appearing after the break are assumed to be unimportant and are not retained. A scree test that displays several large breaks, researchers should look for the last big break before the eigenvalues begin to level off. Only the components that appear before this large break should be retained. The scree test can provide reasonably accurate results, provided the sample is large enough, above 200 and most of the variables communalities are large, [4].

2.5 Proportional of Variance accounted for

This is equaled to eigenvalue for the component of interest divided by the total eigenvalues of the correlation matrix. Total eigenvalues of the correlation matrix is equal to the total number of variable being analyzed because each variable contributes one unit of variance to the analysis. Assuming there exits the following components: First component accounts for 39% of the total variance, second component accounts for 32%, third component accounts for 13%, fourth one account for 7% and fifth one accounts for 5% while the last one accounts for 4%. Assuming that researcher wants to retain any component that accounts for at least 30% of the variance in the data set, using this criterior we would retain components 1 and 2. For at least, 10% of the total variance, we could retain components 1, 2 and 3. Alternatively, criterior to retain enough components so that the cumulative percent of variance accounted for is equal to some minimal value. For instance components 1, 2, 3 and 4 accounted for approximately 39%, 32%, 13% and 7% of the total variance respectively, adding these percentages together results in a sum of 91%. This means that the cumulative percent of variance accounted for by components 1, 2, 3, 4 is 91%.

2.6 Interpretability Criteria

This method involves interpreting the substantive meaning of the retained components and verifying that this interpretation makes sense in terms of what is known about the constructs under investigation. In achieving this, the following rules must be adhered to:

(1) Are there at least 3 variables (items) with significant loadings on each retained component? satisfactory solution is obtained if given component is measured by less than three components

(2) Do the variables that load on a given components share the same conceptual meaning. For example; if 5 questions on a survey all load on component 1. Do all five questions seem to be measuring the same construct?

(3) Do the variables that load on different components seem to be measuring different construct? For example if 3 questions load on component 1, and five other questions load on component2 , do the first three questions seem to be measuring a construct that is conceptually different from the constructs measured by the last three questions?

(4) Does the rotated factor pattern demonstrate simple structure? Simple structure means that the pattern possess two characteristics: most of

the variables have relatively high factor loadings on only one component, and near zero loadings on the other components (b) most components have relatively high factor loading for some variables, and near zero loadings for the remaining variables

2.7 Factor Analysis Extraction Method

The next decision is rotation method. The goal of rotation is to simplify and clarify the data structure. As with extraction method, there are a variety of choices. Varimax rotation is by far the most common choice. Varimax, quartimax, and equamax are commonly available orthogonal methods of rotation; direct oblimin, quartimin, and promax are oblique. Orthogonal rotations produce factors that are uncorrelated while oblique methods allow the factors to correlate. It is practically okay for researchers to use orthogonal rotation because it produces more easily interpretable result. Oblique rotation output is only slightly more complex than orthogonal rotation output. There is no widely preferred method of oblique rotation; all tend to produce similar results, [6].

Costello et al, [7] concluded that the total variance accounted for after rotation is only given for an orthogonal rotation. It is computed using sum of squares loadings, which cannot be added when factors are correlated, but with an oblique rotation the difference between principal components and factor analysis still appears in the magnitude of the item loadings

3. Assumptions Underlying Principal Component Analysis

In multivariate analysis, the data underlying in PCA is preformed on a matrix of Pearson correlation coefficients, and should satisfy the following assumption according to [8]

• Interval-level measurement. All analyzed variables should be assessed on an interval or ratio level of measurement.

• Random sampling. Each subject will contribute one score on each observed variable. These sets of scores should represent a random sample drawn from the population of interest.

• Linearity. The relationship between all observed variables should be linear.

• Normal distributions. Each observed variable should be normally distributed.

• Bivariate normal distribution. Each pair of observed variables should display a bivariate normal distribution.

4. Conclusion

Principal component analysis is a powerful tool for reducing a number of observed variables into a smaller number of artificial variables that account for most of the variance in the data set of measuring instrument. It is essentially useful when researcher needs a data reduction procedure that makes no assumptions concerning an underlying causal structure that is responsible for covariation in the data. Principal component analysis is often used to construct multiple-item scales from the items that constitute questionnaires; once these scales have been instrumented it is often desirable to assess their reliability by computing coefficient alpha: an index of internal consistency reliability. Various techniques and procedures used in capturing PCA to estimate and assessing measuring model of survey questionnaire have been treated and discussed. Researchers should know that ability to answer research questions is as good as the instrument one developed. A well developed survey items will better provide researchers with quality data with which its internal consistency is justified. Establishing measurement reliability is of inarguable importance in both applied and theoretical research because reliability constitutes a necessary first step toward ensuring construct validity [9,10,11] Reliability is deemed so important that even when authors are not creating a scale but only using established scales, readers nevertheless expect a reliability index to be reported. By far the most frequently reported reliability index is Cronbach's coefficient alpha [12, 13]

References

[1] R. L. Gorsuch, "Common Factor-Analysis Versus Component Analysis - Some Well and Little Known Facts", Multivariate Behavioral Research, Vol.25, No.1, 1990, pp. 33-39.

[2] J. K. Ford, R. C. MacCallum, and M. Tait, "The Application of Exploratory Factor-Analysis in Applied-Psychology - a Critical-Review and Analysis", Personnel Psychology, Vol.39, No.2, 1986, pp. 291-314.

[3] H. F. Kaiser, "The application of electronic computers to factor analysis", Educational and Psychological Measurement, Vol.20, 1960, pp. 141-151.

[4] J. Stevens, Applied multivariate statistics for the social sciences. Hillsdale, NJ: Lawrence Erlbaum Associates, 1986.

[5] R. B. Cattell, "The scree test for the number of factors", Multivariate Behavioral Research, Vol.1, 1966, pp. 245-276.

[6] L.R. Fabrigar, D.T. Wegener, R.C. MacCallum, and E. J. Strahan, "Evaluating the use of exploratory factor analysis in psychological research", Psychological Methods, Vol.4, No.3, 1999, pp.272-299.

[7] Costello, B. Anna and Jason Osborne, "Best practices in exploratory factor analysis: four recommendations for getting the most from your analysis", Practical Assessment Research & Evaluation, Vol.10, No.7, 2005.

[8] L. Hatcher, and E. A Stepanski, Step-by-step approach to using the SAS system for univariate and multivariate statistics Cary, NC: SAS Institute Inc. 1994.

[9] L.R. Aiken, Psychological testing and assessment (11th edition), Boston: Allyn & Bacon. 2002.

[10] A. Anastasi, and S. Urbina, Psychological testing (7th edition.), NewYork: Prentice Hall. 1996.

[11] L.J. Cronbach, "Coefficient alpha and the internal structure of tests" Psychometrika, Vol.16, 1951, pp. 297–334.

[12] T.P. Hogan, A. Benjamin, and K.L. Brezinski, "Reliability methods", Educational and Psychological Measurement, Vol.60, 2000, pp.523–531.

[13] R.A. Peterson, "A meta-analysis of Cronbach's coefficient alpha", Journal of Consumer Research, Vol.21, 1994, 381–391.