Evaluation of coordination contracts for a two stage Supply Chain under price dependent demand

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Abstract

Supply chains are characterized by many activities and actors that generally pursue conflicting objectives. Coordination between them may be then necessary to align the individual objectives with the global supply chain objective and achieve optimal performance.

In this paper, we propose a model that aims to assess the relative performance of three well known Coordination Contracts for a two level Supply Chain under price dependent demand. It is shown that a suitable design of the contracts could secure global system efficiency and improve the profit of all the Supply Chain actors.

Keywords: *Supply Chain Management, Coordination, Contracts, Revenue Sharing, Buyback Contracts, Quantity discount*

1. Introduction

A Supply Chain system is usually comprised of many organizations that are often separate and independent economic entities. Although centralized decisions may lead to optimal global Supply Chain performance, these decisions are not always in the best interest of every individual member of the Supply Chain. Independent Supply Chain members are usually more keen in optimizing their individual objectives rather than that of the entire system. This could result in a poor global performance for the Supply Chain as a whole [1] (Whang, 1995).

Thus a key issue in SCM is to develop mechanisms that motivate the independent actors to achieve coordination and optimize Supply Chain Global Performance.

Research has initially focused on centralized Supply Chain. In this setting, the Supply Chain control is assured by a single decision-maker who is given all the contractual powers. In practice, the Supply Chain involves many independent decision makers who pursue generally different objectives.

Coordination contracts are among the main tools that actors of a decentralized Supply Chain can use to reach coherent decisions among them. [2] (Ilaria Giannoccaro*, Pierpaolo Pontrandolfo (2004)).

Coordination contracts have been widely used in practice to increase overall Supply Chain performance and to share risk and rewards between Supply Chain actors.

This paper focuses on evaluating relative performance of different SC contracts. These contracts are used to coordinate a two-stage SC facing a stochastic price dependent customer demand. The issue of the desirability of the contract to the SC actors is also considered.

The paper is organized as follows. In Section 2, a review of the SC contracts is given. We then propose a reference model for evaluating the performance of the different SC contracts in Section 3. In section 4, SC Contracts performance and desirability are discussed through a numerical application.

2. Literature Review

Many forms of Supply Chain contracts have been implemented in industries and analyzed or studied by researchers. Most models are based on the Single Period Newsvendor Problem where coordination is reached through setting the optimal order quantity that maximizes the overall profit of the Supply Chain.

Many extensions to this model have been explored by researcher and usually consider a subset of the following classes of parameters:

- *Decision-Making Process :* Two aspects are mainly considered in literature :
  - Distribution of decision among Supply Chain actors (ex: Centralized vs Decentralized SC)
  - Compliance regime: This issue is related to the right of the SC actors to comply partially or totally with coordination contract terms (Ex: Under a forced compliance regime, the supplier must fulfill totally his customer order.) [3] Cachon, G.P. and Lariviere, M. A., 2001
- *Supply Chain structure:* (Number of tiers and SC network nodes). Most contracts models addressed in literature are two-tier SC (bilateral monopoly) composed of a single supplier and a single retailer serving a final demand. Some research papers have addressed Coordination issues of three level Supply Chain with single actors at each level [4] (Ding Ding, Jian Chen 2008). Some others studied the coordination problem in a context of a Two

level Supply Chain with single vendor and multiple retailer [5] (Darwish & Odah 2010)

- *Certainty/Uncertainty of demand:* The uncertainty of Supply Chain environment refers generally to market demand. The two broad categories are deterministic and probabilistic market demands.
- *Environment dependence of Supply Chain decision:* Market demand is usually sensitive to product selling price and marketing efforts. In this case, Coordination between Supply Chain actors is considered with regard to these internal Supply Chain parameters that impact the market demand. He and all studied Coordination of a two tiers two node Supply Chain in a context of effort and price dependent demand. [6] He, Y., Zhao, X., Zhao, L., and He, J., 2009. [7] Petruzzi and Dada (1999) examine an extension of the newsvendor problem in which stocking quantity and selling price are set simultaneously in context of price dependent demand.
- *Contract period:* The contract period is the duration of time that the contracting agents are assumed to uphold the contract. Distinction can be made between single and multiple replenishment periods.
- *Risk Aversion:* The Supply Chain agent can be risk neutral or show an aversion to risk. [8] Xianghua & all (2004) explore the coordination of SCM with risk averse agents.
- *Information structure:* It pertains to the agents 'knowledge in comparison to the collective knowledge of agents in the Supply Chain. When all the information about Supply Chain is simultaneously known by every agent, the information structure is said to be complete or symmetric. On the other hand, if some agents have some information that the other agents do not, the information structure is incomplete or asymmetric.

In a recent publication, Behzad and all (2010) [15] provide a detailed overview of coordinating contract in literature and present the state of art research in this field.

Two broad classes of coordination contracts have been identified in literature: Quantity dependent Contracts (Quantity Discounts, Quantity flexibility) and Price dependent Contracts (Wholesale price, Buyback or Return policies, Revenue Sharing, Sales rebate, Quantity Discount). The scope of this paper is restricted to three of the well known contracts:

- **Buy Back Contract:** Under this type of contracts, a manufacturer, or an upstream distributor sets the wholesale price and commit to refund a downstream channel member for excess inventory return at the end of the season. The most generous policy promises to refund the full wholesale price for all returned products. [9] (Pasternack, B. A. (1985)) shows that a policy allowing for unlimited returns at partial credit will be system optimal for appropriately chosen wholesale price and refund value. [10] Cachon 2003 shows also that partial return and partial compensation can achieve coordination. [11] Padmanabhan, V., & Png, conclude that, in a context of a single upstream and many downstream actors and a price dependent demand, Return policies is beneficial to the upstream actor when production costs are low and demand is not highly uncertain and competition intensifies at the downstream level.
- **Revenue Sharing:** Under this agreement, the downstream agent (Distributor, retailer) commits to share its revenue with the upstream agent (manufacturer, Distributor) in exchange of a smaller wholesale price. The successful application of this type of contracts has been reported in the American Video rental industry [12] (Cachon and Lariviere, 2005). Cachon and lariviere show that in the case of a single distributor and a single retailer, Revenue Sharing Contracts coordinate the Supply Chain and can allocate profits arbitrarily between actors. [2] Ilaria Giannoccaro*, Pierpaolo Pontrandolfo developed a Revenue Sharing mechanism that coordinate efficiently a three stage Supply Chain and improves profits of the stakeholders.
- **Quantity Discount:** Distinction can be made between incremental and All unit discount. The Quantity Discount is a coordination mechanism that consists of offering a price discount to the buyer so that he orders the global optimal quantity, and improve the profit of both supplier and buyer. This contract has mainly been studied in contexts of deterministic demands. [13] (Jianli Li and Liwen 2006) explore how to use an all unit discount coordination mechanism to achieve coordination within a multi-period supplier–buyer system selling one type of product and facing a probabilistic customer demand. [10] Cachon 2003 shed the light on the coordinating ability of Quantity Discount contract under the assumptions of a single period newsvendor problem.

- **Quantity flexibility:** This coordination mechanism allows downstream actor to change his order quantity within a predefined range as he gains more insight of market demand. On the other side the upstream commits to guaranty product availability within the same range. [14] Tsay 1999 shows that under certain conditions Quantity flexibility contracts can be considered to share risk of market uncertainty between Supply Chain actors and achieve system wide optimal outcome.

3. Reference Model

Assumptions and notations

In this model, we consider a two stage Supply Chain composed of an upstream and a downstream firm. We will refer to the upstream firm as the manufacturer and the downstream firm as the retailer

There is one selling season with stochastic demand and a single opportunity for the retailer to order inventory from the manufacturer before the selling season begins.

The manufacturer produces a single short life cycle product at marginal cost c_m and sells it to a Retailer at a wholesale price w.

The Retailer decides the retail price p and order quantity q. He also incurs marginal variable cost c_r. In the event of stock out, unmet demand is lost, resulting in the margin being lost to the retailer, but without additional penalty (such as a loss of good-will). The retailer can return full-unsold inventory back to the supplier. All The Unsold items are salvaged at price **s** by the retailer at the end of the season. The manufacturer and retailer are both in a monopoly position to serve the end customer. The firms are assumed risk neutral and thus each firm pursue profit maximization.

Information is symmetric for all parties i.e. all the information regarding demand and cost parameters is common knowledge to both parties.

When a contract is accepted, the manufacturer must fulfill the order placed by the retailer.

At the end of the selling season, the market demand is realized, which is stochastic, downward sloping in the retail price. We assume market demand for the product D is stochastic price-dependent as $D(p) = d(p) + \varepsilon$, where $d(p)$ is a deterministic decreasing function of p that captures the dependency between demand and price. $d(p)$ is a linear curve in p and is represented by the following function : $d(p) = a - bp \; (a > 0, b > 0)$. ε is a random variable with known distribution that shows randomness in demand and is price-independent, we assume that ε follows Normal distribution with mean μ and standard deviation σ, i.e., $\varepsilon \sim N(\mu, \sigma^2)$ and that the variance of σ^2 is small relative to a . Let f be the probability density function of ε and F its cumulative distribution function where f is strictly positive.The expected value of demand for a given price p is $d(p)$.

It is also assumed that the retailer sets his price at the same time as his stocking decision and the price is fixed throughout the season.

All these assumptions are moderately reasonable and standard in the literature.

Notations:

c_m Marginal Cost of the manufacturer
c_r Marginal Cost of the retailer
c Total Marginal Cost ($c_{u+}c_d$)
p Unit selling price
w Unit wholesale price charged by supplier
s Unit salvage value at the downstream side
Q_r^* Retailer optimal quantity
Q_{sc}^* Supply Chain optimal quantity
π_m Expected profit of the manufacturer
π_r Expected profit of the retailer
q Quantity ordered by retailer
$S(q,p)$ Expected sale for an order quantity of q and a retail price of p
π_i^j The profit realized where i = {r:retailer; m: manufacturer} and j ={NC: Market Setting, SC: optimal case, B: buyback contract, RS: Revenue Sharing contract, QD: Quantity Discount contract}

Buy Back Contract:
r: Unit Buyback price offered by the manufacturer to the retailer for unsold items

Revenue Sharing Contract:
Φ Fraction of the retailer revenue transferred to upstream actor

Additional Assumptions

It is assumed that $p > w > s$; $c_m < w$, $w + c_r < p$; $w > r+s$ and $c_r + c_m > s$

4. Detailed model

In a first step, the Supply Chain performance is assessed for two basic cases:

The first case corresponds to Market-like setting, where the retailer sets his price at the same time as his stocking decision independently of the manufacturer. In the second case, a unique decision maker determine the optimal stocking and pricing policy that maximizes the global Supply Chain profit given the market demand.

In a second step, the Supply Chain performance is evaluated under 3 different types of coordination contracts (Buy Back Contracts, Revenue Sharing, and Quantity Discounts), relatively to the basic cases.

4.1. Centralized setting :

A unique decision regarding the stocking policy and the retail price is made for the whole Supply Chain. The optimal Buyer Order quantity and the retail price are determined by differentiating Equation (5).

The profit of the downstream actor is:

$$\pi_r\ (q,p) = \begin{cases} p * D(p) - (c_r\ + w) * q + s * \big(q - D(p)\big) & if\ D(p) \le q \\ p * q - (c_r\ + w) * q & if\ D(p) > q \end{cases}$$

(1)

The profit of the upstream actor is :

$\{\pi_m(q,p) = (w - c_m)q$ **(2)**

Let's consider a new variable z such **z = q –d (p)**, the new retailer profit formula becomes :

$$\pi_r\ (z,p) = \begin{cases} p * [d(p) + \varepsilon] - (c_r\ + w)[d(p) + z] + s * [z - \epsilon] \\ \qquad if\ \varepsilon \le z \\ (p - c_r\ - w) * [d(p) + z] \qquad if\ \varepsilon > z \end{cases}$$

(3)

The Supply Chain profit is in this case:

$$\pi(z,p) = \begin{cases} p * [d(p) + \varepsilon] - c * [d(p) + z] + s * [z - \epsilon] & if\ \varepsilon \le z \\ (p - c) * [d(p) + z] & if\ \varepsilon > z \end{cases}$$

(4)

The optimal stocking and pricing policy consists of selling at price p_{SC}^* and ordering q_{SC}^*=z_{SC}^*+d(p_{SC}^*) where z_{SC}^* and p_{SC}^* maximize expected profit

The expected profit of the Supply Chain is :

$$E\big(\pi(z,p)\big) = \int_{-\infty}^{z} (p * [d(p) + u] - c * [d(p) + z] + s * [z - u])f(u)du + \int_{z}^{+\infty} \big((p - c) * [d(p) + z]\big)f(u)du$$

$E\big(\pi(z,p)\big) = (p - c)[d(p) + \mu] - (c - s) \int_{-\infty}^{z}(z - u)f(u)du - (p - c)\ \int_{z}^{+\infty}(u - z)f(u)du$ **(5)**

We refer in the remaining of this section, to the solution approach proposed by Petruzzi and Dada 1999.

Defining $\Lambda(z) = \int_{-\infty}^{z}(z - u)f(u)du$, $\theta(z) = \int_{z}^{+\infty}(u - z)f(u)du$, $\psi(p) = (p - c)[d(p) + \mu]$, we can write the expected Supply Chain profit as:

$$E\big(\pi(z,p)\big) = \psi(p) - (c - s)\Lambda(z) - (p - c)\theta(z)$$

Let's consider the first and second derivative with respect to z and p:

$\frac{\partial E(\pi(z,p))}{\partial z} = -(c - s) + [p^\circ - s - \frac{\theta(z)}{2b}][1 - F(z)]$ (15)

$\frac{\partial^2 E(\pi(z,p))}{\partial z^2} = -(p - s)f(z)$ (16)

$\frac{\partial E(\pi(z,p))}{\partial p} = 2b(p^\circ - p) - \theta(z)$ (17)

$\frac{\partial^2 E(\pi(z,p))}{\partial p^2} = -2b$ (18)

By extension of Petruzzi and Dada solution to the case where **ε** has an unbounded probability distribution function, there is one optimal solution to the maximization problem of $E\big(\pi(z,p(z))\big)$ if the following conditions hold:

a) If F(.) is a distribution function satisfying the conditions 2r(z)²+dr/dz>0 where r(.) =f(.)/[1-F(.)] is the hazard rate
b) $\lim_{+\infty} \frac{\partial E(\pi(z,p(z)))}{\partial z} < 0$
c) If the conditions for (a) and (b) are met and $\lim_{-\infty} \frac{\partial E(\pi(z,p(z)))}{\partial z} > 0$, there is a unique z_{SC}^* that satisfies $\frac{dE(\pi(z,p(z))}{dz} = 0$

The optimal pricing an stocking policy consists then of stocking q_{SC}^*=d(p_{SC}^*)+ z_{SC}^*units and to sell them at price $p_{SC}^* = p^\circ - \frac{\theta(z_{SC}^*)}{2b}\ where\ p^\circ = \frac{a+bc+\mu}{2b}$

Given the assumptions of our model, the conditions a) and b) are fulfilled and the equation (5) has a unique optimal solution (cf. Proof).

Proof:

i. As ε has a normal distribution, It has a strictly increasing failure rate dr/dz>0. It follows that 2r(z)²+dr/dz>0 and then a) is satisfied
ii. $\lim_{+\infty} \frac{\partial E(\pi(z,p(z)))}{\partial z} = -(c - s)$

As $c > s$, it follows that $\lim_{+\infty} \frac{\partial E(\pi(z,p(z)))}{\partial z} < 0$

iii. By rewriting eq (5) as :

$$\frac{\partial E\big(\pi(z,p(z))\big)}{\partial z} = \{-(c - s) + [p^\circ - s][1 - F(z)]\} + \left\{-\frac{\theta(z)}{2b}[1 - F(z)]\right\}$$

When $z \to -\infty$ the first term converges to $-(c - s)$

Let's consider the second term:

$$-\frac{\theta(z)}{2b}[1 - F(z)] = -\frac{\theta(z)}{2b}\int_{z}^{+\infty} f(u)du = -\frac{1}{2b}\left[\int_{z}^{+\infty} uf(u)du - z\int_{z}^{+\infty} f(u)du\right]\int_{z}^{+\infty} f(u)du$$

When $z \to -\infty$, $\int_{z}^{+\infty} uf(u)du \to \mu\ and\ \int_{z}^{+\infty} f(u)du \to 1$

Then the second term converges to +∞ when $z \to -\infty$

From ii and iii it follows that There exist some A and B such that:

$for\ z < A\ , \lim_{-\infty} \frac{\partial E(\pi(z,p))}{\partial z} > 0$ And $for\ z > B\ , \lim_{+\infty} \frac{\partial E(\pi(z,p))}{\partial z} < 0$

As a corollary of Petruzzi and Dada proof for the case of bounded Probability Distribution Function it can be deduced that there exists a unique optimal solution to the maximization problem of $E\big(\pi(z, p(z))\big)$.

4.2. Market like setting (Non Coordinated case):

The retailer and manufacturer act independently and each aims to maximize his profit. The optimal retailer profit is determined by differentiating eq.(6)

$E\big(\pi_r\ (z,p)\big) = \big(p - c_r\ - w\big)[d(p) + \mu] - (c_r\ + w - s) \int_{-\infty}^{z}(z-u)f(u)du - (p - c_r\ - w)\ \int_{z}^{+\infty}(u - z)f(u)du$ **(6)**

Eq(6) is similar to eq(5) if c is substituted by $c_r\ + w$ in eq(5). The conditions a), b) and c) are satisfied as in the coordinated case and the optimal pricing and stocking decision (z_{NC}^*, p_{NC}^*) are determined by differentiating (6).

As $c_m < w$, it can be shown that $(z_M^*, p_M^*) \neq (z_{SC}^*, p_{SC}^*)$. Thus, the expected profit of the downstream actor is reduced if the Supply Chain is centrally coordinated.

Eq(2) shows that the upstream actor's profit is proportional to the Buyer order quantity and then he expects more profit if more quantity is ordered by the Downstream actor.

Even if Central Coordination scheme yields the highest global profit, it is not in the best interest of the downstream actor to be part of it as he will experience a reduction of his profit.

4.3. Revenue Sharing Contract :

Under this contract, the retailer benefits from a reduced wholesale price w_{RS}while sharing a fraction ***Φ*** of the revenue he generates with the manufacturer.

From (1) and (2), the expected profit of the global Supply Chain and the retailer actor can be written as follows:

$E(\pi(q,p)) = (p - s)S(q,p) - (c - s)q$ **(7)**

$E(\pi_r^{RS}(q,p)] = \mathbf{\Phi}(p - s)S(q,p) - \big(c_r\ + w_{RS} - \mathbf{\Phi}s\big)q$ **(8)**

If the wholesale price and the sharing factor are set such that:

$w_{RS} =\ \mathbf{\Phi}c - c_r\ \ where\ \mathbf{\Phi} > \frac{c_r}{c}$ **(8')**

Eq.8 changes into :

$E(\pi_r^{RS}(q,p)) = \mathbf{\Phi}(p - s)S(q,p) - \mathbf{\Phi}(c - s)q$

$E(\pi_r^{RS}(q,p)) = \mathbf{\Phi}\mathrm{E}(\pi(q,p))$**(9)**

Then optimal stocking and pricing policy is then optimal for the downstream actor.

The upstream actor profit is:

$E(\pi_r^{RS}(q,p)) = E(\pi(q,p)) - E(\pi_r^{RS}(q,p)) = (1 - \mathbf{\Phi})\mathrm{E}(\pi(q,p))$**(10)**

The Downstream actor profit is an increasing function of $\mathbf{\Phi}$ while the upstream actor profit is decreasing in $\mathbf{\Phi}$. This revenue contract coordinates the Supply Chain and arbitrarily allocates its profit.

This Revenue Sharing contract should be desirable if the Chain partners obtain higher profits than in the Market like Settings:

$\frac{E(\pi_i^{RS}(q_{RS}^*, p_{RS}^*))}{E(\pi_i^{M}(q_{NC}^*, p_{NC}^*))} \geq 1\ where\ \mathrm{i}\ =$
$\{\mathrm{r{:}\ Retailer;\ m{:}\ manufacturer}\}$ **(10')**

4.4. Buy Back Contract :

Under this contract, the retailer salvages the leftover inventory and gets a credit of ***r*** for each unsold unit at the end of the season. It is assumed that the manufacturer is able to monitor the level of the leftover inventory and that the retailer should not profit from it $(w \geq r + s)$.

With a buyback contract the downstream actor profit is :

$E(\pi_r^B(q,p)) = (p - s - r)S(q,p) - \big(w_B\ - r + c_r\ - s\big)q$ (11)

A necessary condition with respect to price for the contract to coordinate the Supply Chain at the downstream level is that:

$\frac{\partial E(\pi_d^B(q,p))}{\partial p} = S(q,p) + (p - s - r)\frac{\partial S(q,p)}{\partial p} = 0$ (12)

A comparison of the first order condition -with respect to price- for the global Supply Chain

$\frac{\partial E(\pi(q,p))}{\partial p} = S(q,p) + (p - s)\frac{\partial S(q,p)}{\partial p} = 0$ (13)

Implies that eq.12 holds if r=0

By comparing the necessary first order condition -with respect to quantity- for the retailer and the global Supply Chain, $w_B = c_m$ should hold for the Buyback contract to coordinate the Supply Chain.

Eq.11 changes into:

$E\big(\pi_r^B(q,p)\big) = (p - s)S(q,p) - (c - s)q = E(\pi(q,p))$

Under these conditions, the retailer monopolizes the whole Supply Chain profit. This situation cannot be acceptable to the supplier.

As a consequence, the Buy Back contract doesn't coordinate the Supply Chain when the demand is price dependent.

4.4.1. Special case (Price is set first):

When the retailer sets the price before the order quantity is decided, Buy-Back Contracts coordinates the Supply Chain if the wholesale price w_B and the repurchase price r are defined in the following way:

$$\begin{cases} r = (1 - \alpha)(p - s) \\ w_b = r + \alpha(c - s) - (c_r - s) \end{cases} where\ \ 0 < \alpha < 1$$

By substituting b and w_b in (11), the retailer profit can be expressed as a fraction of the Supply Chain profit:

$E\big(\pi_r^B(q,p)\big) = \boldsymbol{\alpha}\mathrm{E}(\pi(q,p))$

The retailer's profit is increasing in **α** and the supplier's profit is decreasing in **α.** So the parameter **α** acts to allocate the Supply Chain's profit between the two firms.

The feasible values of **α** are determined with respect to the constraints

$$\begin{cases} \dfrac{E\left(\pi_i^B(q_B^*, p_B^*)\right)}{E\left(\pi_i^M(q_{NC}^*, p_{NC}^*)\right)} \geq 1 \; where \text{ i } = \begin{Bmatrix} \text{r: Retailer;} \\ \text{m: manufacturer} \end{Bmatrix} \\ w_b \geq r + s \; equivalent \; to \; \alpha > \dfrac{c_r}{c-s} \end{cases}$$

4.5. Quantity Discount Contract :

Many Quantity Discount scheme has been addressed in literature. Distinction can be made between incremental and all unit Quantity Discount. We consider all unit Quantity Discount in what follows. With Quantity Discount contract, the wholesale price is a decreasing function of the ordered quantity.

The retailer profit function has the form:

$E(\pi_r^{QD}(q,p)) = (p-s)S(q,p) - \left(w_{QD}(q) + c_r - s\right)q$ (14)

Let $p_{SC}^*(q)$ be the optimal stocking policy for a given q for the global Supply Chain.

As

$$\frac{\partial E(\pi_r^{QD}(q,p))}{\partial p} = S(q) + (p - v - b)\frac{\partial S(q,p)}{\partial p} = \frac{\partial E(\pi \quad (q,p))}{\partial p}$$

It follows that

$$\frac{\partial E(\pi_r^{QD}(q, p_{SC}^*(q))}{\partial p} = \frac{\partial E(\pi \quad (q, p_{SC}^*(q))}{\partial p} = 0$$

So the first optimality condition with regard to p is satisfied .

Let's replace p by $p_{SC}^*(q)$ in (14), the retailer profit is reduced to a function of the single variable q:

$$E(\pi_r^{QD}(q, p_{SC}^*(q))) = (p-s)S(q, p_{SC}^*(q)) - \left(w_{QD}(q) + c_r - s\right)q$$

By choosing $w_{QD}(q)$ such that :

$w_{QD}(q) = ((1-\alpha)((p_{SC}^*(q_{SC}^*) - s)\left(\frac{S\left(q, p_{SC}^*(q_{SC}^*)\right)}{q}\right) + \alpha(c-s) - (c_r - s)$ Where $\alpha \geq 0$ **(15)**

The retailer profit can be written as:

$$E\left(\pi_r^{QD}(q, p_{SC}^*(q))\right) = (p-s)S\left(q, p_{SC}^*(q)\right) - \alpha(c-s)q - (1-\alpha)(p_{SC}^*(q_{SC}^*) - s)S(q, p_{SC}^*(q_{SC}^*))$$

Let's evaluate the expected retailer profit at the global Supply Chain optimal price $p_{SC}^*(q_{SC}^*)$:

$$E\left(\pi_r^{QD}\left(q, p_{SC}^*(q_{SC}^*)\right)\right) = (p_{SC}^*(q_{SC}^*) - s)S\left(q, p_{SC}^*(q_{SC}^*)\right) - \alpha(c-s)q - (1-\alpha)(p_{SC}^*(q_{SC}^*) - s)S\left(q, p_{SC}^*(q_{SC}^*)\right)$$

$$E\left(\pi_r^{QD}\left(q, p_{SC}^*(q_{SC}^*)\right)\right) = \alpha(p_{SC}^*(q_{SC}^*) - s)S\left(q, p_{SC}^*(q_{SC}^*)\right) - \alpha(c-s)q$$

$$E\left(\pi_r^{QD}\left(q, p_{SC}^*(q_{SC}^*)\right)\right) = \alpha E\left(\pi\left(q, p_{SC}^*(q_{SC}^*)\right)\right)$$

Thus given that the optimal global Supply Chain price is chosen, the Quantity Discount contract coordinates the Supply Chain and allocate the Supply Chain profit between the retailer and manufacturer.

The positivity condition for the optimal wholesale price and the sharing factor, results in the following inequality constraints for the sharing factor α:

$0 < \alpha < 1 \; and \; (B - A)\alpha > (C - A)$ **(16)** where $A = (p_{SC}^*(q_{SC}^*) - s)\left(\frac{S\left(q_{SC}^*, p_{SC}^*(q_{SC}^*)\right)}{q_{SC}^*}\right)$, $B = (c - s)$ and $C = (c_r - s)$

The desirability issue of the Quantity Discount contract can be addressed through determining the feasible values of α that let the profit of both manufacturer and retailer higher than in the Non Coordinated case:

$\frac{E(\pi_i^{QD}(q_{QD}^*, p_{QD}^*))}{E(\pi_i^M(q_{NC}^*, p_{NC}^*))} \geq 1 \; where \; i =$ {r: Retailer; m: manufacturer} (17)

5. An application

The proposed model has been applied to a numerical example and various scenarios were simulated using Matlab Software.

	Parameter	Value
Salvage value	s	20
Retailer marginal cost	c_r	15
Manufacturer marginal cost	c_m	50
Wholesale price (Market like Setting)	w	65
Demand = *D* = *a-bp* + *ε*	a	2000
	b	20
ε ~ Normal (μ,σ)	μ	0
	σ	30

Eight scenarios have been considered to illustrate the results of coordination contracts simulation:

Contract	Scenario	Description	Decision Variables
Centralized Setting	**S1**	Centralized Control	
Market like Setting	**S2**	Non coordinated Supply Chain	
Revenue Sharing	**S3**	Minimum profit for retailer	Φ_{min} = 0,28, w = 3,21
	S4	Maximum profit for retailer	Φ_{max} = 0,498, w= 17,38
Quantity Discount	**S5**	Minimum profit for retailer	α_{min} = 0,28
	S6	Maximum profit for retailer	α_{max}= 0,498
Buy Back contract (Special Case $\boldsymbol{p = p^*_{SC}}$)	**S7**	Minimum profit for retailer	(r = 41,28,w=60,28)
	S8	Maximum profit for retailer	(r = 31,07,w=58,48)

The different scenarios have been designed such that the manufacturer and retailer profits are higher than in the Market like setting scenario, and constraints are fulfilled for each coordination contract type:

- Scenarios S3 resp. (S5,S7) corresponds to the situation where the retailer gets exactly the same profit as in the Market like setting, under Revenue Sharing contract resp. (Quantity Discount, Buy back contract)
- Scenarios S4 resp. (S6,S8) corresponds to the situation where the retailer gets the maximum profit under each of the coordination contracts : Revenue Sharing resp. (Quantity Discount, Buy back contract)

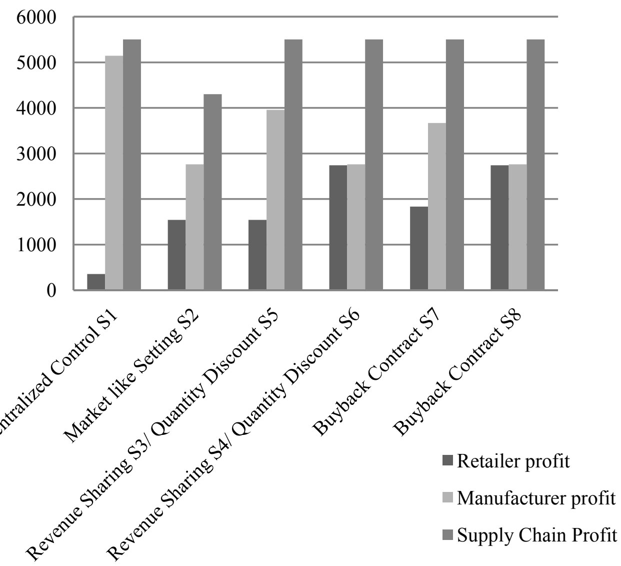

Fig. 1 Actor's profit for different coordination scenarios

<u>Market like Setting</u>

The order quantity and price are determined by differentiating eq.(6) ($p^*_{NC} = 89.11$; $Q^*_{NC=}184$).

When channel coordination is not pursued, Supply Chain efficiency $\frac{E(\pi^{SC})-E(\pi^{RS})}{E(\pi^{SC})}$ amounts to 12%.

However Retailer profit is higher than in the centralized setting (1544 vs 355).

Unless, some mechanism is implemented to achieve coordination and allocate the extra Supply Chain profit between the retailer and manufacturer, the retailer will find no interest in ordering the optimal order quantity $Q^*_{SC=}343$.

<u>Centralized Control</u>

By differentiating eq.(5), the optimal order quantity and price amounts to ($p^*_{SC} = 81.92$; $Q^*_{SC=}343$).

The Supply Chain profit is optimal in this case. This setting is more attractive to the manufacturer who captures the biggest part of the Supply Chain profit (94%) compared to the retailer (6%).

<u>Revenue Sharing</u>

The proposed coordination contract achieves coordination. The minimum and maximum values of the sharing ratio Φ are determined using the inequalities (10') and (8'). The Wholesale price w_{RS} can be deduced from (8'). The ranges of the sharing ratio and wholesale price are respectively [28%; 49.8%] and [3.21; 17.38]. The two ranges suggest a contract design that not only ensures channel coordination, but also an improvement of the profit of all actors.

Quantity Discount

For the chosen Quantity Discount contract scheme, the retailer's share of Supply Chain profit ranges from 28% to 49,8% using Inequalities (16) and (17). The stakeholders' negotiation capabilities can decide the allocation of the profit between the retailer and Manufacturer. The wholesale price is a function of the order quantity and is expressed through eq.(15) (cf. Graph for a fraction of 39%)

Buy Back Contract (Special Case)

The Buyback contract fails to coordinate Supply Chain when quantity ordered and retail price are set simultaneously. However if price is set first to the optimal Supply Chain price $p = p^*_{SC}$ before order quantity is defined, this contract coordinates Supply Chain and allocate extra profit between retailer and manufacturer. The expected profits gained by SC actors for different choices of the contract parameters are compared to those obtained under the market-like setting. For this application, the retailer (resp Manufacturer) Supply Chain profit share falls in the range [33%;49.8%] resp [50,2%;67%].

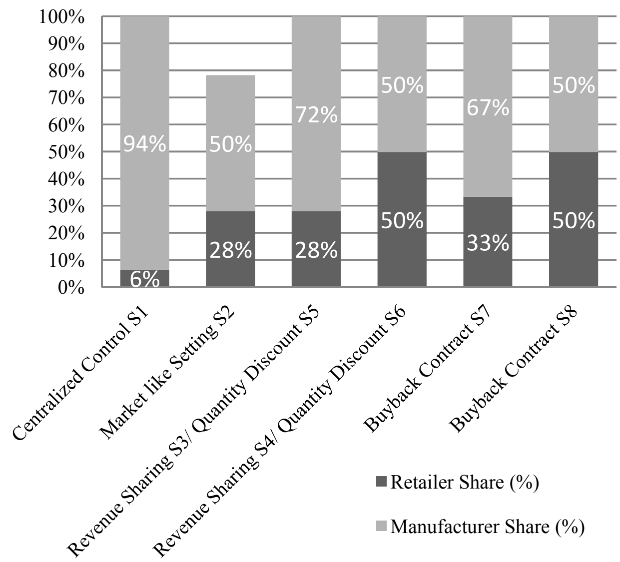

Fig .2 Actors ' share of optimal Supply Chain profit

6. Conclusions and Discussion

This paper has addressed the problem of coordination under the hypothesis of decentralized control and price dependent demand for a two level Supply Chain. Our comparative analysis of 3 coordination contracts shows that a suitable design of the Revenue Sharing and Quantity Discount contracts can lead the Supply Chain actors to select the price and order quantity that are optimal for the global Supply Chain. It also demonstrates that a correct choice of the contract parameters improves the individual profit of all the stakeholders.

Buy Back contract shows its limits in coordinating the Supply Chain when demand is price dependent. However it can still achieve coordination and divide extra profit between Supply Chain actors if the later agree to choose the optimal price as a retail price.

Through an application, it has been shown that a proper design of the coordination contracts could be a convincing incentive for the Supply Chain actors to adopt them.

This research work has shed the light on the potential benefits of coordination contracts. The model studied can be developed by making further assumptions regarding some parameters such as Supply Chain structure, contract periodicity, information structure and external factors affecting the supply chain (ex: Effort dependent demand,…).

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