The application of methods of nonlinear dynamics for ECG in Normal Sinus Rhythm

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Abstract
The ECG signals were processed in three steps: First by reconstructing their phase portraits, second by calculating their spectra and finally by computing their Largest Lyapunov exponents. This study presents a framework to assess nonlinear parameters of ECG signals that may be useful for further exploration of physiological and path-physiological significance of dynamics in electrocardiograms. This paper devotes to identify the nonlinear properties of human heart beat dynamics for 18 characteristic examples of normal subjects with normal sinus rhythm. A new phase space characterization of a class of ECG signals with normal sinus rhythm through is presented. Such approach led to find out a common visual feature namely an anchor-shaped Poincar´e plot resulting from an appropriate choice of the delay time. Any value of the delay time is acceptable, but the shape of the embedded time series depends critically on the choice of its value. The results reported in this study may be useful not only for the classification of ECG states, but can serve as a benchmark to which pathological cases can be compared.

Keywords: ECG, Normal sinus rhythm, nonlinear dynamics, Largest Lyapunov exponent

1. Introduction

In recent literature, approaches to understanding and intervening in the cardiovascular system are being developed using the new methods from nonlinear system theory. The normal development of heart rate controlling system was associated with characteristic changes in linear and nonlinear measures. Non stationarity in the Electrocardiogram (ECG) signals manifest both in RR interval (the time elapsing between two consecutive R waves in the ECG) timing changes and in morphological changes Although the former changes are often thought of as rhythm disturbances and the latter as beat abnormalities. Many scientists turn to the analysis of chaotic dynamics of various phenomena including functioning of alive organisms. It is often difficult to decide whether the dynamics of a biological system is chaotic or not, the question whether the human cardiac system is chaotic or not has been an open one. In chaotic behavior, the initial close orbits of the strange attractor will diverge exponentially with time, with extreme sensitivity to initial conditions. Much evidence shows that the human heart is not a perfect oscillator but it is chaotic system, so ECG signals have been considered as chaotic signals and this has been successfully proved by computing Lyapunov exponents [1][2]. In former studies, strictly periodic cardiac rhythm is not accompanied by healthy condition but, on the contrary, by pathological states. Besides, investigation of continuous ECG with methods of nonlinear system theory is originated from the hypothesis that cardiac rhythms are associated with chaotic dynamics [3][4]. A characteristic feature of a dynamical system exhibiting deterministic chaos is that it is possible to predict the behavior of the system with some degree of confidence in the short-term even though long-term prediction is impossible. Profound inspection of abnormalities in the form of ECG signals allows the medical doctors to identify a larger number of distinct diseases. Some methods of recognition of possible diseases rely on a statistical analysis of a large number of heart beats [5]. The complexity of cardiac dynamics decreases significantly in the morning and this may contribute to the ominously increased rate of cardiac death in the morning hours [6]. The main metrics used to characterize the nonlinear behavior of heart beat signal (Heart Rate Variability and continuous ECG signals) are correlation dimension, Lyapunov Exponent, Kolmogorov entropy, fractal parameters [7]. Largest Lyapunov exponent can be a diagnostic criterion allowing to distinguish between different groups of patients [8]. For instance, The Largest Lyapunov Exponent (LLE) of coronary artery disease patients with sinus rhythms is less than that of healthy people [9]. Use of Lyapunov exponents in studying biological systems has been increasingly frequent. Its have been employed particularly in an application of Adaptive neuro-fuzzy inference system for classification of ECG signals [10], to characterize the states of cardiovascular system in [11] and, more generally, to analyze ECG time series in [12]. As shown in [9], the investigation of the Lyapunov exponents of synchronous 12-lead ECG signals leads to conclude that the Lyapunov exponents of human heart rate are not the same for different lead ECG signals for all the tests for both Coronary Artery Disease patients and healthy people. But more importantly, the Lyapunov exponents have the same sign for all the synchronous eads. It is worth noting that the LLE is a nonlinear measure that reflects the ’overall’ properties of the instantaneous heart rhythm[13].A typical chaotic system has at least one positive Lyapunov exponent. Numerous methods for alculating the Lyapunov exponents have been developed during the past decade, among the approaches for calculation of the Lyapunov exponents may be mentioned particularly the Jacobian and the direct method [14]. In general, phenomena observed in finite time intervals are called transient phenomena. Transient chaos is one of transient phenomena that can be found in a class of systems whose asymptotic behavior is regular. Lyapunov exponents is among the most useful tools applied for measuring the sensitivity to initial conditions in the case of

asymptotic chaos. Finite-time Lyapunov exponents were used for characterizing sensitivity to initial conditions in the case of transient chaos[15]. The embedding dimension plays an essential role for the identification and the prediction of nonlinear dynamical systems including chaos. Analysis of heart rate variability focused on the RR-interval fluctuations data. Measurements of heart rate variability by conventional and new nonlinear methods is a useful research tool for documenting changes in neural regulation in relation to arrhythmia events in various clinical settings [16]. In [17] it was reported that heart rate variability can be used as a reliable indicator of the heart state. It becomes less random with aging, it has also been shown that the parameter values fall into different ranges for different health conditions. A heart rate variability analyzer was designed in [18] to assess nonlinear metrics like, LLE, fractal dimension, correlation dimension, approximate entropy and slope of Poincar´e plots. Many attempts to find a relationship between some nonlinear indices and the biological aspects of heart rate regulation have been reported in[19]. Phase space plots facilitate the visualization of beat-to-beat dynamics in the heart rate. The measures of nonlinear dynamics through Poincar´e maps, symbolic dynamics and renormalized entropy have been investigated in [20]. In this study, an approach based on phase space plots is used to determine a new characterization of a certain class of 18 human heart beat recordings of subjects who have had no significant arrhythmias. Such approach led to find out a common visual feature in phase space of the so called recordings. Simultaneously with the phase space plots, Spectral analysis of data strings of the ECG signals was performed in order to provide additional details related to such class of recordings. Investigation of chaotic nature of the ECG recordings was then done by computing their corresponding LLE.

2. Materials and Methods

2.1. ECG Description

An ECG in normal sinus rhythm, P-waves are followed after a brief pause by a QRS complex, then a T-wave. The time intervals between heartbeats usually vary in a complex and irregular manner. However, even without arrythmia, the variability of sinus rhythm in a healthy individual is very complex. Normal sinus rhythm not only indicates that the rhythm is normally generated by the sinus node and traveling in a normal manner in the heart, but also that the heart rate is within normal limits. In a normal heart rhythm, the sinus node generates an electrical impulse which travels through the right and left atrial muscles producing electrical changes which is represented on the electrocardiogram (ECG) by the P-wave. The ventricular contraction is represented electrically on the ECG by the QRS complex of waves. cardiac cycle after a short pause repeats itself, and so on. There is no one normal heart rate, but this varies by age. It is normal for a newborn to have a heart rate up to 150 beats per minute, while a child of five years of age may have a heart rate of 100 beats per minute. The adult's heart rate is even slower at about 60-80 beats per minute. The database of MIT-BIH normal sinus rhythm, includes 18 long-term ECG recordings (sampling frequency is 128 Hz) of subjects referred to the Physiobank databases[21]. Subjects included in this database were found to have had no significant arrhythmias; they include 4 men, aged 26 to 45, and 14 women, aged 20 to 50. From such recordings we calculate an estimate value of the heart beat rate expressed in beats per minute (BPM) see table 1.

Table 1: MIT-BIH Normal Sinus Rhythm Database

Record	Gender	Age	BPM
nsrdb16265	M	32	98
nsrdb16272	F	20	62
nsrdb16273	F	28	85
nsrdb16420	F	38	93
nsrdb16483	M	42	88
nsrdb16539	F	35	78
nsrdb16773	M	26	74
nsrdb16786	F	32	72
nsrdb16795	F	20	61
nsrdb17052	F	45	67
nsrdb17453	F	32	81
nsrdb18177	F	26	92
nsrdb18184	F	34	82
nsrdb19088	F	41	83
nsrdb19090	M	45	84
nsrdb19093	F	34	70
nsrdb19140	F	38	87
nsrdb19830	F	50	102

2.2. Poincar´e plot analysis

The Poincar´e plot is a tool developed by Henri Poincar´e for analyzing complex systems. Such technique taken from nonlinear dynamics, also named return map, is the simplest technique to describe the nonlinear dynamics of a phenomenon.

Table 2: Spectral parameters of ECG signals; *a*0: Fourier constant term; *fM*: the predominant frequency; *AM*: The amplitude of the predominant higher harmonic; *LLE*: Largest Lyapunov Exponent

Record	a_0	$f_M(Hz)$	A_M	LLE
nsrdb16265	14.020	6.406	11.840	all < 0
nsrdb16272	9.147	1.063	5.595	all < 0
nsrdb16273	9.870	4.531	14.910	all < 0
nsrdb16420	10.430	7.906	1.802	some $LLE > 0$
nsrdb16483	8.436	1.500	17.090	all < 0
nsrdb16539	6.971	4.125	1.838	all < 0
nsrdb16773	5.779	1.188	21.450	all < 0
nsrdb16786	9.989	3.594	15.653	all < 0
nsrdb16795	7.099	2.281	2.289	some $LLE > 0$
nsrdb17052	6.975	1.094	1.410	all < 0
nsrdb17453	10.570	1.344	3.427	some $LLE > 0$
nsrdb18177	9.427	3.313	2.153	all < 0
nsrdb18184	7.392	0.250	5.344	all < 0
nsrdb19088	5.526	1.500	0.852	all < 0
nsrdb19090	4.917	2.969	1.869	all < 0
nsrdb19093	23.440	1.156	50.360	some $LLE > 0$
nsrdb19140	3.008	4.625	6.420	some $LLE > 0$
nsrdb19830	24.330	0.250	1.164	all < 0

The Poincar´e plot had been used to portray the heart rate fluctuations because it provides summary information as well as detailed beat-to-beat information on the behavior of the heart [18]. Poincar´e plot was used as a quantitative visual technique whereby the shape of the phase plane plot was categorized into functional classes that indicate the degree of the heart failure in a subject[17]. Since the phase space representation is a convenient method for graphical characterization of ECG signals, it is used in this study as a visual technique to recognize the hidden correlation patterns of a time series signal. In fact, a three dimensional phase space will be described by plots in which the temporal samples of ECG are plotted as a function of the previous ones. A phase plane plot is obtained by plotting the value at time (t+τ) vs. the value at time t where τ is an arbitrary delay which should be sufficient to reconstruct the attractor, efficiency with a limited amount of data is enhanced by particular choices of the time delay τ. In the following, we use three dimensional representations to distinguish between different ECG signals. A phase space plot is obtained by plotting the values of the continuous
ECG with three axes (e(t), e(t+2τ), e(t+τ)).

2.3. Frequency domain measures

The spectral analysis of heart rate variability had been applied to populations with cardiovascular diseases. It is well known that a typical heart beat has the extremes P, Q, R, S and T, the beat-beat interval is the time between two consecutive R-peaks, this can be considered as the 'period' of heart oscillation. Spectral analysis of ECG signals aims to put into evidence features that may lead to categorize certain particular physiological signals through their spectral composition. An ECG frequency spectrum can be thick or sparse, so the role of higher harmonics can be of great interest to explain and interpret particular healthy or diseased situations. The spectral composition of an ECG signal depends on its clinically relevant parameters such as time interval between waves, duration of each wave or composite waveforms, peak amplitudes. Aiming to analyze Electrocardiogram (ECG) signals given as discrete time signals, one can investigate their frequency content and other particular parameters such as the Fourier constant term, the frequency and the magnitude of the predominant harmonic. Thus, in the following, spectral analysis is performed by computing the FFT of a data string of 512 sample points lasting 4ms.

2.4. Largest Lyapunov exponent

Spectral and correlation analysis have been widely used for
the processing of time series far before the concept of dynamical chaos has been settled[8]. Lyapunov exponents are a quantitative measure of the sensitive dependence of nonlinear system variables on the initial conditions. Its define the average rate of divergence of initially nearby trajectories in phase plan e for distinguishing among the various types of orbits based on their sensitive dependence on the initial conditions, and are used to determine the stability of any steady-state behavior, including chaotic solutions. Starting from two neighbor points $X0$ and $\Delta X0$ which will generate two different orbits in the phase space, the divergence between such orbits is measured by the Lyapunov exponents. In chaotic states the separation between the orbits, which is a function of time will behave erratically. In the following The Largest Lyapunov exponents will be estimated from the observed time series in order to investigate the 'chaocity' of the studied class of ECG signals. If the Largest Lyapunov exponent of an ECG signal is positive, then it is said to have chaotic characteristics, this may mean that the electrical activity of the human heart is not periodic but chaotic. The Largest Lyapunov exponent equals zero for strict periodic motion and infinite for pure noise.

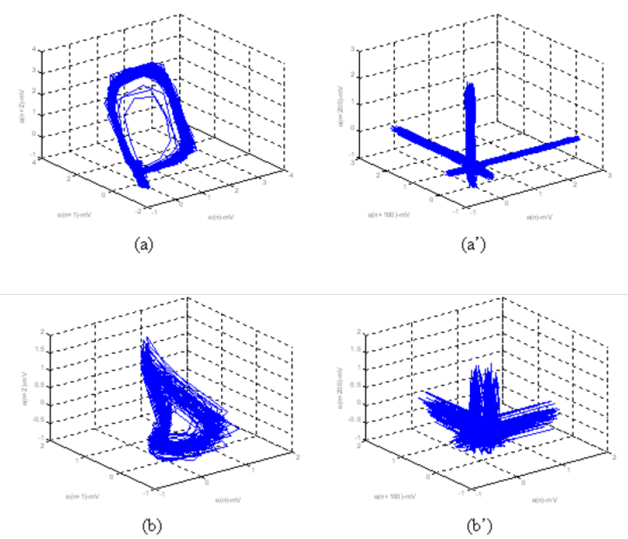
2.5. Embedding dimension and delay time

Aiming to perform the phase space reconstruction, it is worth noting that the selection of a suitable pair of embedding dimension and time delay is based on two

different points of view. One is that such parameters are not correlated with each other and the second viewpoint is that these parameters are closely related, for most researchers the latter is more practical and reasonable than the first one in the engineering practice[23]. To perform an adequate embedding, the first parameter to be estimated is the embedding dimension d. Practically, the method used to determine the embedding dimension consists in guessing suitable value for d, by successively embedding in higher dimensions and looking for consistency of results. Selecting an embedding dimension devotes to make sufficiently many observations of the system state so that the deterministic state of the system can be determined unambiguously. In former studies, autoregressive modeling
was used to extract features in order to classify certain cardiac arrhythmias. A nonlinear autoregressive model is described by:

$$X_t = F_d(X_{t-1}, X_{t-2}, ..., X_{t-d})$$

The embedding dimension is defined as follows[22]: A time series X_t is said to have the embedding dimension d_0 with the delay time τ_0 if and only if there exist non-negative integers $d_0 < \infty$, and any $\tau_0 < \infty$ such that: $F_d(X_{t-\tau}, X_{t-2\tau}, ..., X_{t-d\tau})$, $F_{d0}(X_{t-\tau0}, X_{t-2\tau0}, ..., X_{t-d0\tau0})$ *a.e* for any $d < d_0$, and any $\tau > 0$, and $F_d(X_{t-\tau}, X_{t-2\tau}, ..., X_{t-d\tau}) = F_{d0}(X_{t-\tau0}, X_{t-2\tau0}, ..., X_{t-d0\tau0})$ *a.e*, for any $(d, \tau) \in B(d_0, \tau_0)$ where $B(d_0, \tau_0) = \{(d, \tau) \mid \{\tau_0, 2\tau_0, ..., d_0\tau_0\} \subset \{\tau, 2\tau, ..., d\tau\}\}$.



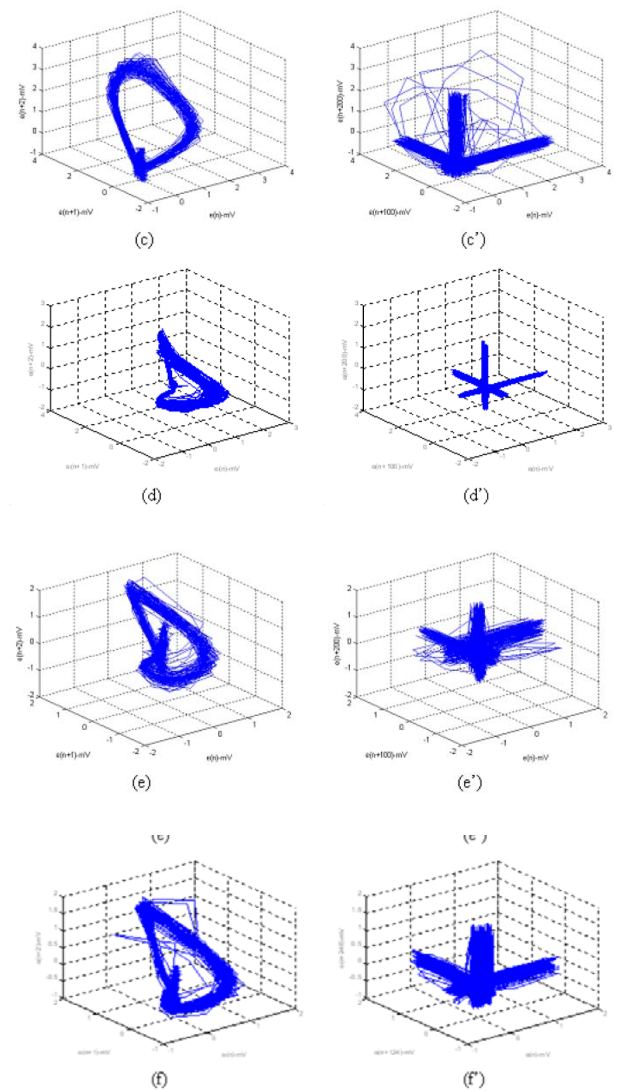
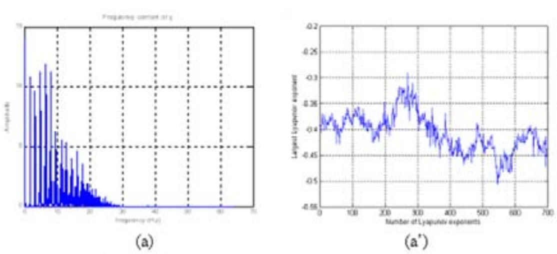


Figure 1: phase plots for the ECG signals: attractor and reconstruction of the attractor. The left panel is a single 7680 point trajectory plotted in e(t), e(t+1) and e(t+2) coordinates (embedding lag of 1), the right hand panel is a delay embedding in three dimension (embedding lag of 100 in figures a',b',c',d' and e' and 124 in figure f') of the e(t) coordinate



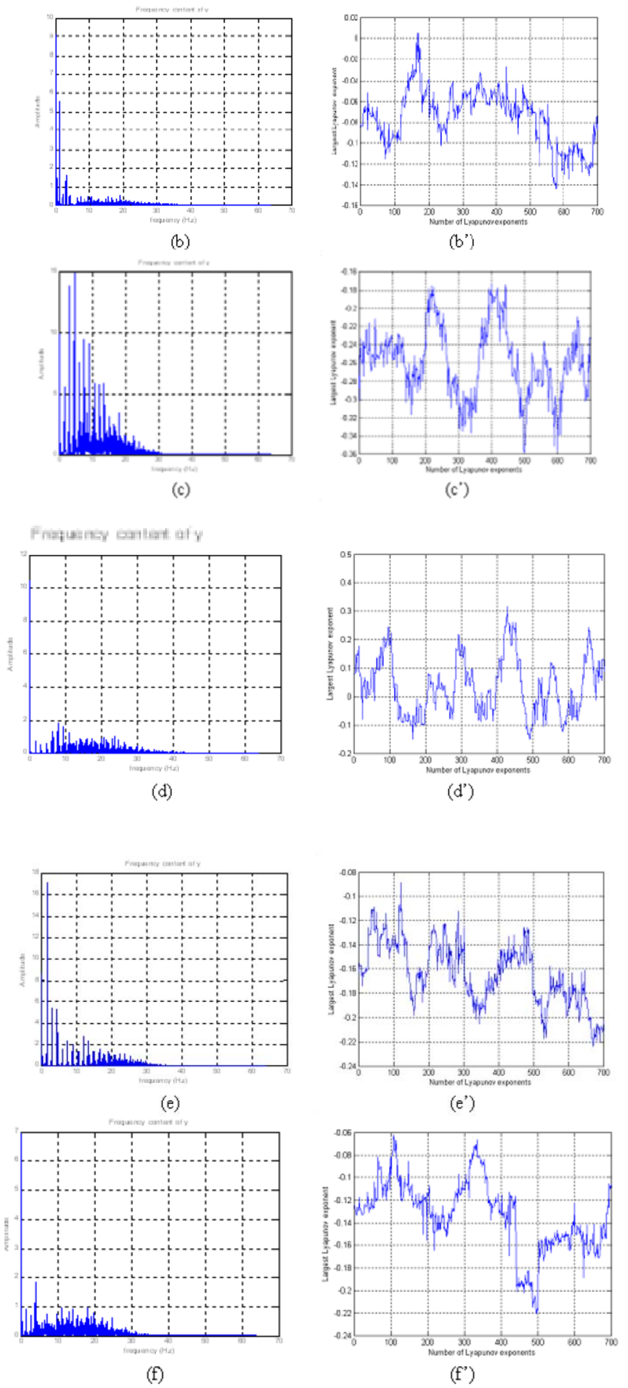
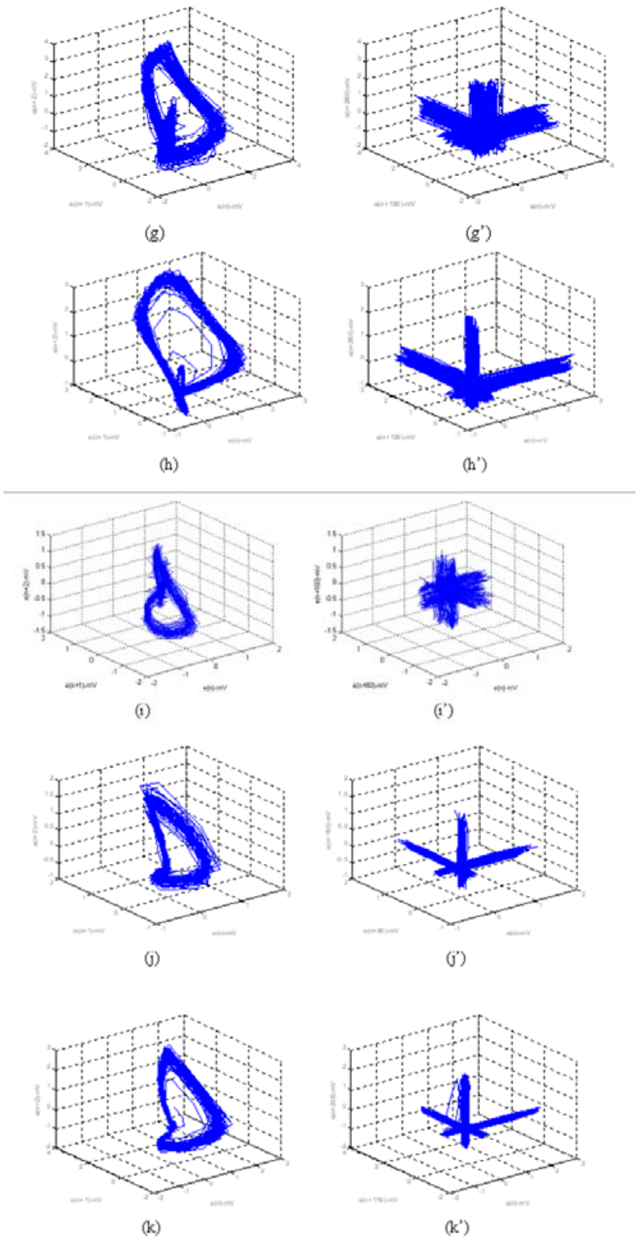


Figure 2: The left panel is the power spectrum distribution (FFT of a data string of 512 sample points lasting 4ms), the right hand panel is the Lyapunov exponents of the data string of the ECG (sliding window of 610 samples)

3. Results and Evaluation

Despite that the signals belong to the same class of ECG namely, the normal sinus rhythm, their phase space plots are quite different. The three dimensional phase space display the attractors of the heart beat dynamics. of the ECG signals was reconstructed using the delay time embedding method, where the delay can be chosen one or more samples. Figures 1,2 and3 show the reconstructed phase space graphs of 18 data sets using one minute string data of ECG signals. A three dimensional phase space graph was constructed using e(t), e(t+τ) and e(t+2τ). The choice of τ is important to find out a visual characterization of human ECG classes. For τ = 1, several different shapes of phase portraits were determined see Figures 1-(a-f),2-(g-l),3-(m-r). An appropriate choice of τ leads to put into evidence a common visual shape of the three dimensional representation.



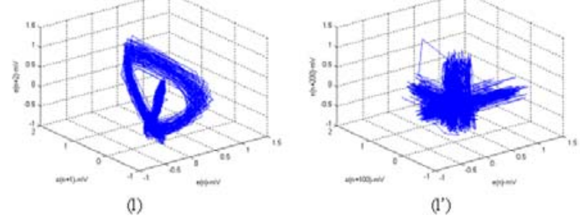
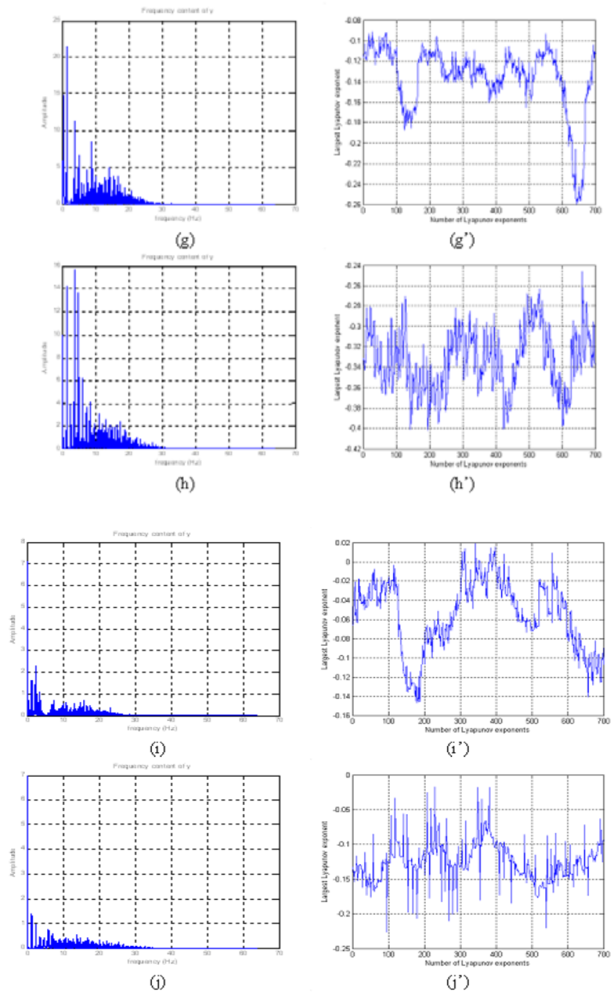


Figure 3: phase plots for the ECG signals: attractor and reconstruction of the attractor. The left panel is a single 7680 point trajectory plotted in e(t), e(t+1) and e(t+2) coordinates (embedding lag of 1), the right hand panel is delay embedding in three dimension (embedding lag of 100 in figures g',h' and l', 80 in figures i' and j', and 119 in figure k') of the e(t) coordinate.



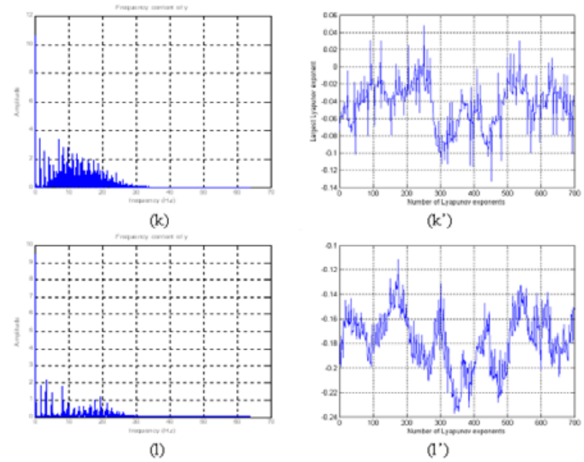
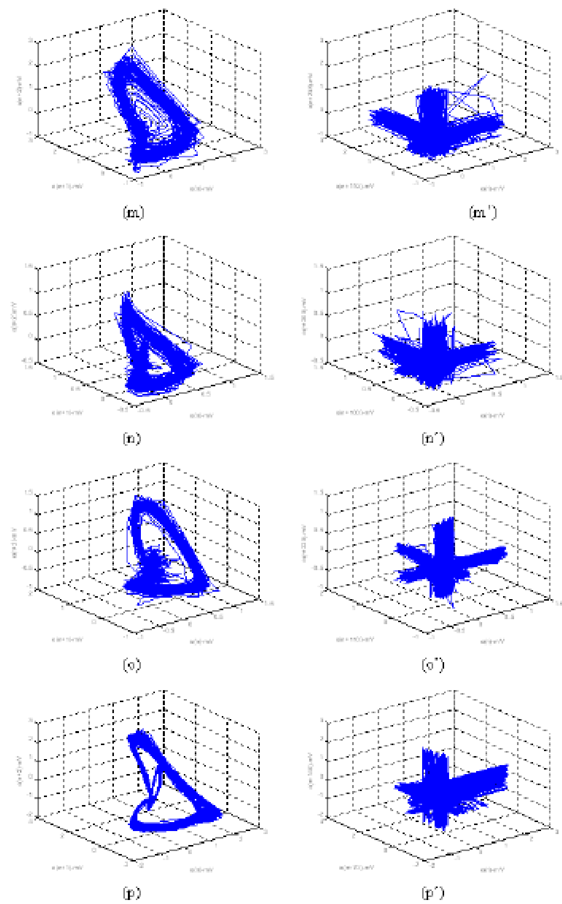


Figure 4: The left panel is the power spectrum distribution (FFT of a data string of 512 sample points lasting 4ms)), the right hand panel is the Lyapunov exponents of the data string of the ECG (sliding window of 610 samples).



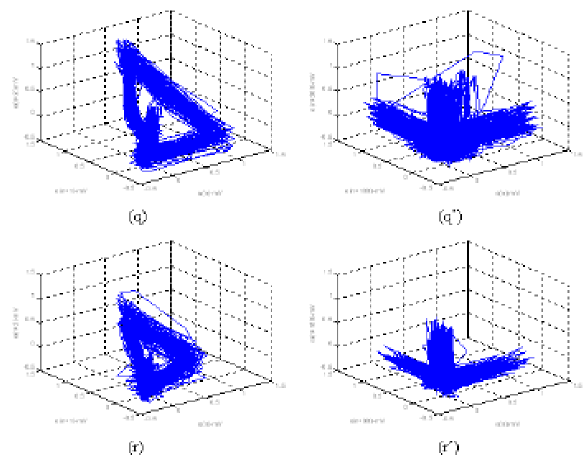
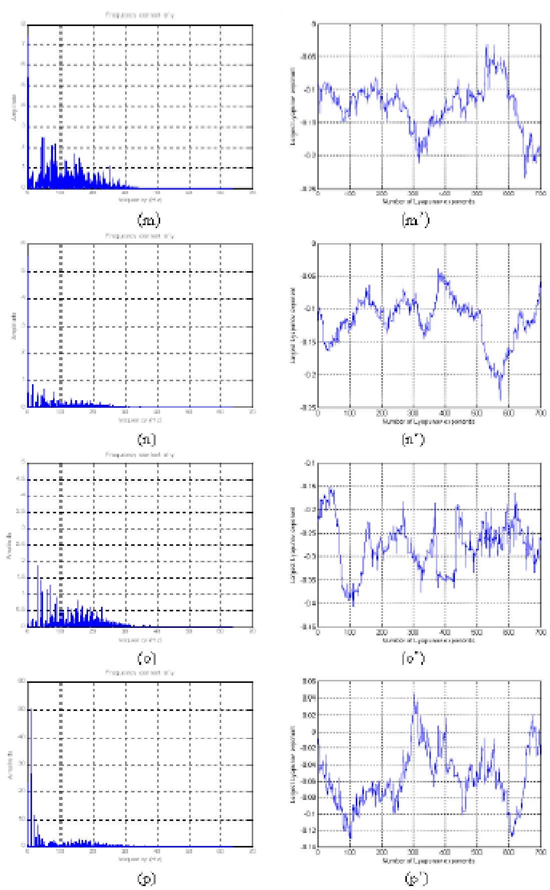


Figure 5: phase plots for the ECG signals: attractor and reconstruction of the attractor. The left panel is a single 7680 point trajectory plotted in e(t), e(t+1) and e(t+2) coordinates (embedding lag of 1), the right hand panel is a delay embedding in three dimension (embedding lag of 100 in figures n' and q', 70 in figures p', 90 in figure r', 110 in figure o' and 119 in figure m') of the e(t) coordinate.



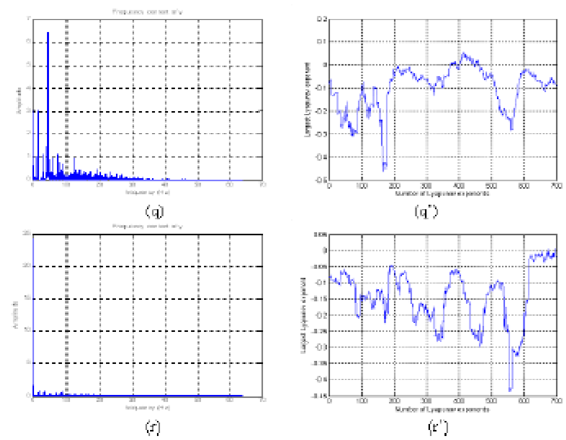


Figure 6: The left panel is the power spectrum distribution (FFT of a data string of 512 sample points lasting 4ms), the right hand panel is the Lyapunov exponents of the data string of the ECG (sliding window of 610 samples)

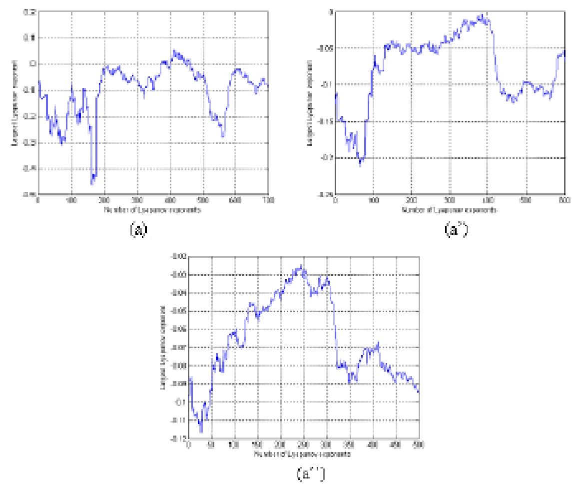


Figure 7: The Lyapunov exponents of the ECG beats (record. nsrd19140)(a) sliding window of 610 samples, a') sliding window of 1610 samples, (a") sliding window of 2610 samples

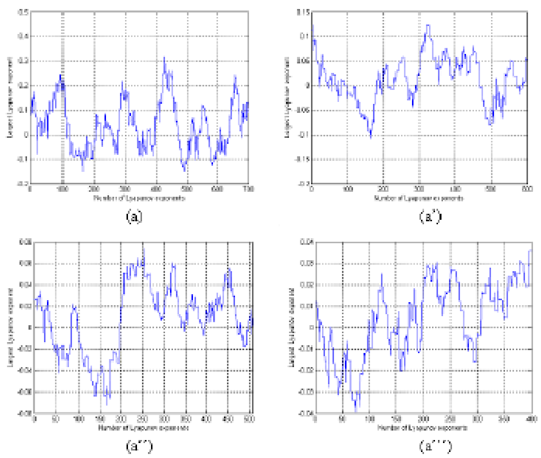


Figure 8: The Lyapunov exponents of the ECG beats(record. nsrd16420) (a) sliding window of 610 samples, (a') sliding window of 1610 samples, (a") sliding window of 2610 samples,(a"') sliding window of 3610 samples.

Such visual phase plots resemble to three invariant cross axes parallel to the phase space axes see Figures 1-(a'-f'),2-(g'-l'),3-(m'-r'). Such particular geometric shape of phase space may related to a certain class of healthy people ECG dynamics, nevertheless , we should not neglect the fact that ECG signals may be noise corrupted.

We develop the spectral point of view, through the parameters defined in table2, namely the Fourier constant term $a0$, the predominant frequency fM and the corresponding amplitude AM. Each 1-minute ECG segment includes 7680 samples, the FFT is applied to the 4096 initial samples (32*s*). The Largest Lyapunov exponents were computed by using a sliding window of 610 discrete data (contains nearly 8 heart beats). For each string of data(610 samples), 700 Lyapunov exponents of the ECG signal samples are computed and plotted in Figures 4-(a'-f'),5-(g'-l'),6-(m'-r'). For most cases treated in this study, the Largest Lyapunov Exponent are negatives, meaning that the ECG signals with normal sinus rhythm are generally not chaotic. Nevertheless, few cases exhibit positive Largest Lyapunov exponents thereby confirming the chaotic nature of the associated data strings see Figures 2-d',4-(i',k'),6-(p',q').
Regarding the power spectra distribution of the considered ECG signals, all of the power is contained in frequencies below 30 Hz with peak power occurring in the range of 0.25 to 8 Hz. The ratio of $a0$ to AM is in the range of 0.27 to 21.(see table 2)
Plots from Figure 7.(a) show that some narrow width intervals of 610 samples having positive LLEs may possess a chaotic behavior, it is seemingly a 'local chaocity'. whereas intervals of larger widths such as 1610 and 2610 samples (see Figure 7.(a') Figure 8: The Lyapunov exponents of the ECG beats(record. nsrd16420) (a) sliding window of 610 samples, (a') sliding window of 1610 samples, (a") sliding window of 2610 samples,(a"') sliding window of 3610 samples and (a")) give rise to negative LLEs only. Certainly a signal having a larger LLE is supposed to be 'more chaotic' than a signal having a lower one. Thereby, the chaotic behavior put into evidence in few data strings is vanishing by enlarging the interval widths, because initially the LLEs have positive low values. whereas when the LLEs are initially large, increasing the width of the intervals for calculating these values only weaken the latter which remain positives see Figure 8 (a.610 samples, a'.1610 samples,a".2610 samples, and a"'.3610 samples).

4. Discussion

It was the aim of this study to find out a common feature that may be a specific characteristic of a certain category of ECG signals namely that corresponding to human heart beats with normal sinus rhythm. In a typical QRS of normal duration,virtually all of the power is contained in frequencies below 30 Hz with peak power occurring in the range of 4 to 12 Hz, in our study a peak power occurred at 0.25Hz.
Results show, in certain cases in Figures 2-d',4-(i',k'),6-(p',q'), that the LLEs were above zero, and this validated that the non linear dynamics really exist in cardiovascular system.
When the LLE decreases the complexity of the attractors decrease and the heart electrical activity becomes more regular. For the majority of the signals examined in this study, the Lyapunov exponents are negatives, except for few cases which possess certain intervals with positive Lyapunov exponents. The results of this study demonstrate that every subject in the studied group had an anchor-shaped Poincar´e plot resulting from an appropriate choice of the delay time. The inter-beat time can be considered as a 'period' of the ECG signal although the latter cannot be perfectly periodic. In this study, it has been obvious to note that the delay time is close to the value of the so-called ECG 'period'. Thus, an open question in future researches, is to find out a relationship between the delay time and the heart beat 'period'. The chaotic behavior may characterize ECGs of healthy people as well as those of diseased ones. Furthermore, certain long-term ECG recordings may include local distortions, disturbances or noise of any origin. The problem that arises is how wide the range to choose to calculate the Largest Lyapunov Exponent? In fact, it must be pointed out that Low distortions occurring on the ECG, not necessarily of pathological origin, can give rise to chaotic behavior in a narrow range (positive LLE).The extension of the width of

such interval leads to having a negative LLE meaning that the overall behavior is not chaotic. The Fourier constant term may be too high compared to the amplitude of the predominant harmonics; in figure 6.(r), the ratio of $a0$ to AM equals 20.94, the frequency of the predominant harmonic is 0.25 Hz. The estimation of the heart beat rate expressed in beats per minute (102 BPM) of such subject is the highest among all the others (see table 1).

5. Summary and Conclusion

Applying nonlinear signal processing techniques to signals like ECG provides very useful information for detection of cardiac abnormalities. The proposed technique have been shown to be effective for the characterization of a certain class of ECG signals namely those of normal sinus rhythm. In this study we are concerned with the evolution of the dynamics in phase space and the related properties in temporal and frequential domains. The main findings highlighted are essentially the following: - characterization of a class of ECG signals with Normal Sinus Rhythm through anchor-shaped Poincar´e plot resulting from an appropriate choice of the delay time. - chaotic behavior is not a specific property of the ECG in healthy people. – spectral analysis can provide additional information to characterize the dynamics of the ECG. given the small number of cases studied, these results need to be confirmed by larger and prospective clinical investigation. The obtained results are compatible with a non linear complex behavior in heart rate variability of human being. The different structure of the return map of the chosen group could express some information that we are not able to define otherwise.

The methods described in this study may be useful not only for the characterization of the particular class of ECGs studied, but can serve as a benchmark to which pathological cases can be compared. Future researches should be devoted to investigate more profoundly eventual relationships between nonlinear indices and the biological aspects of heart beat dynamics.

Table .3: List of figures

Record	Figures
nsrdb16265	Figure1-(a),(a'),Figure2-(a),(a')
nsrdb16272	Figure1-(b),(b'),Figure2-(b),(b')
nsrdb16273	Figure1-(c),(c'),Figure2-(c),(c')
nsrdb16420	Figure1-(d),(d'),Figure2-(d),(d')
nsrdb16483	Figure1-(e),(e'),Figure2-(e),(e')
nsrdb16539	Figure1-(f),(f'),Figure2-(f),(f')
nsrdb16773	Figure3-(g),(g'),Figure4-(g),(g')
nsrdb16786	Figure3-(h),(h'),Figure4-(h),(h')
nsrdb16795	Figure3-(i),(i'),Figure4-(i),(i')
nsrdb17052	Figure3-(j),(j'),Figure4-(j),(j')
nsrdb17453	Figure3-(k),(k'),Figure4-(k),(k')
nsrdb18177	Figure3-(l),(l'),Figure4-(l),(l')
nsrdb18184	Figure5-(m),(m'),Figure6-(m),(m')
nsrdb19088	Figure5-(n),(n'),Figure6-(n),(n')
nsrdb19090	Figure5-(o),(o'),Figure6-(o),(o')
nsrdb19093	Figure5-(p),(p'),Figure6-(p),(p')
nsrdb19140	Figure5-(q),(q'),Figure6-(q),(q')
nsrdb19830	Figure5-(r),(r'),Figure6-(r),(r')

References

[1] Ubeyli E.D., 'Detecting variabilities of ECG signals by Lyapunov exponents', Neural Computing and Applications 18, pp 653-662.

[2] Babloyantz A., and Destexhe A., 'Is the normal heart a periodic oscillator' Biological Cybernetics, 1988, 58:203.

[3] Fell J., Mann K., Roschke J., and Gopinathan S., 'Nonlinear analysis of continuous ECG during sleep I. Reconstruction', Biological Cybernetics

[4] Fell J., Mann K., Roschke J., and Gopinathan S., 'Nonlinear analysis of continuous ECG during sleep II. Reconstruction', Biological Cybernetics 82, pp. 485-491, 2000.

[5] Bartsch R., Hennig T., Heinen A., Heinrichs S., and Maass P., 'Statistical analysis of fluctuations in the ECG morphology', Physica A 354, pp 415- 431, 2005.

[6] Yum M.K.,'Non-linear cardiac dynamics and morning dip: an unsound circadian rhythm', Clin Physiol. 1999 Jan;19(1):56-67

[7] Xinbao N., Chunhua B., Jun W., and Ying C., 'Research progress in nonlinear analysis of heart electric activities', Chinese Science Bulletin, Volume 51, N 4 pp 385-393, 2006.

[8] Pavlov A.N., Janson N.B., Anishchenko V.S., Gridnev V.I., and Dovgalevsky P.Y. 'Diagnostic of cardio-vascular disease with help of largest Lyapunov exponent of RR-sequences', Chaos, Solitons and Fractals, volume 11, Issue 5, pp. 807-814, April 2000.

[9] Zhenzhou W., Zheng L., Yixiang W., Xinbao N., and Yuzheng L., 'Lyapunov exponents for synchronous

12-lead ECG signals', Chinese Science Bulletin Vol. 47 N 21 November 2002.
[10] Ubeyli E.D., 'Adaptive neuro-fuzzy inference system for classification of ECG signals using Lyapunov exponents', Computer Methods and Programs in Biomedicine 93, pp 313-321.
[11] Xia L., Tianliang K., Jinghua L., and Xin T., 'Largest Lyapunov index of EHRV for analyzing status of cardiovascular system', IFMBE Proceedings 19, pp. 361-362, 2008.
[12] Wang N. and Jiong R., 'Principal components cluster analysis of ECG time series based on Lyapunov exponents spectrum', Chinese Science Bulletin, Volume 49, N 18 pp 1980-1985, 2004.
[13] Mrowka R., Patzak A., Schubert E., and Persson P. B., 'Linear and nonlinear properties of heart rate in postnatal maturation', Cardiovascular Research 31, pp. 447-454, 1996.
[14] Govindan R.B., Narayanan K., and Gopinathan M.S., 'On the evidence of deterministic chaos in ECG: Surrogate and predictability analysis', Chaos, volume 8, N2, june 1998.
[15] Klonowski W., Olejarczyk E., and Stepien R.,'Chaocity and Dimensional Complexity of EEG-Signal, International Symposium on Nonlinear Theory and its Applications, Special Session 'Nonlinear Dynamics in Brain and Nervous System', NOLTA'2001, Miyagi, Japan, October 28 - November 1, 2001.
[16] Lombardi F., Makikallio T.H., Myerburg R.J., and Huikuri H.V., 'Sudden cardiac death: role of heart rate variability to identify patients at risk', Cardiovascular Research 50, pp. 210-217, 2001.
[17] Acharya R.U., Kannathal N., Sing O.W., Ping L.Y., and Chua T., 'Heart rate analysis in normal subjects of various age groups', Biomedical Engineering Online 2004, 3:24.
[18] Mohan A., James F., and Fazil S., 'Design and Development of a Heart Rate Variability Analyzer', Journal of Medical Systems, Springer Science and Business Media, LLC 2010.
[19] Guzzetti S, Signorini MG., Cogliati C., Mezzetti S., Porta A., Cerutti S., and Malliani A., 'Non-linear dynamics and chaotic indices in heart rate variability of normal subjects and heart-transplanted patients. Cardiovasc Res 1996;31:441446.
[20] Voss A., Kurths J., Kleiner H.J., Witt A., Wessel N., Saparin P., Osterziel K.J., Schurath R., and Dietz R., 'The application of methods of nonlinear dynamics for the improved and predictive recognition of patients threatened by sudden cardiac death', Cardiovascular Research 31, pp. 419-433, 1996.
[21] Goldberger A. L. et al.,'PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals', Circulation,2000 June 13(23), Vol. 101,pp.215-220.
[22] Kaoru F., and Takashi Y., ' Estimating the embedding dimension and the delay time from chaotic time series with dynamic noise', Journal of the Japan Statistical Society, Vol. 31, No. 1, pp 27-38, 2001.
[23] Hong-guang M., and Chong-zhao H., 'Selection of Embedding Dimension and Delay Time in Phase Space Reconstruction', Front. Electr. Electron. Eng. China (2006) 1: pp 111-114.

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