Segmenting and supervising an ECG signal by combining the CWT & PCA

Hanen Chaouch[1], Khaled Ouni[2] and Lotfi Nabli[3]

Automatic and Signal processing and Image Laboratory, National school of engineering, Monastir, Tunisia

Abstract

In this paper we go about the segmentation and analysis of an electrocardiographic (ECG) signal. Firstly, it consists in working out the locations of different characteristic waves of this signal: the QRS complex and the waves P and T, successively and in the order of magnitude and their amplitude. Secondly, we go about the analysis of this signal by the linear principal component analysis (LPCA) based on the results found in the first part. The importance of this algorithm comes in the context of the ECG supervision and then the cardiovascular system. This algorithm integrates the multi-scale wavelet analysis and principal component analysis. This analysis allows us, first, to apply the continuous wavelet transform (CWT) on the totality of the signal. After that, based on the property of the CWT regularity, the waves will be detected after a thresholding operation. To evaluate the segmentation algorithm two parameters are introduced, the sensitivity Se and the predictive value P+. The results have given an average of Se=99.93% and P+=99.96%, which indicates that our segmentation algorithm is sufficiently reliable in comparison to the real database. In the supervision of the ECG, we use the detected parameters to construct the data-matrix of the LPCA. The defects are detected and located by this tool in order to determine the existing failure in the used signal. Comparing our results with those of the expert, we find that the LPCA gives a concrete state of the cardiovascular system.

Keywords: *Multi-scale analysis, continuous wavelet transform, ECG, segmentation, detection, location, principal component analysis.*

1. Introduction

The analysis of the electrocardiographic signal has been the target of many works since a dozen of years. Some researchers have been interested in the segmentation of the ECG, which consists in defining the different locations of the characteristic waves of this signal, the QRS complex and the waves P and T [1, 2, 3 and 4]. These works have introduced different tools of signal processing, mainly the wavelet analysis. Others have made use of the diagnosis of the cardiovascular system starting from the ECG wave characteristics [5, 6 and 7]. Our proposed approach comes in the context of segmenting and analyzing the waves of the electrocardiographic signal. In the segmentation part, we have introduced the multi-scale continuous wavelet transform [8, 9 and 10]. The principal of this method is based on the result of the wavelet coefficients in different levels of resolution. A thresholding operation is then applied in order to detect the different ECG waves. These waves are detected in the order of magnitude and their amplitude. In a second part, detecting and locating the defective parameters on an ECG is an asset to help diagnose the cardiovascular system. The principal component analysis (PCA) is applied at this level. The PCA is a method of reducing the classic linear dimension which consists in projecting the samples on the maximum variance axes of data. This method is frequently used in detecting and locating defects and afterward in supervising the industrial and biological processes [11, 12, 13 and 14]. The PCA rests on two parts. The first step is the detection of defects which uses the PCA to model the behavior of the process in a normal state. Comparing the observed behaviour with that given by the PCA, the defects are then detected. Many methods have been used for the detection of defects, and the most frequent ones are the square prediction error SPE and the Hotteling statistics T^2 [15, 16, 17 and 18], which will be also introduced in this approach. To locate the variables in defect, many methods are introduced [19, 20 and 21]. In this paper, the defects are located by the method of calculating contributions.

2. Regularity analysis by wavelet transform

The wavelet transform can be used as a tool for analyzing and measuring the uniform and local regularity of a signal. To characterize the singular structures, we should quantify precisely the local regularity of the signal $x(t)$. The Lipschitz exponents provide measurements of a uniform consistency not only on intervals but also in any v point. The Lipschitz exponents are also called the Hôlder exponents. The localized Lipschitz exponents can arbitrarily vary from one abscissa to the other. The uniform

Lipschitz exponents provide a more global measurement of the consistency, which is laid over a whole interval.

2.1 Multi-scale differential operator

To measure the local regularity of a signal, the number of nil moments has a great importance.

$$\int_{-\infty}^{+\infty} t^k \psi(t)\,dt = 0 \quad pour \quad 0 \le k < n \qquad (1)$$

It has been shown that a wavelet at n nil moments can be written as the derivative in the n order of a θ function. The corresponding wavelet transform is a multi-scale differential operator. When the wavelet is at a rapid decrease, we get a connection between the uniform regularity of the function and the decrease in its wavelet transform. A Ψ wavelet at a rapid decrease has n nil moments only if a θ function exists at a rapid decrease as:

$$\psi(t) = (-1)^n \frac{d^n\theta}{dt^n}(t) \qquad (2)$$

The wavelet transform is expressed by:

$$W_x(u,s) = s^n \frac{d^n}{du}\left(x * \bar{\theta}_s\right)(u) = s^n \frac{d^n}{du}\left(x * \frac{1}{\sqrt{s}}\theta\left(\frac{-t}{s}\right)\right) \qquad (3)$$

This theorem shows that a wavelet at n nil moments can be written as the derivative in the n order of a θ function [9 and 10]. Ψ can not have more n nil moments when $\int_{-\infty}^{+\infty}\theta(t)\,dt = K \neq 0$

This shows that a connection between the uniform regularity of $x(t)$ and the decrease in its fine-scale wavelet transform $|W_x(u,s)|$. There exists a relationship between the uniform consistency of an *f* signal and the decrease in the coefficients of its fine-scale wavelet transform.

2.2 Detection of singularities by wavelet transform

The detection of singularities of a signal is a basic operation because these points often correspond to the important events of the signal. These moments can be determined by the wavelet transform thanks to local maxima (Maxima of the wavelet transform). The decrease in $|W_x(u,s)|$ can in fact be controlled by values of its local maxima. The term 'maximum module' is used to describe the points such as $|W_x(u,s)|$ being locally maximum. This implies that:

$$\frac{\partial W x(u,s)}{\partial u} = 0 \qquad (4)$$

The singularities are located on the abscissa where the maxima modules of the fine-scale wavelet coefficients converge. To understand the properties of these maxima, we write the wavelet transform as a multi-scale differential operator. If a wavelet has exactly n nil moments and a compact support, θ is also at a compact support as $\psi = (-1)^n\, \theta^n$ with $\int_{-\infty}^{+\infty}\theta(t)\,dt \neq 0$; the wavelet transform is written as follows:

$$W\,x(u,s) = s^n \frac{d^n}{du^n}(x * \bar{\theta}_s)(u)\,. \qquad (5)$$

for n=1, $W\,x(u,s) = s\frac{d}{du}(x*\bar{\theta}_s)(u)$

and for n=2, $W\,x(u,s) = s^2\frac{d^2}{du^2}(x*\bar{\theta}_s)(u)$

If the wavelet has only one nil moment, the maxima modules of the wavelets are the maxima of the first derivative of $x(t)$ smoothed by $\bar{\theta}_s$. If the wavelet has two nil moments, the maxima module correspond to the maxima of the second derivative. If the wavelet has no local maximum of the fine scales, then $x(t)$ is locally regular. The Lipschitzian regularity is calculated starting from the decrease in the amplitude of the maximum modules determined at the scale level. We can measure this regularity by calculating the parameter:

$$\alpha_k = \log_2\left|W\,x(u_{k+1}, s_{k+1})\right| - \log_2\left|W\,x(u_k, s_k)\right|$$

In the dyadic case and at the level of the resolution j, this parameter becomes:

$$\alpha_j = \log_2\left|W\,x(n^{j+1}, 2^{j+1})\right| - \log_2\left|W\,x(n^j, 2^j)\right|$$

2.3 Coefficient products

The coefficients of the wavelet decomposition include the lines of the maxima modules in proximity to the signal singularities. This allows spotting the singularities of a signal by detecting its maxima. For a multi-scale analysis, the coefficient product of the wavelet transform of an $x(n)$ signal of some dyadic scales is given in the following equation:

$$p(n) = \prod_{j=j_0}^{j=j_1} w_{2^j}\, x(n). \qquad (6)$$

The resulting signal $p(n)$ shows peaks in the present transitions of the signal and presents weak values elsewhere [10]. In this non linear operation on $x(n)$, the producing singularities through the scales of peaks whose coefficients are of the wavelet transform are reinforced by the product $p(n)$, while those caused by fluctuations will be detected. The peaks of the signal will line up through , and not for, all the scales because on increasing the smoothing effect, the response will spread and then the singularities will interfere. Thus, choosing a quite high scale value makes the alignment of the peaks in the product $p(n)$ lost. The number of the $p(n)$ terms allows preserving the singularity sign. After determining the coefficients for three successive resolution levels, we calculate their product in order to amplify data and attenuate the weak amplitudes.

2.3 Thresholding

The operation of thresholding is applied on the given product of the wavelet coefficients. The threshold value is determined in function of the maximum module of the coefficients product by:

$$S = a * \max|P| \quad (7)$$

with a as a coefficient that belong to the interval [0, 1]. After testing many threshold values by changing the value of a, we notice that for $a = 0.2$ the obtained results are the best. We must cancel the signal part (product of the wavelet coefficients) which is less than the applied threshold and take into account the part that exceeds it; this is the principal of adapted thresholding in this algorithm.

3. Detection and location of defects (LPCA)

Consider a process whose normal functioning is presented by a matrix of data X with n measurements and m variables. Determining the number of the principal components ℓ is based on the benchmark proposed by Qin.S[21] which integrates the principal of minimizing the variance of reconstruction error. The reconstruction error variance of the i[th] component of $x(k) = [x_1 ......x_m]^T \in \Re^m$ is given by: $\rho_i = \text{var}\left\{\zeta_i^T (x(k) - x(k)^i)\ \right\}$.

where ζ_i is the i[th] column of the identity matrix and $x(k)$ represents the measurement vectors whose i[th] component has been reconstructed in the following way:

$$x(k)_i = \frac{\left\lfloor C_{-i}^T 0 C_{+i}^T \right\rfloor}{1 - C_{ii}} x(k) \quad (8)$$

where $C = \rho_\ell \rho_\ell^T = [C_1 C_2 ......C_m]$, C_i is the i[th] column of the C matrix and the signs (+i) and (-i) represent the vector made up of the first $(i-1)$ and the last $(m-i)$ elements of the vector C_i. The minimisation criterion used for determining the number of principal components is given by:

$$J(\ell) = \sum_{i=1}^{m} \frac{\rho_i}{\text{var}\left\{\zeta_i^T x_k\ \right\}} = \sum_{i=1}^{m} \frac{\rho_i}{\zeta_i^T \sum \zeta_i} \quad (9)$$

3.1 Detection

In this approach two statistics are introduced, those of the Hotteling T² and the SPE, and are determined as follows:

$$T^2(k) = \sum_{i=1}^{\ell} \frac{t_i^2(k)}{\lambda_i} \quad (10)$$

$$SPE(k) = \sum_{j=1}^{m} (e_j(k))^2 \quad (11)$$

with $e_j(k)$ being the j[th] residue given by:

$$e_j(k) = x_j(k) - \hat{x}_j(k)$$

where $x_j(k)$ is the j[th] element of the measuring vector, $x(k)$ is the i[th] principal component and t_i is the i[th] proper value of the correlation matrix $\sum$, which represents the variance of t_i. m is the number of quality indictors and ℓ is the number of components, with $\ell < m$ which is the estimation of x by the PCA model given by:

$$\hat{x} = Cx$$

where ρ_ℓ is the matrix formed by the ℓ first proper vectors of the matrix $\sum$. The process will be considered functioning abnormally if one of these following inequalities at least is true:

$$SPE \prec \delta_\alpha^2$$

$$T^2 \prec \chi_\alpha^2(\ell)$$

where δ_α^2 and $\chi_\alpha^2(\ell)$ are respectively the thresholds of T^2 and SPE.

3.2 Location

This step follows the detection step to highlight the variables in defect. The method used at this level is based on calculating the contribution of variables. This method introduces the principal of quantifying the contribution of each variable at the detection statistics. Particularly, we will take an interest in calculating the contribution of principal component variables; each principal component is expressed by:

$$t_i = \rho_i^T x = \sum_{j=1}^{m} \rho_{ij} x_j \quad (12)$$

with ρ_i as the proper vector corresponding to the value λ_i. Qin.S.[21] propose a simultaneous use of the principal components and contribution of the initial variables. They propose, at the time of detecting a defect, to analyse the standardised principal components having an important variation. The total contribution of the variable x_j on the q highest components is given by:

$$Cont_j = \sum_{i=1}^{q} Cont_{ij} \quad (13)$$

with

$$Cont_{ij} = \frac{t_i}{\lambda_j} \rho_{ij} x_j \quad (14)$$

ρ_{ij} is j^{th} component of the proper vector ρ_i.

4. Application on an ECG signal

4.1 ECG characteristic waves and intervals

The usual waves in an ECG are P, QRS and T waves.P waves represent depolarization of the atria. QRS represents the depolarization on the ventricles. T wave is due to the ventricular repolarisation. The repolarisation wave of the atria is not usually visible. If present, it is known as the Ta wave. A segment in an electrocardiogram is the region between two waves. PR segment begins at the end of the P wave and ends at the onset of the QRS complex. ST segments starts from the end of the QRS and terminates at the onset of the T wave. TP is segments between the end of the T wave and the beginning of the next P wave. It is the true isoelectric interval in the electrocardiogram. An interval in an ECG includes one segments and one or more waves. PR interval starts at the beginning of the P wave and ends at the onset of the QRS. It denotes the conduction of the impulse from the upper part of the atrium to the ventricle. In this paper, we introduce the MIT/BIH database to apply our approach.

4.2 Algorithm

In this work we put forward a method for the segmentation of different characteristic waves of the electrocardiographic signal based on the multi-scale wavelet analysis. This segmentation makes it possible to prepare a data matrix for the application of the PCA at the level of supervising the ECG described as indicated in the following diagram:

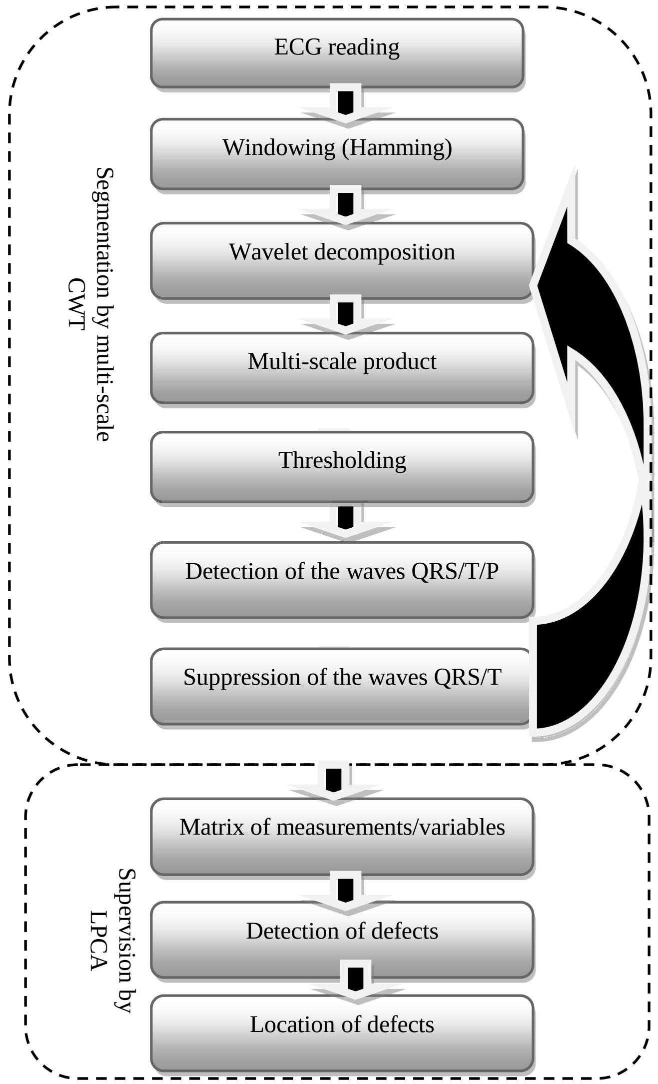

Fig. 1 Proposed algorithm.

4.3 Segmentation

4 .3.1 Detection of the ECG waves

The proposed method for detecting the ECG waves is based on the principle of the multi-scale analysis of the CWT. Our detection algorithm consists of five basic steps which are: the windowing, the wavelet decomposition, the multi-scale product, the thresholding and the detection (fig1).

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- Windowing: This operation enables segmenting the ECG signal into a window set due to its length which is hard to carry out and which takes a significant time for execution. Moreover, the used Hamming window makes it easy to highlight the high amplitude, which will help to better detect the ECG waves afterwards.

- The wavelet decomposition at different resolution levels is then applied on the signal. The used parent wavelet is the first derivative of a Gaussian. The principal is based on the zero crossing of the maxima in the first derivative (Gaussian)(fig.2).

- Once the wavelet coefficients are determined, the proposed algorithm effects its product in order to highlight the high amplitudes and to reduce the weak amplitudes. This product allows us to facilitate the detection thereafter by eliminating noise.

- A thresholding operation is afterward effected on the wavelet coefficient product. The used threshold is calculated in function of the maximum module of the continuous wavelet transform (as indicated in 2.4).

- After detecting the R wave, the two Q and S waves are determined by the onset and the offset of the maximum module of the R wave. To be able to decet the T wave, it is essential to delete the high amplitude QRS complex. The P wave is detected by eliminating the T wave and the QRS complex (fig.4).

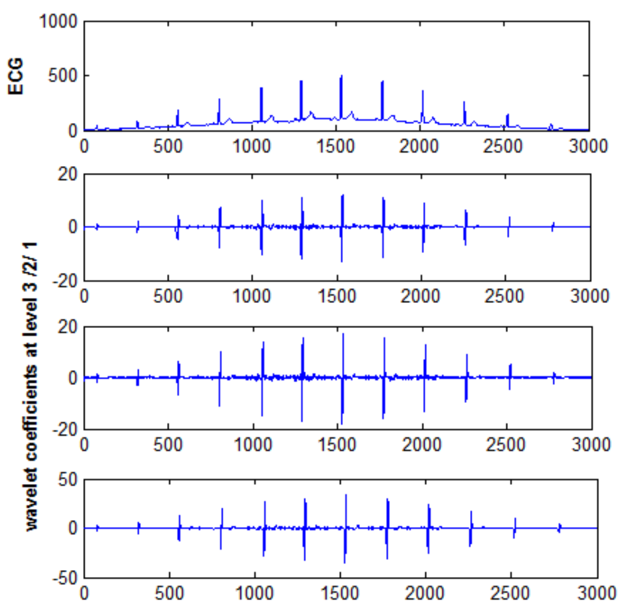

Fig. 2 Wavelet decomposition on three scales

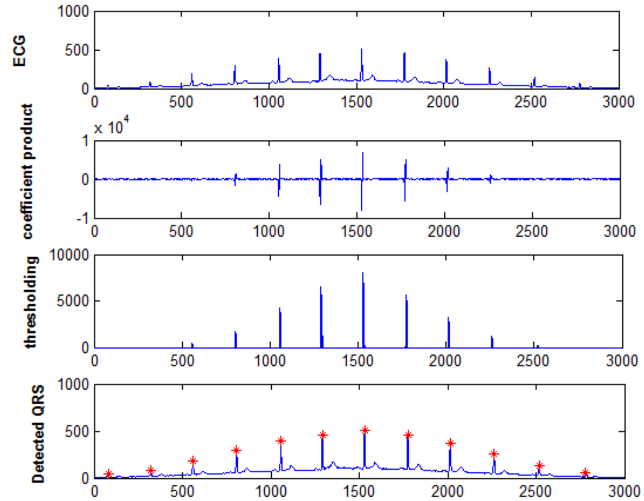

Fig. 3 Detection of the QRS complex

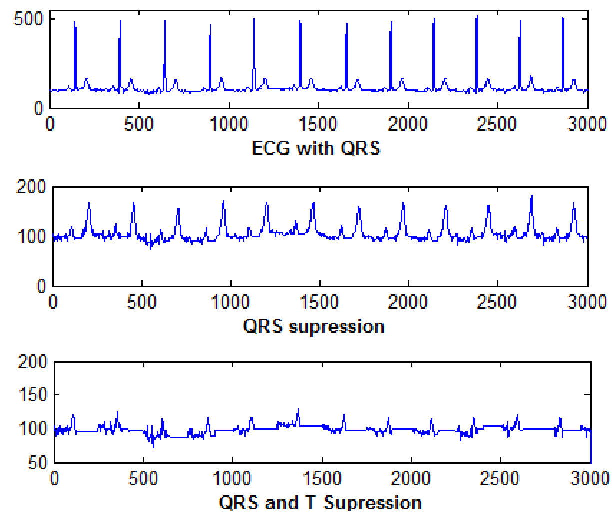

Fig. 4 Wave suppression

4.3.2 Assessment

To assess the obtained results of the detection method, two parameters are used: the sensitivity Se and the positive predictivity P+ which are determined as follows:

$$Se = TP / (FN + TP) \qquad (15)$$

$$P+ = TP / (FP + TP) \qquad (16)$$

where TP is the number of beats correctly detected, FN represents the number of wrong detections and FP is the number of undetected beats. The following table presents the detailed results for each wave:

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Wave	*Number of beats*	*TP*	*FP*	*FN*	*Se%*	*P+%*
QRS	1865	1865	0	2	99.89	100
T	1863	1862	1	1	99.95	99.95
P	1887	1887	1	1	99.95	99.95

Table 1: Detection results of the ECG waves

4.4 ECG signal analysis by LPCA

4.4.1 Data matrix

Once the ECG waves are detected (previous part), the matrix of data or measurements is determined. This matrix is composed of 500 measurements of the following variables: the amplitudes of the waves P, Q, R, S and T, and the intervals PQ, QS, ST and RR which are calculated starting from the locations of the detected waves. The choice of these variables is due to the bringing-in of information which is about the state of the ECG and the cardiovascular system afterwards. The variables introduced in this algorithm are presented as indicated on figure 5:

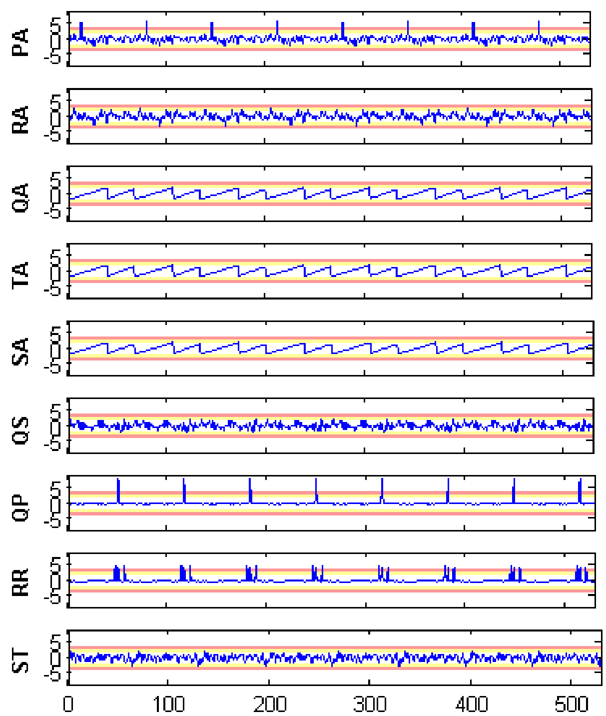

Fig. 5 Progress of ECG parameters

It is vital to centre and reduce these variables before applying the PCA for detecting and locating the defects (fig.6).

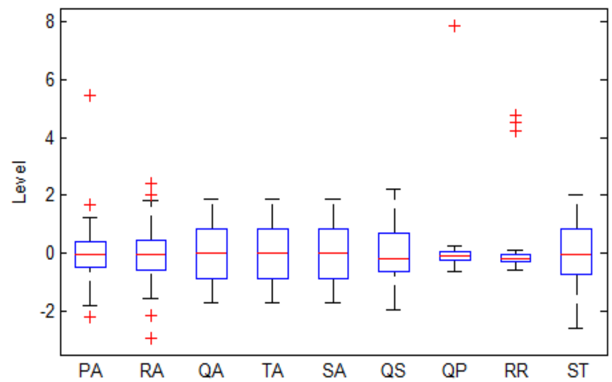

Fig. 6 Variables of reduced and centred data

4.4.2 Defect detection

Figure 7 and 8 describe the detection of the defects by the PCA by introducing respectively the statistics T² an SPE. According to these figures, we notice that there is a big contradiction at the level results. In fact, the T² Hotteling method detects many defects on the totality of the signal while the SPE statistic does not show any defects. Comparing these results at the real state of data (normal ECG) shows us that the SPE method is the most reliable method that is why we are going to use only this method for the location to get good results.

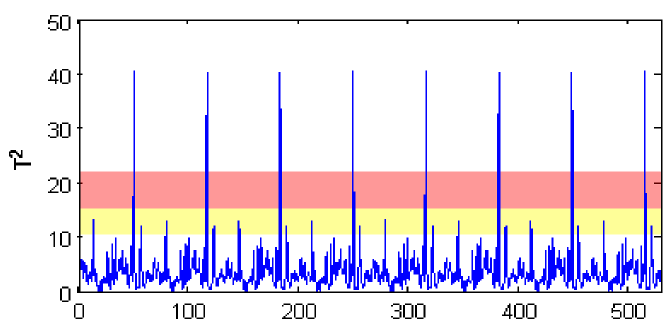

Fig. 7 Detecting defects by T²

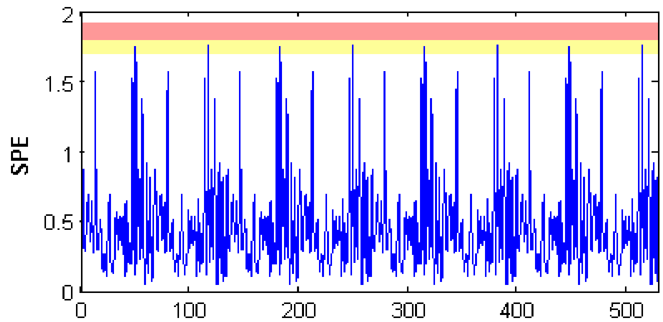

Fig. 8 Detecting defects by SPE

After that, we generate defects on one of the variables by increasing a dozen of measurements using the same amplitude. The SPE statistic allows us to detect the excited defect as shown in the following figure.

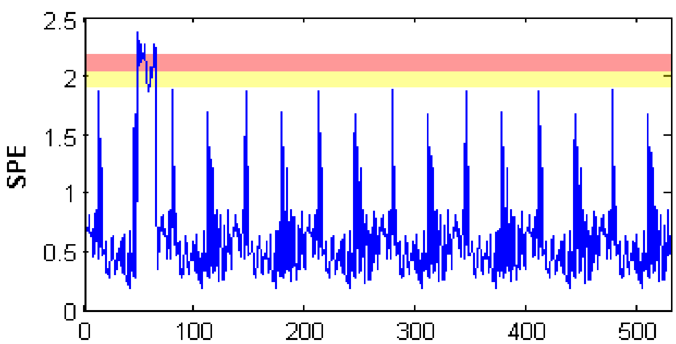

Fig. 9 Detecting a defect generated on a variable by SPE

4.4.3 Defect location

This step of the PCA enables us to locate the defect detected previously by calculating the contribution. This defect is generated in view of the fact that there exists no defect on the original signal according to the SPE statistic. Figure 10 shows that the defect is at the level of the second variable, which is real because the defect generation is on the same variable. Otherwise, according to figure 10, we notice that the contributions has given a real location of defect; but the contributions of other variables are too close to the contribution of the variable in defect, which proves that this location method is limited.

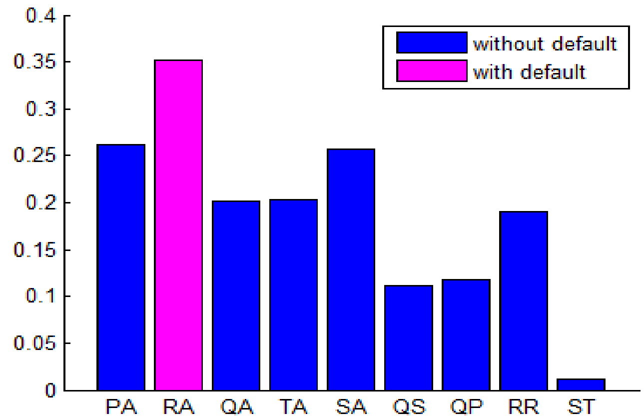

Fig. 10 Locating the defect generated by SPE

5. Conclusion

In this paper, an analysis of the ECG signal is proposed. This analysis consists of two principal steps: the segmentation and the supervision of the ECG signal. In the segmentation algorithm, we have used the multi-scale analysis of the continuous wavelet transform. This analysis is set up at three resolution levels. To detect the weak amplitude waves P and T, we need to delete the QRS complex and then the P wave. This method is evaluated by two parameters, Se and P+, which are in our case in the order of 99.9%. In fact, the segmentation of the ECG allows us to prepare the data matrix for applying the PCA to supervise this signal. Knowing that the used ECG is that of a normal subject, the defect detection by the PCA has shown that the SPE statistic is the most reliable at this level. To assess our method in a better way, we have stimulated a defect at the level of the RA variable, which is well-located by calculating the SPE contribution. In other words, this method is due to the fact that the variables are very close and that other methods are considered to better locate the defect as the reconstruction principle.

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First Author Chaouch Hanen received his enigineering diploma of industrual instrumentation and maintenance from INSAT-Tunis in 2007. In 2009, she obtained her mastery in measurment and instrumentation from the same university. She is currently Assistant in the Superior Institute of Applied Maths and Informatics in Kairouan (ISMAIK).Hier research interests include Modelling, Control, Monitoring and Command biologic Systems.

Second Author Ouni khaled received his License of Automatically Industrial Data Processing from ESST-Tunis. In 2007, he obtained his Mastery in Safety and Security of the Industrial Systems from ISSAT- Sousse.He is currently Assistant in the Superior Institute of Applied Sciences and Technology in Kairouan (ISSATK). His research interests include Modeling, Control, Monitoring and Command Manufactory Systems.

Third Author Nabli Lotfi received his Mastery of Sciences and DEA from ENSET- Tunis - Tunisia in 1989 and 1991 respectively. In 2000, he obtained his doctorate degree in Industrial Automation: Automatic and Industrial Computing from the University of Sciences and Technologies of Lille France. He is currently Assistant professor of Electrical Engineering at the National School of Engineers of Monastir (ENIM) - Tunisia and a Master of conference candidate. His research interests include Modeling, Control, Monitoring and Command Manufactory Systems.