

# (i,j)-Quasi Semi Weakly $g^*$ - Closed Functions in Bitopological Spaces

1 C.Mukundhan †

Department of Mathematics

Assistant Professor, Faculty of Science, L.N.V. College of Arts and Science,

Podanur, Coimbatore – 641 201, Tamil Nadu, India.

2 N.Nagaveni ‡

Associate Professor, Department of Mathematics

Coimbatore Institute of Technology,

Coimbatore. Tamil Nadu, India.

## Abstract

The primary purpose of this paper is to introduce and study two new types of functions on bitopological spaces called (i,j)- quasi semi weakly  $g^*$ -open and (i,j)- quasi semi weakly  $g^*$ -closed.

Mathematical subject classification: Primary 54 A10, 54 C10; Secondary 54 C08, 54 D 15.

**Key Words :** Bitopological spaces, (i,j)- semi weakly  $g^*$ -open , (i,j)- quasi semi weakly  $g^*$  -closed, Pairwise open, Pairwise closed, Pairwise –semi weakly  $g^*$  -closed, Pairwise semi weakly  $g^{**}$ -closed, Pairwise continuous , Pairwise semi weakly  $g^*$  -continuous , Pairwise semi weakly  $g^*$ - irresolute, Pairwise normal, Pairwise semi weakly  $g^*$ -normal, Pairwise semi weakly  $g^{**}$ - normal.

## 1. Introduction

The Concept of a bitopological space  $(X, \tau_1, \tau_2)$  was first introduced by Kelly [1] , where  $X$  is a non-empty set and  $\tau_1, \tau_2$  are topologies on  $X$ . The authors [3, 4] defined the notions of  $swg^*$ -open sets and  $swg^*$ -continuity.

Pervin [6] investigated connectedness in bitopological space. Khedr, El.Areefi and Noiri [2] defined pre-continuity and semi pre continuity in bitopological spaces. In this paper, we introduce and study the concepts of quasi  $SWG^*$ - open and quasi- $swg^*$ -closed functions on bitopological spaces.

Throughout this paper,  $(X, \tau_1, \tau_2)$  or simply  $X$  denote a bitopological space. The intersection (resp.union) of all  $\tau_i$  - closed sets containing  $A$  (resp.  $\tau_i$ -open sets contained in  $A$ ) is called the  $\tau_i$ -closure (resp.  $\tau_i$ -interior) of  $A$ , denoted by  $\tau_i\text{-cl}(A)$  (resp.  $\tau_i\text{-int}(A)$ ).

## 2. Preliminaries

**Definition 2.1:** Let  $A$  be subset of a topological space  $(X, \tau)$ . It is called semi weakly  $g^*$ -closed [3] denoted by  $swg^*$ -closed set if  $gcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open.

**Definition 2.2:** Let  $X$  and  $Y$  be topological spaces. A map  $f: X \rightarrow Y$  is said to be semi weakly  $g^*$ -continuous [4] ( $swg^*$ -continuous), if the inverse image of every open set  $Y$  is  $swg^*$ -open in  $X$ .

**Definition 2.3:** Let  $(i, j) \in \{1, 2\}$  be fixed integers. In a bitopological space  $(X, \tau_1, \tau_2)$  a subset  $A \subseteq X$  is said to be  $(i, j)$ -semi weakly  $g^*$ -closed [5] (briefly  $(i, j)$ - $swg^*$ -closed), if  $j-gcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U \in \tau_i$ -semi-open.

**Definition 2.4:** A space  $(X, \tau_1, \tau_2)$  is said to be pairwise normal [7], if for each  $\tau_1$ -closed set  $A$  and  $\tau_2$ -closed set  $B$  disjoint from  $A$ , there is a  $\tau_1$ -open set  $U$  containing  $A$  and a  $\tau_2$ -open set  $V$  containing  $B$  such that  $U \cap V = \emptyset$ .

### 3. $(i, j)$ -QUASI SEMI WEAKLY $g^*$ -OPEN AND QUASI SEMI WEAKLY $g^*$ -CLOSED FUNCTIONS

**Definition 3.1:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be  $(i, j)$ -quasi semi weakly  $g^*$ -open if the image of every  $(i, j)$ -semi weakly  $g^*$ -open set in  $X$  is  $\sigma_i$ -open in  $Y$ .

**Remark 3.2:** It is clear that every  $(i, j)$ -quasi  $swg^*$ -open function is both pairwise open and pairwise  $swg^*$ -open. The converse is not true as seen from the following example.

**Example 3.3 :** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{X, \emptyset, \{a, b\}\}$ ,  $\sigma_1 = \{Y, \emptyset, \{a\}, \{a, b\}\}$ , and  $\tau_2 = \{X, \emptyset, \{b, c\}\}$ ,  $\sigma_2 = \{Y, \emptyset, \{b, c\}, \{c\}\}$ . Clearly the function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is pairwise open and pairwise  $swg^*$ -open. However,  $f$  is not quasi  $(i, j)$ - $swg^*$ -open because  $\{a, b\}$  is  $(2, 1)$ - $swg^*$ -open in  $(X, \tau_1, \tau_2)$ , but not  $\sigma_2$ -open.

**Theorem 3.4:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is a function. Then the following are equivalent:

- (i)  $f$  is  $(i, j)$ -quasi  $swg^*$ -open;

- (ii) For each subset  $U$  of  $X$ ,  $f((i, j) - g \text{ int}(U)) \subseteq \sigma_i - \text{int}(f(U))$ ;
- (iii) For each  $x \in X$  and each  $(i, j) - swg^*$ -neighbourhood  $U$  of  $x$  in  $X$ , there exists a  $\sigma_i$ -neighbourhood  $V$  of  $f(x)$  such that  $V \subseteq f(U)$ .

**Proof :** (i)  $\Rightarrow$  (ii): Let  $f$  be an  $(i, j)$ -quasi  $swg^*$ -open function. Since  $(i, j) - g \text{ int}(U)$  is an  $(i, j)$ - $swg^*$ -open set contained in  $U$ , we obtain that  $f((i, j) - g \text{ int}(U)) \subseteq f(U)$ . As  $f((i, j) - g \text{ int}(U))$  is  $\sigma_i$ -open,  $f((i, j) - g \text{ int}(U)) \subseteq \sigma_i - \text{int}(f(U))$ .

(ii)  $\Rightarrow$  (iii): Let  $x \in X$  and  $U$  be an  $(i, j)$ - $swg^*$ -neighbourhood of  $x$  in  $X$ . Then there exist an  $(i, j)$ - $swg^*$ -open set  $V$  in  $X$  such that  $x \in V \subseteq U$ . Thus by (ii), we have  $f(V) = f((i, j) - g \text{ int}(V)) \subseteq \sigma_i - \text{int}(f(V))$ , and hence,  $f(V) = \sigma_i - \text{int}(f(V))$ . Therefore it follows that  $f(V)$  is  $\sigma_i$ -open such that  $f(x) \in f(V) \subseteq f(U)$ .

(iii)  $\Rightarrow$  (i): Let  $U$  be an  $(i, j)$ - $swg^*$ -open set in  $X$ . Then by (iii), for each  $y \in f(U)$ , there exists a  $\sigma_i$ -neighbourhood  $V_y$  of  $y$  such that  $V_y \subseteq f(U)$ . As  $V_y$  is a  $\sigma_i$ -neighbourhood of  $y$ , there exists a  $\sigma_i$ -open set  $W_y$  such that  $Y \in W_y \subseteq V_y$ . Thus  $f(U) = \cup \{W_y : Y \in f(U)\}$  is  $\sigma_i$ -open. Hence,  $f$  is  $(i, j)$ -quasi  $swg^*$ -open.

**Theorem 3.5:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is  $(i, j)$ -quasi  $swg^*$ -open, if and only if for any subset  $B$  of  $Y$  and for any  $(i, j)$ - $swg^*$ -closed set  $F$  in  $X$  such that  $f^{-1}(B) \subseteq F$ , there exists a  $\sigma_i$ -closed set  $G$  containing  $B$  such that  $f^{-1}(G) \subseteq F$ .

**Proof:** Suppose that  $f$  is  $(i, j)$ -quasi  $swg^*$ -open. Let  $B \subseteq Y$  and  $F$  be an  $(i, j)$ - $swg^*$ -closed set in  $X$  such that  $f^{-1}(B) \subseteq F$ . Now, put  $G = Y - f(X - F)$ . It is clear that  $B \subseteq G$  as  $f^{-1}(B) \subseteq F$ , and that  $f^{-1}(G) \subseteq F$ . Also  $G$  is  $\sigma_i$ -closed, since  $f$  is  $(i, j)$ -quasi  $swg^*$ -open. Conversely, let  $U$  be an  $(i, j)$ - $swg^*$ -open set in  $X$ , and put  $B = Y - f(X - U)$ . Then  $X - U$  is an  $(i, j)$ - $swg^*$ -closed set in  $X$  such that  $f^{-1}(B) \subseteq X - U$ . By hypothesis, there exists a  $\sigma_i$ -closed set  $G$  such that  $B \subseteq G$  and  $f^{-1}(G) \subseteq X - U$ . Hence,  $f(U) \subseteq Y - G$ . On the other hand, since  $B \subseteq G$ ,  $Y - G \subseteq Y - B = f(U)$ . Thus  $f(U) = Y - G$  is  $\sigma_i$ -open, and hence,  $f$  is a  $(i, j)$ -quasi  $swg^*$ -open.

**Theorem 3.6:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function. Then the following are equivalent:

- (i)  $f$  is  $(i,j)$ -quasi  $\text{swg}^*$ -open;
- (ii)  $f^{-1}(\sigma_i\text{-cl}(B)) \subset (i,j)\text{-gcl}(f^{-1}(B))$  for every subset  $B$  of  $Y$ ;
- (iii)  $(i,j)\text{-gint}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-int}(B))$  for every subset  $B$  of  $Y$ .

**Proof: (i)  $\Rightarrow$  (ii):** Suppose that  $f$  is  $(i, j)$  - quasi  $\text{swg}^*$ -open. Now for any subset  $B$  of  $Y$ ,  $f^{-1}(B) \subset (i, j)$  -  $\text{gcl}(f^{-1}(B))$ . Therefore by Theorem 3.5, there exists a  $\sigma_i$  - closed set  $G$  such that  $B \subset G$  and  $f^{-1}(G) \subset (i,j)\text{-gcl}(f^{-1}(B))$ . Hence,  $f^{-1}(\sigma_i\text{-cl}(B)) \subset f^{-1}(G) \subset (i,j)\text{-gcl}(f^{-1}(B))$ .

**(ii)  $\Rightarrow$  (i):** Let  $B \subset Y$  and  $F$  be an  $(i, j)$ - $\text{swg}^*$ -closed set in  $X$  such that  $f^{-1}(B) \subset F$ . Put  $G = \sigma_i\text{-cl}(B)$ , then  $B \subset G$ ,  $G$  is  $\sigma_i$  - closed, and  $f^{-1}(G) \subset (i, j)$  -  $\text{gcl}(f^{-1}(B)) \subset F$ . Thus by theorem 3.5,  $f$  is  $(i, j)$  - quasi -  $\text{swg}^*$ -open.

**(ii)  $\Leftrightarrow$  (iii):** It is Clear, because  $f^{-1}(\sigma_i\text{-cl}(B)) \subset (i,j)\text{-gcl}(f^{-1}(B))$  for every subset  $B$  of  $Y$  is equal to  $(i,j)\text{-gint}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-int}(B))$  for every subset  $B$  of  $Y$ .

**Theorem 3.7:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  be two functions such that  $g \circ f: X \rightarrow Z$  is  $(i,j)$ -quasi  $\text{swg}^*$ -open. If  $g$  is a pairwise continuous injection, then  $f$  is  $(i, j)$  - quasi- $\text{swg}^*$ -open.

**Proof:** Let  $U$  be an  $(i, j)$  -  $\text{swg}^*$ -open set in  $X$ . Then  $(g \circ f)(U)$  is  $\eta_i$ -open as  $g \circ f$  is  $(i, j)$ -quasi - $\text{swg}^*$ -open. Since  $g$  is a pairwise continuous injection,  $f(U) = g^{-1}((g \circ f)(U))$  is  $\sigma_i$ -open. Hence,  $f$  is  $(i, j)$ -quasi - $\text{swg}^*$ -open.

**Definition 3.8:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be  $(i,j)$ -quasi -  $\text{swg}^*$  - closed if the image of each  $(i,j)$ - $\text{swg}^*$ -closed set in  $X$  is  $\sigma_i$ -closed in  $Y$ .

**Remark 3.9:** It is clear that every  $(i, j)$ -quasi-  $\text{swg}^*$ -closed function is both pair wise closed and pairwise  $\text{swg}^*$ -closed. The converse is not true as seen from the following example.

**Example 3.10:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{X, \phi, \{c\}\}$ ,  $\sigma_1 = \{Y, \phi, \{b, c\}, \{c\}\}$ , and  $\tau_2 = \{X, \phi, \{a\}\}$ ,  $\sigma_2 = \{Y, \phi, \{a\}, \{a, b\}\}$ . Clearly the function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is pairwise closed and pairwise  $\text{swg}^*$ -closed. However,  $f$  is not quasi - $(i,j)$ - $\text{swg}^*$ -closed because  $\{c\}$  is  $(2,1)$ - $\text{swg}^*$ -closed in  $(X, \tau_1, \tau_2)$ , but not  $\sigma_2$ -closed.

**Theorem 3.11:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is  $(i,j)$ -quasi  $\text{swg}^*$ -closed if and only if  $\sigma_i\text{-cl}(f(A)) \subset f((i,j)\text{-gcl}(A))$  for every subset  $A$  of  $X$ .

**Proof:** Suppose that  $f$  is  $(i,j)$ -quasi - $\text{swg}^*$  -closed, there exist  $\sigma_i\text{-cl}(f(A)) \subset f((i,j)\text{-gcl}(A))$  for every subset  $A$  of  $X$ . Conversely, every  $\sigma_i\text{-cl}(f(A)) \subset f((i,j)\text{-gcl}(A))$  is  $(i,j)$ -quasi  $\text{swg}^*$ -closed.

**Theorem 3.12:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function. Then the following are equivalent:

- (i)  $f$  is  $(i,j)$ -quasi  $\text{swg}^*$ -closed;
- (ii) For any subset  $B$  of  $Y$  and for any  $(i,j)$ - $\text{swg}^*$  -open set  $G$  in  $X$  such that  $f^{-1}(B) \subset G$ , there exists a  $\sigma_i$ -open set  $U$  containing  $B$  such that  $f^{-1}(U) \subset G$ ;
- (iii) For each  $y \in Y$  and for any  $(i,j)$ - $\text{swg}^*$ -open set  $G$  in  $X$  such that  $f^{-1}(\{y\}) \subset G$ , there exists a  $\sigma_i$ -open set  $U$  containing  $\{y\}$  such that  $f^{-1}(U) \subset G$ .

**Proof:**

**(i)  $\Rightarrow$  (ii):** Suppose  $f$  is  $(i, j)$ -quasi - $\text{swg}^*$  closed set. Now there exist for any subset  $B$  of  $Y$  and for  $(i,j)$ - $\text{swg}^*$ -open set  $G$  in  $X$  such that  $f^{-1}(B) \subset G$ , there exist a  $\sigma_i$ -open set  $U$  containing  $B$  such that  $f^{-1}(U) \subset G$ .

**(ii)  $\Rightarrow$  (iii):** For any subset  $B$  of  $Y$  and for any  $(i,j)$ - $\text{swg}^*$  -open set  $G$  in  $X$  such that  $f^{-1}(B) \subset G$ , there exists a  $\sigma_i$ -open set  $U$  containing  $B$  such that  $f^{-1}(U) \subset G$ . Also there exist for each  $y \in Y$  and for any  $(i,j)$ -  $\text{swg}^*$ -open set  $G$  in  $X$  such that  $f^{-1}(\{y\}) \subset G$ , there exists a  $\sigma_i$ -open set containing  $\{y\}$  such that  $f^{-1}(U) \subset G$ .

**(iii)⇒(i):** For each  $y \in Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$  such that  $f^{-1}(\{y\}) \subset G$ , there exists a  $\sigma_1$ -open set  $U$  containing  $B$  such that  $f^{-1}(U) \subset G$ . Then  $f$  is (i,j)-quasi swg\*-closed.

**Definition 3.13:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called pair wise swg\*\*\*-closed if the image of every (i,j)-swg\*-closed set in  $X$  is (i,j)-swg\*-closed in  $Y$ .

**Theorem 3.14:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function. Then the following are equivalent:

- (i)  $f$  is pair wise swg\*\*\*-closed;
- (ii) For any subset  $B$  of  $Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$  such that  $f^{-1}(B) \subset G$ , there exists an (i,j)-swg\*-open set  $U$  in  $Y$  such that  $B \subset U$  and  $f^{-1}(U) \subset G$ ;
- (iii) For each  $y \in Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$  such that  $f^{-1}(\{y\}) \subset G$ , there exists an (i,j)-swg\*-open set  $U$  in  $Y$  such that  $y \in U$  and  $f^{-1}(U) \subset G$ ;
- (iv)  $(i,j)\text{-gcl}(f(A)) \subset f((i,j)\text{-gcl}(A))$  for every subset  $A$  of  $X$ .

**Proof:**

**(i) ⇒ (ii):** Let  $f$  be a pair wise swg\*\*\*-closed. By definition 3.13. A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called pair wise swg\*\*\*-closed if the image of every (i,j)-swg\*-closed set in  $X$  is (i,j)-swg\*-closed in  $Y$ , there exists for any subset  $B$  of  $Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$  such that  $f^{-1}(B) \subset G$ , there exists an (i,j)-swg\*-open set  $U$  in  $Y$ , such that  $B \subset U$  and  $f^{-1}(U) \subset G$ .

**(ii)⇒(iii):** For any subset  $B$  of  $Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$  such that  $f^{-1}(B) \subset G$ , there exists an (i,j)-swg\*-open set  $U$  in  $Y$  such that  $B \subset U$  and  $f^{-1}(U) \subset G$ . There exist for  $y \in Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$ . Such that  $f^{-1}(\{y\}) \subset G$ , Also there exists an (i,j)-swg\*-open set  $U$  in  $Y$  such that  $y \in U$  and  $f^{-1}(U) \subset G$ .

**(iii)⇒(iv):** Let each  $y \in Y$  and for any (i,j)-swg\*-open set  $G$  in  $X$  such that  $f^{-1}(\{y\}) \subset G$ , there exists an (i,j)-swg\*-open set  $U$  in  $Y$  such that  $y \in U$  and  $f^{-1}(U) \subset G$ . That implies  $(i,j)\text{-gcl}(f(A)) \subset f((i,j)\text{-gcl}(A))$  for every subset  $A$  of  $X$ .

**(iv) ⇒(i):** Let  $(i,j)\text{-gcl}(f(A)) \subset f((i,j)\text{-gcl}(A))$  for every subset  $A$  of  $X$ . There exist a  $f$  is pair wise swg\*\*\*-closed.

**Theorem 3.15:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  are two (i,j)-quasi-swg\*-closed functions, then  $g \circ f: (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$  is (i,j)-quasi swg\*-closed.

**Proof:** If  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  are two (i,j)-quasi-swg\*-closed. Let  $U$  be an (i,j)-swg\*-closed in  $X$ . Then  $(g \circ f)(U)$  is  $\sigma_1$ -closed as  $g \circ f$  is (i,j)-quasi-swg\*-closed.

**Theorem 3.16:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  be any two functions. Then if  $f$  is pairwise swg\*-closed and  $g$  is (i,j)-quasi-swg\*-closed the  $g \circ f$  is pairwise closed.

**Proof:** If  $f$  is pairwise swg\*-closed and  $g$  is (i,j)-quasi-swg\*-closed then  $g \circ f$  is pair wise closed.

**Theorem 3.17:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  be any two functions. Then if  $f$  is pairwise swg\*\*\*-closed and  $g$  is (i,j)-quasi-swg\*-closed then  $g \circ f$  is (i,j)-quasi-swg\*-closed.

**Proof:** If  $f$  is pairwise swg\*\*\*-closed and  $g$  is (i,j)-quasi-swg\*-closed then  $g \circ f$  is (i,j)-quasi-swg\*-closed.

**Definition 3.18:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called pairwise swg\*-irresolute, if  $f^{-1}(V)$  is (i,j)-swg\*-open in  $(X, \tau_1, \tau_2)$  for every (i,j)-swg\*-open set  $V$  in  $Y$ .

**Definition 3.19:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called pairwise swg\*-continuous, if  $f^{-1}(V)$  is (i,j)-swg\*-open in  $(X, \tau_1, \tau_2)$  for every  $\sigma_1$ -open set  $V$  in  $Y$ .

**Theorem 3.20:**

Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  be two functions such that  $g \circ f: X \rightarrow Z$  is (i,j)-quasi-swg\*-closed. Then

- (i) If  $f$  is a pairwise  $\text{swg}^*$ -irresolute surjection, then  $g$  is  $(i, j)$ -quasi  $-\text{swg}^*$ -closed.
- (ii) If  $g$  is a pairwise  $-\text{swg}^*$ -continuous injection, then  $f$  is pairwise  $\text{swg}^{**}$ -closed.

**Proof:** (i) Suppose that  $F$  is  $(i, j)$ - $\text{swg}^*$ -closed set in  $Y$ . Then  $f^{-1}(F)$  is  $(i, j)$ - $\text{swg}^*$ -closed in  $X$  as  $f$  is pairwise  $\text{swg}^*$ -irresolute. Since  $g \circ f$  is  $(i, j)$ -quasi  $-\text{swg}^*$ -closed and  $f$  is subjective  $(g \circ f (f^{-1}(F))) = g(F)$  is  $\eta_i$ -closed. Hence  $g$  is  $(i, j)$ -quasi- $\text{swg}^*$ -closed.

(ii) Suppose that  $F$  is an  $(i, j)$ - $-\text{swg}^*$ -closed set in  $X$ . Since  $g \circ f$  is  $(i, j)$ -quasi  $\text{swg}^*$ -closed,  $(g \circ f)(F)$  is  $\eta_i$ -closed, but  $g$  is a pairwise  $\text{swg}^*$ -continuous injection, so  $g^{-1}(g \circ f(F)) = f(F)$  is  $(i, j)$ - $\text{swg}^*$ -closed in  $Y$ . Hence  $f$  is pairwise  $\text{swg}^{**}$ -closed.

**Theorem 3.21:** Let  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \sigma_1, \sigma_2)$  be a function. Then  $g$  is  $(i, j)$ -quasi  $-\text{swg}^*$ -closed if and only if  $g(X)$  is  $\sigma_i$ -closed, and  $g(V) - g(X-V)$  is  $\sigma_i$ -open in  $g(X)$  whenever  $V$  is  $(i, j)$ - $\text{swg}^*$ -open in  $X$ .

**Proof: Necessity:** Let  $g$  is  $(i, j)$ -quasi  $\text{swg}^*$ -closed. Then  $g(X)$  is  $\sigma_i$ -closed as  $X$  is  $(i, j)$ - $\text{swg}^*$ -closed and  $g(V) - g(X-V) = g(X) - g(X-V)$  is  $\sigma_i$ -open in  $g(X)$  when  $V$  is  $(i, j)$ - $\text{swg}^*$ -open in  $X$ .

**Sufficiency:** Suppose that  $g(X)$  is  $\sigma_i$ -closed and  $g(V) - g(X-V)$  is  $\sigma_i$ -open in  $g(X)$  when  $V$  is  $(i, j)$ - $\text{swg}^*$ -open in  $X$ , and let  $C$  be  $(i, j)$ - $\text{swg}^*$ -closed in  $X$ . Then  $g(C) = g(X) - (g(X-C) - g(C))$  is  $\sigma_i$ -closed in  $g(X)$ , and therefore,  $\sigma_i$ -closed. Hence,  $g$  is  $(i, j)$ -quasi  $-\text{swg}^*$ -closed.

**Corollary 3.22:** Let  $g: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a surjection. Then  $g$  is  $(i, j)$ -quasi  $-\text{swg}^*$ -closed if and only if  $g(V) - g(X-V)$  is  $\sigma_1$ -open whenever  $V$  is  $(i, j)$ - $\text{swg}^*$ -open in  $X$ .

**Definition 3.23:** A Space  $(X, \tau_1, \tau_2)$  is said to be pairwise  $\text{swg}^*$ -normal if for any disjoint subset  $F_1 \in (1, 2)$ - $\text{SWG}^*C(X)$  and  $F_2 \in (2, 1)$ - $\text{SWG}^*C(X)$ , there exist disjoint subsets  $U \in \tau_1$  and  $V \in \tau_2$  such that  $F_1 \subset U$  and  $F_2 \subset V$ .

**Theorem 3.24:** Let  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be two spaces, where  $X$  is pairwise  $\text{swg}^*$ -normal and let

**Theorem 3.24:** Let  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be two spaces, where  $X$  is pairwise  $\text{swg}^*$ -normal and let  $g: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a pairwise  $\text{swg}^*$ -continuous,  $(i, j)$ -quasi  $\text{swg}^*$ -closed surjection. Then  $Y$  is pairwise normal.

**Proof:** Let  $X$  is pairwise  $-\text{swg}^*$ -normal. Let  $K$  be  $\sigma_1$ -closed and  $M$  be  $\sigma_2$ -closed disjoint subsets of  $Y$ . Then  $g^{-1}(K) \in (1, 2)$ - $\text{SWG}^*C(X)$ ,  $g^{-1}(M) \in (2, 1)$ - $\text{SWG}^*C(X)$  and  $g^{-1}(K) \cap g^{-1}(M) = \emptyset$ . Since  $X$  is pairwise  $-\text{swg}^*$ -normal, there exist disjoint sets  $V \in \tau_1$  and  $W \in \tau_2$  such that  $g^{-1}(K) \subset V$  and  $g^{-1}(M) \subset W$ . Thus  $K \subset g(V) - g(X-V)$  and  $M \subset g(W) - g(X-W)$ . It follows also from corollary 3.22 that  $g(V) - g(X-V) \in \sigma_1$  and  $g(W) - g(X-W) \in \sigma_2$ , and clearly  $(g(V) - g(X-V)) \cap (g(W) - g(X-W)) = \emptyset$  because  $V \cap W = \emptyset$ . Hence,  $Y$  is pairwise normal.

**Theorem 3.25:** Let  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be two spaces, where  $X$  is pairwise  $\text{swg}^*$ -normal and let  $g: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a pairwise  $\text{swg}^*$ -irresolute  $(i, j)$ -quasi  $\text{swg}^*$ -closed surjection. Then  $Y$  is pairwise  $\text{swg}^*$ -normal.

**Proof:** Let  $X$  is pairwise  $\text{swg}^*$ -normal. Let  $K$  be  $\sigma_1$ - $\text{swg}^*$ -closed and  $M$  be  $\sigma_2$ - $\text{swg}^*$ -closed disjoint subsets of  $Y$ . Then  $g^{-1}(K) \in (1, 2)$ - $\text{SWG}^*C(X)$ ,  $g^{-1}(M) \in (2, 1)$ - $\text{SWG}^*C(X)$  and  $g^{-1}(K) \cap g^{-1}(M) = \emptyset$ . Since  $X$  is pairwise  $\text{swg}^*$ -normal, there exists disjoint sets  $V \in \tau_1$  and  $W \in \tau_2$  such that  $g^{-1}(K) \subset V$  and  $g^{-1}(M) \subset W$ . Thus  $K \subset g(V) - g(X-V)$  and  $M \subset g(W) - g(X-W)$ . It follows from corollary 3.22 that  $g(V) - g(X-V) \in \sigma_1$  and  $g(W) - g(X-W) \in \sigma_2$ , and clearly  $(g(V) - g(X-V)) \cap (g(W) - g(X-W)) = \emptyset$  because  $V \cap W = \emptyset$ . Hence  $Y$  is pairwise  $\text{swg}^*$ -normal.

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