# Adaptive Synthesis Filter Banks for Image Compression

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**Abstract**

The subject of this paper is the design of adaptive synthesis filter bank for enhancement of image reconstruction by exploiting the phase diversity of synthesis filters in subband coding system. This paper for the first time, presents a comprehensive as well as an efficient approach of handling a large number of synthesis filters and reducing the computation complexity of the synthesis filter bank. The synthesis section comprises of linear phase filters along with a set of delay filters, where each filter has different phase characteristics to encounter reconstruction error especially at object edges in image compression applications.

***Keywords:*** *Adaptive synthesis filter banks, reconstruction error, delay filters, phase characteristics.*

## 1. Introduction

The selection of filter bank in wavelet compression is crucial, affecting image quality and system design [1-4]. In subband/wavelet image coding systems, analysis and synthesis filters are a major component in many compression algorithms. These algorithms have achieved high performance. Typically, the configuration for such compression algorithms involves a bank of analysis filters which is then followed by subband quantization and encoding. Decoding is performed using a corresponding set of synthesis filters and the subbands are merged together.

For image and video coding**,** quantization is usually applied to the subband signals, which causes quantization errors to occur in the decoded image. With linear filters, these errors are most visible around sharp edges [5], where they appear as ringing artifacts, (Gibbs phenomenon). Even with linear filters designed to reduce ringing, there is still significant noise around sharp edges.

The problem of inability of conventional filter banks to accurately preserve edge characteristics at low coding rates [6]. In particular, when images are coded at low bit rates; ringing distortion at object edges is often observed. This is due to the step response characteristics of the analysis/ synthesis filters and is a consequence of their having good magnitude response properties. If filters with monotonic step response characteristics are used (which precludes of

their having good magnitude response characteristics) aliasing distortion becomes very visible. Time-varying filter banks have the ability to simultaneously reduce the aliasing distortion and ringing distortion at low bit rates. This can be done by switching back and forth between analysis filters with different spectral and temporal (step response) characteristics such that in regions where no major transitions occur, the filter set with good magnitude response characteristics is used. When transition regions or object edges are encountered, the system switches to the filter set with good step response properties.

An interesting variation of the analysis-synthesis filter bank in form of adaptive filters was originally introduced by Nayebi [7], by using adaptive analysis and adaptive synthesis filters where the constituent filters were allowed to be adaptive. It was shown that exact reconstruction could be achieved in an adaptive environment by having the analysis filters and synthesis filters change in accordance with a set of reconstruction/design conditions. The formulation was later refined by Sodagar [8], where a post filter was introduced that made design and implementation much simpler. One of the key problems associated with this type of adaptive filter bank is that synchrony between the analysis and synthesis filters must be maintained. This issue was explored by Arrowood [9] that synchronization could be done in either a backward adaptive or forward adaptive mode. But in either way, it adds a layer of computational overhead to the encoder, which in many situations is unattractive. Furthermore, the post filter becomes more complex as the interval between switching decreases and the number of switches increases. As a result, adaptive analysis and adaptive synthesis filter banks have not been widely adopted.

Lettsome [10-11] developed an analysis-synthesis system with a non-adaptive (i.e. conventional) analysis section and an adaptive synthesis section. Because the analysis section is fixed, there was no issue of maintaining synchrony, which further simplified the operation of the filter bank. The synthesis filters were used to exploit the phase diversity in the synthesis section by means of a

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combination of minimum, maximum and linear phase synthesis filters while minimizing the reconstruction error. These filters were used in only first level of subband structure by using all the possible reconstruction combinations and in the remaining levels linear phase filters were employed. It was shown that exact reconstruction could be possible and at the same time reconstruction quality could be enhanced.

In this paper, we explore the phase diversity of the synthesis section in a comprehensive manner, by designing a set of (n-1) delay filters along with linear phase filters (where n is total number of synthesis filters and it is odd) and optimizing the synthesis filters in order to minimize the reconstruction error. We introduce a new method which effectively deals large number of delay synthesis filters and reduces computation complexity. The new synthesis filter bank is deployed in level 1, level 2 and level 3 of subband structure and linear phase filters are used in the remaining levels.

In the next section, we present a general design formulation of adaptive synthesis filter bank. In the subsequent section, we explain the implementation of proposed synthesis filter bank with various design examples. Finally, results are discussed and a conclusion is made along with suggestions for future work.

## 2. Adaptive Synthesis Filter banks

In new adaptive synthesis filter bank, synthesis section consists of "n" filters (where n is an odd integer) and it is composed of (n-1) delay filters and linear phase filters. Odd length filters have been considered because even length filters exhibit fractional delay in compression.

### 2.1 Block Diagram

The block diagram of the new system is shown in Fig 1. The analysis section is the conventional one, while the synthesis filters are switched adaptively and selectively in the reconstruction process with minimum reconstruction error on pixel by pixel basis.

### 2.2 Diagram Procedure

If 'L' is the length of given analysis filters, we can design '2L − 2' synthesis filters by using the following time domain equation

$$\mathbf{A} = \mathbf{SB}$$

where **A** is a block Toeplitz matrix of analysis filter coefficients, **S** is a matrix of synthesis filter coefficients, and **B** is the reconstruction matrix. In a more expanded form, above equation can be expressed as

$$\underbrace{\begin{bmatrix} \mathbf{P}_0^T & 0 & \cdots & 0 & 0 \\ \mathbf{P}_1^T & \mathbf{P}_0^T & \vdots & 0 & 0 \\ \mathbf{P}_2^T & \mathbf{P}_1^T & \vdots & \vdots & \\ \mathbf{P}_3^T & \mathbf{P}_2^T & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & \\ \mathbf{P}_{K-1}^T & \mathbf{P}_{K-2}^T & \cdots & \mathbf{P}_0^T & 0 \\ 0 & \mathbf{P}_{K-1}^T & \ddots & \mathbf{P}_1^T & h_0(0) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \mathbf{P}_{K-1}^T & h_0(L-1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{K-2} \\ \mathbf{Q}_{K-1} \end{bmatrix}}_{\mathbf{S}} = \underbrace{\begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_R \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}}. \qquad (1)$$

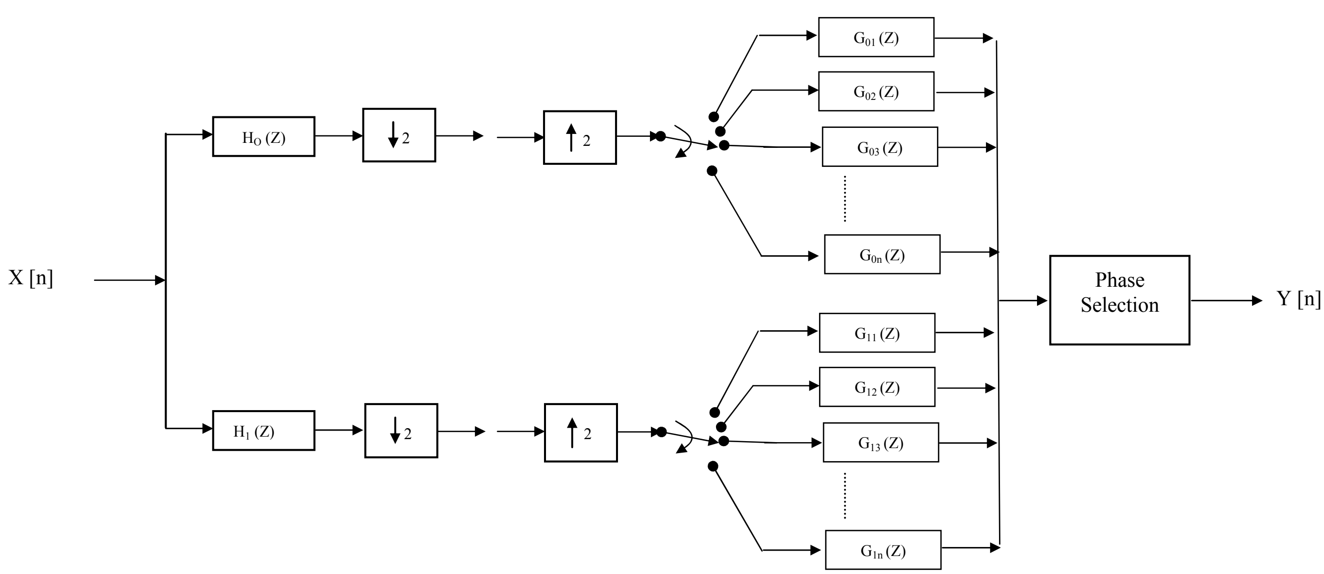


Fig. 1 Block diagram conventional analysis and adaptive synthesis filter bank.

Since odd length filters have implicit different length low pass and high pass filters. To mitigate this situation, let **K** is the length of longer filter and shorter filter is adjusted to **K** by zero padding at the back end. The length parameter of **P** and **Q** in the above equation is set to **K**. So **A** and **S** are defined in terms of

$$\mathbf{P} = [\mathbf{P}_0|\mathbf{P}_1|\ldots|\mathbf{P}_{K-1}]$$

and

$$\mathbf{Q} = [\mathbf{Q}_0|\mathbf{Q}_1|\ldots|\mathbf{Q}_{K-1}].$$

The submatrices of **P** are defined as

$$\mathbf{P}_i^T = [h_0(i)h_1(i)]$$

where $h_0(i)$ and $h_1(i)$ represent the lowpass and highpass analysis filters. The **Q** matrices are analogues to the **P** matrices in that they contain the lowpass and highpass synthesis filter coefficients. The **Q** matrices are given by

$$\mathbf{Q}_0 = [g_0(0)g_0(1)],$$

$$\mathbf{Q}_1 = [g_1(0)g_1(1)],$$

$$\mathbf{Q}_2 = [g_0(2)g_0(3)],$$

$$\mathbf{Q}_3 = [g_1(2)g_1(3)],$$

and so on until all the synthesis filter coefficients are included. Finally, "$\mathbf{J_R}$" in reconstruction matrix **B** defined as

$$\mathbf{J}_R = \begin{bmatrix} 0 & 0 & \ldots & 1 \\ 0 & & \cdot & 0 \\ \vdots & \cdot & & 0 \\ 1 & 0 & \ldots & 0 \end{bmatrix}.$$

The position of "$\mathbf{J_R}$" in the reconstruction matrix **B** controls the phase characteristics of the synthesis filters. Given a desired sample delay, "$\mathbf{J_R}$" is positioned in the (d−1) location of matrix **B** where "d" is the desired system delay. Thus we can easily design optimal filters with group delays ranging from minimum to maximum phase.

For the general odd-length case, if "R" is decimation factor and "M" is number of bands then dimensions of matrices **A**, **S** and **B** will be 2K – R – (K mod R) rows and rnd(MK/R) columns, rnd(MK/R) rows and R columns and 2K – R – (K mod R) by R respectively.

The delay synthesis filters are computed from common analysis filters by using equation (1) in the form

$$\mathbf{S} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}.$$

and reconstruction error is calculated by the relation

$$\epsilon_r = \|\mathbf{AS} - \mathbf{B}\|_F^2$$

For a given set of parameters, the exact reconstruction solution is unique to those parameters. Consequently, when the system is designed for linear phase filters with exact reconstruction, none of the delay synthesis filters also have exact reconstruction if they all share common analysis filters. However, this reconstruction error of these filters can be minimized by optimizing synthesis filter coefficients. For low pass filters, the sum of the odd coefficients and the sum of the even coefficients both are made approximately equal to 0.7071. Similarly, for high pass filters, the sum of the odd coefficients and the sum of the even coefficients are made approximately equal to 0.7071 and -0.7071 respectively.

A cost function is used to further optimize the reconstruction error and frequency domain characteristics of low pass

$$s_0^2 = \frac{1}{N}\sum_{k=1}^{\frac{N}{2}}(|G_0(k)| - \sqrt{2})^2 + \frac{1}{N}\sum_{k=\frac{N}{2}+1}^{N}|G_0(k)|^2$$

and high pass synthesis filters.

$$s_1^2 = \frac{1}{N}\sum_{k=1}^{\frac{N}{2}}|G_1(k)|^2 + \frac{1}{N}\sum_{k=\frac{N}{2}+1}^{N}(|G_1(k)| - \sqrt{2})^2.$$

If we express the frequency domain error for the synthesis filters as

$$s^2 = 1/2 * s_0^2 + 1/2 * s_1^2$$

then the total error function minimized in the design process is given by

$$\epsilon_t = \alpha * \epsilon_r + (1-\alpha) * s^2, \quad 0 < \alpha < 1$$

where "$\alpha$" is a weighting factor. This weighting factor allows for tradeoffs between the reconstruction error and passband-stopband error.

To handle "m" number of delay synthesis filters (where m is an even number), the delay filters are divided into "m/2" groups and each group comprises of two delay filters along with linear phase filters. If "$S_d$" and "$S_p$" are delay and linear phase synthesis filters respectively and "d" represents the delay, where

**d = 1, 2, 3, ………… m**

then synthesis filters are grouped as

$$\mathbf{G_1 = S_1, S_m, S_p}$$
$$\mathbf{G_2 = S_2, S_{m-1}, S_p}$$

$$\mathbf{G_3 = S_3, S_{m-2}, S_p}$$

$$\vdots \qquad \vdots$$

$$\mathbf{G_{m/2} = S_{m/2}\,, S_{(m/2)+1}, S_p}$$

In the reconstruction process synthesis filters are selected in an adaptive mechanism i.e. at edges delay filters are used and in the smooth regions, reconstruction is accomplished by linear phase filters. The selection is based on comparison of three outputs of each group on pixel by pixel basis. Let $y_o(i)$, $y_n(i)$ and $y_h(i)$ are the outputs of the lowest delay, linear phase and highest delay filters respectively, then if

$$\mathbf{y_o(i) \approx y_n(i)}$$

and

$$\mathbf{y_h(i) \neq y_n(i)}$$

reconstruction is accomplished by the highest delay filters. Similarly, if

$$\mathbf{y_h(i) \approx y_n(i)}$$

and

$$\mathbf{y_o(i) \neq y_n(i)}$$

reconstruction is done by the lowest delay filters. In case, if none of the delay filter of each group meet the above criteria, then reconstruction is accomplished by linear phase filters.

Since for "n" synthesis filters, there will be "$n^2$" possible reconstruction combinations. However, this computational complexity can be reduced to "2n-1" by combining each set of delay filters with corresponding linear phase filters only because other combinations do not significantly contribute in optimal image reconstruction.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots\cdots & a_{1m} & a_{1p} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots\cdots & a_{2m} & a_{2p} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots\cdots & a_{3m} & a_{3p} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \cdots\cdots & a_{mm} & a_{mp} \\ a_{p1} & a_{p2} & a_{p3} & a_{p4} & \cdots\cdots & a_{pm} & a_{pp} \end{pmatrix}_{n \times n}$$

Placing of the delay filters is made before the linear phase filters by keeping the delay filters of the groups

$$\mathbf{G_{m/2}, G_{(m/2)-1}, G_{(m/2)-2}, \ldots\ldots\ldots\ldots\ldots\ldots\ G_1}$$

in the order of

$$\mathbf{1^{st}, 2^{nd}, 3^{rd}, \ldots\ldots\ldots\ldots\ldots\ m/2}$$

positions respectively in the synthesis filter bank and the same placing configuration is implemented up to "level 3" of the subband tree and in all the remaining levels, linear phase filters are used.

## 3. Design Examples

In this section, we first describe a detail implementation of new adaptive synthesis filter bank for n = 5, which comprises of "the lowest delay, low delay, linear phase, high delay and the highest delay" synthesis filters. Popular bi-orthogonal Daubechies 9/7 filters are selected because of their superior performance. Since the analysis section is non-adaptive, so same linear phase analysis filters are employed and proposed adaptive synthesis filter bank is used in synthesis section. The length of Daubechies 9/7 analysis filters is 16, so we can design 14 different types of synthesis filters. The sizes of matrices **A, S** and **B** will be 15 x 9, 9 x 2 and 15 x 2 respectively.

For the Daubechies 9/7 filters, the associated frequency response for the lowest delay, low delay, high delay, highest delay and linear phase low pass synthesis filters are shown in Fig 2, 3, 4, 5 and 6.

We have incorporated adaptive synthesis filter bank in conventional popular SPHIT (Set Partitioning in Hierarchical Trees) and modified the synthesis section so

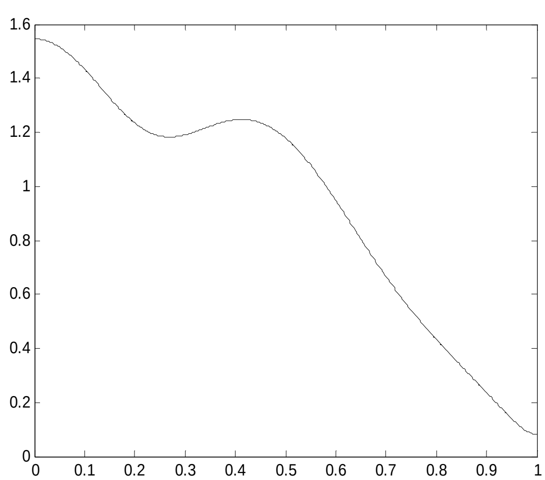


Fig 2. Frequency response for Daubechies 9/7 lowpass the lowest delay synthesis filter.

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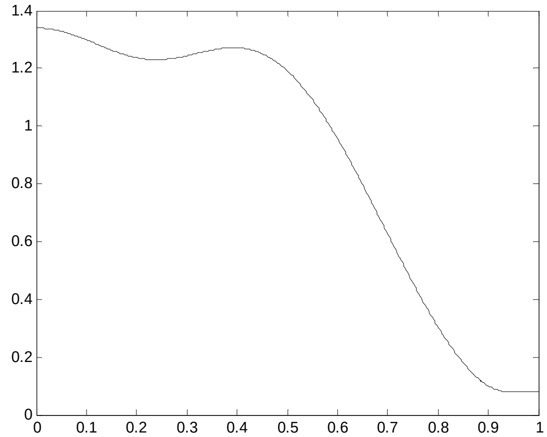


Fig 3. Frequency response for Daubechies 9/7 lowpass low delay synthesis filter.

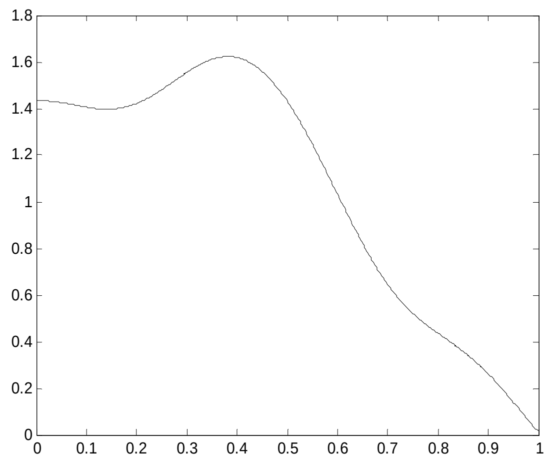


Fig 4. Frequency response for Daubechies 9/7 lowpass high delay synthesis filter.

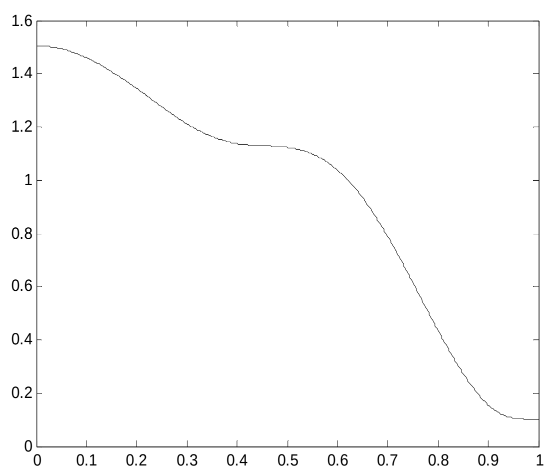


Fig 5. Frequency response for Daubechies 9/7 lowpass the highest delay synthesis filter.

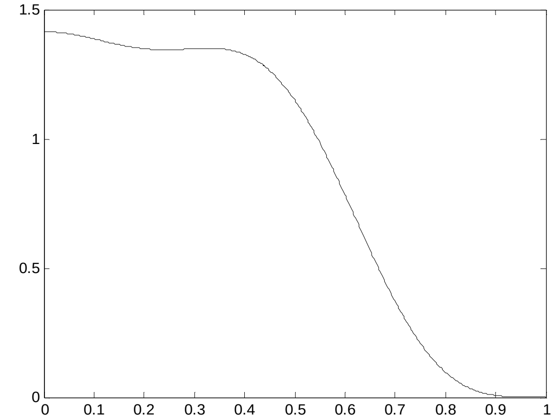


Fig 6. Frequency response for Daubechies 9/7 lowpass linear phase synthesis filter.

that all the delay filters of the groups and linear phase filters take part in the optimal image reconstruction.

For this particular example, synthesis filters are divided into two groups; group 1 comprises of "the lowest delay, the highest delay and the linear phase filters" whereas, the group 2 is composed of "low delay, high delay and linear phase filters". Computational complexity has been reduced from 25 possible reconstruction combinations to 9 for first three subband level. The new reconstruction combinations are: the lowest delay on rows and linear phase on columns, linear phase on rows and the lowest delay on columns, low delay on rows and linear phase on columns, linear phase on rows and low delay on columns, high delay on rows and linear phase on columns, linear phase on rows and high delay on columns, and so on. The reconstruction is first tried with delay filters of group 2, then with delay filters of group 1 and finally with linear phase filters in the synthesis filter bank up to "subband level 3" and linear phase filters are used in the remaining levels.

Similarly, for n = 3, the adaptive synthesis filter bank is implemented with a group of "the lowest delay, the highest delay and linear phase filters", whereas for n = 7, it is implemented with six delay filters along with linear phase filters. For n = 7, synthesis filters are divided into three groups; group 1 "the lowest delay, the highest delay and linear phase filters", group 2 "lower delay, higher delay and linear phase filters" and group 3 "low delay, high delay and linear phase filters" and the placing of the delay filters of the group 3, group 2 and group 1 is done in the order of $1^{st}$, $2^{nd}$ and $3^{rd}$ positions before linear phase filters in the synthesis filter bank.

Table 1: Comparison of New adaptive synthesis filter banks and Lettsome synthesis filter bank for compression ratio = 0.5 bits per pixel.

| **Image** | **Lettsome Filter Bank** | | **New Adaptive Filter Banks** | | | | | |
|---|---|---|---|---|---|---|---|---|
| | *No. of filters (n=3) C Complexity = 9* | | *No. of filters (n=3) C Complexity = 5* | | *No. of filters (n=5) C Complexity = 9* | | *No. of filters (n=7) C Complexity = 13* | |
| | *MSE* | *PSNR* | *MSE* | *PSNR* | *MSE* | *PSNR* | *MSE* | *PSNR* |
| Cameraman | 57.3526 | 30.54 | 57.3567 | 30.54 | 57.2102 | 30.56 | 57.1528 | 30.57 |
| Couple | 44.6570 | 31.63 | 44.6598 | 31.63 | 44.4830 | 31.65 | 44.4686 | 31.65 |
| Goldhill | 37.7827 | 32.35 | 37.7973 | 32.35 | 37.6944 | 32.37 | 37.6657 | 32.37 |
| Lena | 36.1099 | 32.55 | 36.1114 | 32.55 | 36.0218 | 32.57 | 36.0180 | 32.57 |
| Lake | 85.8752 | 28.79 | 85.8833 | 28.79 | 85.6835 | 28.8 | 85.6729 | 28.8 |
| Kiel | 102.9763 | 28.00 | 102.9817 | 28.00 | 102.8752 | 28.01 | 102.8624 | 28.01 |

## 4. Results and Conclusion

Various standard images using the new adaptive synthesis filter banks and Lettsome proposed synthesis filter bank are tested and results are shown in Table 1 for compression ratio 0.5 bits per pixel.

For n = 3, results of new adaptive synthesis filter bank are almost same as that of Lettsome filter bank, however the computational complexity has been reduced from 9 possible reconstruction combinations to 5 and for n= 5, computational complexity is the same as that of Lettsome filter bank, but there are quite objective improvements in results. Similarly, for n = 7, results are further improved but this improvement is very modest. However, the potential of the proposed adaptive filter banks are actually quite high to exploit phase diversity in the synthesis section if we use a slightly more sophisticated optimizing technique for the synthesis delay filters and deploy different filters placing configurations for different levels of subband structure.

Phase diversity of the synthesis section has been exploited by using a new adaptive synthesis filter bank which efficiently handles a large number of synthesis filters as well as reduces the computation complexity. In future, we will improve optimization of the delay synthesis filters and at the same time implement different filters placing configurations for different subband levels.

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