Image Denoising with Modified Wavelet Feature Restoration

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Abstract

Image denoising is the principle problem of image restoration and many scholars have been devoted to this area and proposed lots of methods. In this paper we propose modified feature restoration algorithm based on threshold and neighbor technique which gives better result for all types of noise. Because of some limits of conventional methods in image denoising, several drawbacks are seen in the conventional methods such as introduction of blur and edges degradation. Those can be removed by using the new technique which is based on the wavelet transforms. The shrinkage algorithms like Universal shrink, visue shrink, bays shrink; have strengths in Gaussian noise removal. Our proposed method gives noise removal for all types of noise, in wavelet domain. It gives a better peak signal to noise ratio as compared to traditional methods

Keywords: *Image Noise, threshold, Wavelet.*

1. Introduction

Image Denoising consists of an attempt to recover an image which has been degraded by a linear shift-invariant filtering operation with noise. It has applications in fields such as astronomy, remote sensing and biomedical imaging as well as in everyday life for the enhancement of noisy photos. The existing linear image restoration of algorithms assumes that the Point Spread Function (PSF) is known a priori and it attempts to reverse it in cooperation to reduce noise by utilizing the available information. Although many researchers have worked on this type of problem, it was difficult to work with the unknown noise in many real situations [1].

The image restoration process is restoring an unknown image using partial or no information about the imaging system. It is well known that the image restoration is quite a challenging problem in the field of image processing, especially for those images which are degraded by Gaussian noise. The traditional method for image restoration is to detect the parameters of the PSF firstly from the degraded image, and then to recover the underlying image. However, the restoration of the Gaussian noise image is very difficult, especially in the case of PSF unknown.

In [2] [3] Donoho proposes different thresholding technique, but this technique not keep details like edges, to overcome this we proposes new technique. In this paper, we have proposed the threshold and convolution technique. The input image is applied to different noises to get the noisy image. Different types of noise such as white Gaussian, Salt and Pepper, Speckle and Poisson's added to the image .This image is transformed into the wavelet domain [4][5].The Wavelet features are modified by the proposed technique and process which would be reversed by applying Inverse Wavelet Transform to remove the noise from the image. Figure 1, elaborates the process of denoising Image denoising algorithm consists of a few steps, let us consider an input signal x(t) and noisy signal n(t), add both the signals to get y(t) , i.e.

$$y(t) = x(t) = n(t) \quad (1)$$

where the noise can be Gaussian, Poisson's, speckle and Salt and pepper, then apply wavelet transform to get *w(t)*.

$$y(t) \xrightarrow{\text{Wavelet Transform}} w(t) \quad (2)$$

Modify the wavelet coefficient *w(t)* using different threshold algorithm and take inverse wavelet transform to get denoising image $\hat{x}(t)$.

$$w(t) \xrightarrow{\text{Inverse Wavelet Transform}} \hat{x}(t) \quad (3)$$

The Fig. 1 depicts a denoising scheme. The results obtained from various proposed threshold and convolution methods compare with the PSNR values of denoising images [6]. This paper is organized as chapter I expresses the introduction of image denoising scheme, chapter II illustrates the theory of wavelet transform while chapter III introduces proposed threshold methods and chapter IV briefs the implementation and result.

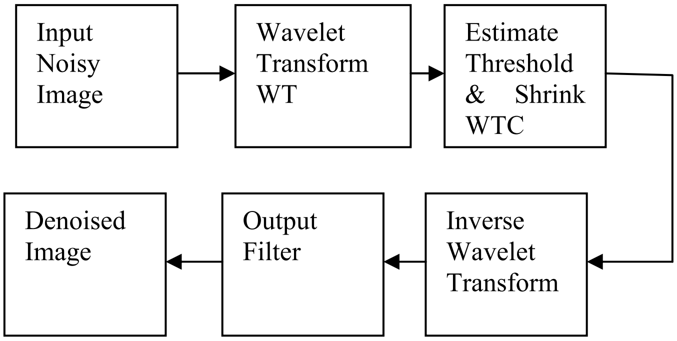

Fig. 1: Block diagram of denoising technique

2. Wavelet Transform

The wavelet expansion set is not unique. A wavelet system is a set of building blocks to construct or represent a signal or function. It is a two dimensional expansion set, usually a basis, for some class one or higher dimensional signals [7] [8] [9].

The wavelet can be represented by a weighted sum of shifted scaling function $\varphi(2t)$ as,

$$\psi(t) = \sum_n h_1(n)\sqrt{2}\varphi(2t-n) \qquad n \in Z \tag{4}$$

For some set of coefficient h_1 (n), this function gives the prototype or mother wavelet $\psi(t)$ for a class of expansion function of the form

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \tag{5}$$

Where 2^j is the scaling of $t, 2^{-j}k$ is the translation in t, and $2^{j/2}$ maintains the L^2 norms of the wavelet at different scales. The construction of wavelet using the set of scaling function $\varphi_k(t)$ and $\psi_{j,k}(t)$ that could span all of $L^2(R)$; therefore function $g(t) \in L^2(R)$ can be written as

$$g(t) = \sum_{k=-\infty}^{\infty} c(k)\varphi_k(t) + \sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty} d(j,k)\psi_{j,k}(t) \tag{6}$$

First summation in the above equation gives a function that is low resolution of *g (t),* for each increasing index j in the second summation, a higher resolution function is added which gives increasing details. The function d(j,k) indicates the differences between the translation index *k*, and the scale parameter *j*. In wavelet analysis expand coefficient at a lower scale level to higher scale level, from equation (7), we scale and translate the time variable to give

$$\begin{aligned}\varphi(2^t t - k) &= \sum_n h(n)\sqrt{2}(2(2^t t - k) - n \\ &= \sum_n h(n)\sqrt{2}\varphi(2^{j+1} t - 2k - n)\end{aligned} \tag{7}$$

After changing variables m=2k+n, the above equation becomes

$$\varphi(2^t t - k) = \sum_m h(m-2k)\sqrt{2}\varphi(2^{j+1} t - m \tag{8}$$

At one scale lower resolution, wavelets are necessary for the detail not available at a scale of j. We have

$$\begin{aligned}f(t) &= \sum_k c_j(k)2^{j/2}\varphi(2^j t + k) \\ &\quad + \sum_k d_j(k)2^{j/2}\varphi(2^t t - k)\end{aligned} \tag{9}$$

Where the $2^{j/2}$ terms maintain the unity norm of the basic functions at various scales. If $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$ are orthonormal, the j level scaling coefficients are found by taking the inner product

$$c_j(k) = \langle f(t), \varphi_{j,k}(t)\rangle = \int f(t)2^{j/2}\varphi(2^j t - k)dt \tag{10}$$

By using equation (10) and interchanging the sum and integral, can be written as

$$c_j(k) = \sum_m h(m-2k)\int f(t)2^{(j+1)/2}\varphi(2^{j+1} t - m)dt \tag{11}$$

But the integral is an inner product with the scaling function at a scale $j+1$ giving

$$c_j(k) = \sum_m h(m-2k)c_{j+1}(m) \tag{12}$$

The corresponding wavelet coefficient is

$$d_j(k) = \sum_m h_1(m-2k)c_{j+1}(m) \tag{13}$$

Fig. (2) shows the structure of two stages down sampling filter banks in terms of coefficients.

A reconstruction of the original fine scale coefficient of the signal made from a combination of the scaling function and wavelet coefficient at a course resolution is derived by

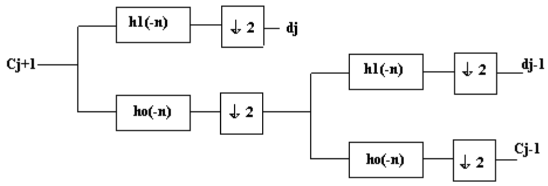

Fig. 2: Two stages down sampling Filter bank

considering a signal in the j+1 scaling function space $f(t) \in v_{j+1}$. This function is written in terms of the scaling function as

$$f(t) = \sum_k c_{j+1}(k) 2^{(j+1)/2} \varphi(2^{j+1} t - k) \qquad (14)$$

In terms of the next scales require wavelet as

$$\begin{aligned} f(t) &= \sum_k c_j(k) 2^{j/2} \varphi(2^j t - k) \\ &+ \sum_k d_j(k) 2^{j/2} \psi(2^j t - k) \end{aligned} \qquad (15)$$

$$c_{j+1}(k) = \sum_m c_j(m) h(k-2m) + \sum_m d_j(m) h_1(k-2m) \qquad (16)$$

Fig. (3) shows the structure of two stages up sampling filter banks in terms of coefficients i.e. synthesis from coarse scale to fine scale one [5] [6] [7].

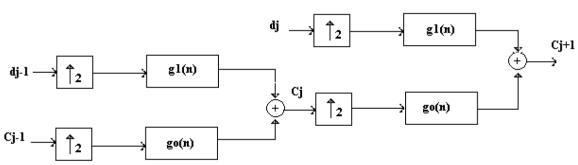

Fig. 3: Two stages up sampling filter

The filter structure analysis can be done by applying one step of the one dimensional transform to all rows, then repeating the same for all columns then proceeding with the coefficients that result from a convolution within both directions, this is one level wavelet decomposition that proceeds similar for two levels for LL components to get a two-level decomposition structure shown in Fig. (4).

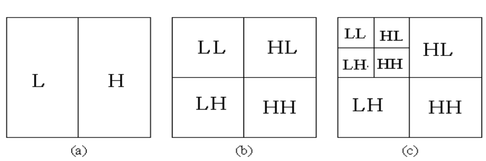

Fig. 4: Two-dimensional wavelet transforms

Calculating wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data. We choose only a subset of scales and positions at which to make our calculation. It turns out, rather remarkably, that if we chose only a subset of scales and positions based on powers of two so-called *dyadic* scales and positions then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the discrete wavelet transform [10] [11] [12].

3. Proposed Thresholding

There are different denoising schemes used to remove noise while preserving original information and basic parameter of the image. Contrast, brightness, edges and background of the image should be preserved while denoising in this technique. Wavelet transform tool is used in denoising of image. Actually, the performance of our algorithm is very close to, and in some cases even surpasses, to that of the already published denoising methods. There are different techniques applied to the feature vector of wavelet. It was good for a few noise functions but our proposed technique gives a better result than the existing method does [13] [14] [15]. The universal threshold scheme was suggested by Donoho and Johnston for Gaussian noise. As the name in itself suggests, the universal threshold scheme is a global thresholding scheme in which a universal threshold is fixed for all the empirical wavelet coefficients. Visu Shrink was introduced by Donoho [2]. It uses a threshold value *t* that is proportional to the standard deviation of the noise. Bayes Shrink is an adaptive wavelet threshold method proposed by Chang, Yu, and Vetterli using a Bayesian estimate of the risk. Threshold calculations are based on the assumption that wavelet coefficients can be described by a generalized Gaussian distribution with shaping parameter. These distribution functions fit the coefficients of the most natural images very well. The threshold chooser based on Stein's Unbiased Risk Estimator (SURE) was proposed by Donoho and Johnston and is called as Sure Shrink [17] [18]. It is a combination of the universal threshold and the SURE threshold. We have proposed a few functions for denoising using wavelet, performance of our methods evaluated by using PSNR [19]. As one may observe, threshold determination is an important question when applying the wavelet thresholding scheme. A small threshold may yield a result close to the input, but the result may be still being noisy. A large threshold on the other hand, produces a signal with a large number of zero coefficients. This leads to overly smooth signal. Paying too much attention to smoothness generally suppress the details and edges of the original signal and cause blurring and ringing artifacts.

3.1 Proposed Threshold 1:

The image features are characterized by mean and standard deviation. The mean smoothen the image data reduces noise. Noise in an image logarithmically reduces. We propose nonlinear threshold operator for removing noise. This threshold is generated as

$$newth = \sqrt{2m \times \log(M)} \tag{17}$$

Where, **M is** the total number of pixel of an image, **m** is the mean of the image. This function preserves the contrast, edges, background of the images. This threshold function is calculated at different scale levels. This proposed threshold performance calculated using peak signal to noise ratio (PSNR). A simple but often used quantitative measure of assessing image distortion due to degradation is the signal to noise ratio (SNR). The disadvantage with this measure of the SNR is that it is a function of the image variance. Even if the mean square error (MSE) between the two images is the same, SNR values can differ if the corresponding variance differs. Another quantitative measure often used in practice is peak SNR (PSNR), which is defined as the ratio of the square of the peak signal to the MSE, expressed in dB

$$PSNR = 10\log_{10}\left(\frac{peak^2}{MSE}\right)$$

$$MSE = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}(x[m,n] - \hat{x}[m,n])$$

Where, $x[m,n]$ is the original image detail, and $\hat{x}[m,n]$ is the recovered image detail.

3.2 Circular kernel

Kernel applied to the wavelet approximation coefficient, to get de-noised image with all parameters is undisturbed. The kernel used here in this technique contains some components are zeros and ones as shown in Fig. (5).

A multi-resolution analysis wavelet structure has been used for this kernel to get result. This kernel helps to preserve the edges and boundary of the images so that better technique as compare to the other threshold methods. This kernel is used at the different decomposition level simply moving this window in an image. This step is called as convolution. The kernel has little degradation and enhancing components will not affect the original information of the image. It preserves the detail of the image. After convolution we have to apply inverse wavelet to get denoisy image. This convolution method gives better peak signal to noise ratio (PSNR).

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Fig. 5: Circular Kernel

3.3 Mean-Max threshold:

This method generates the threshold using mean and max method after wavelet decomposition at different level. Let xi denote the sequence of elements; threshold can be calculated using following technique.

$$MAXMIN = \{x_j\} = MAX\begin{Bmatrix} [MIN(x_1,.......x_k)], \\ [MIN(x_2,.......x_{k+1})],....+ \\ [MIN(x_{i-k+1},.......x_i)] \end{Bmatrix} \tag{18}$$

$$MINMAX = \{x_j\} = MIN\begin{Bmatrix} [MAX(x_1,.......x_k)], \\ [MAX(x_2,.......x_{k+1})],....+ \\ [MAX(x_{i-k+1},.......x_i)] \end{Bmatrix} \tag{19}$$

This methodology gives MIN and MAX of the sub band at different level of the decomposed image. Then wavelet coefficients are threshold by different combination at different decomposition level. This threshold gives better PSNR for different noisy images.

3.4 Nearest neighbor:

This technique gives better result for different kernel structure shown in Fig. (6). In this kernel central pixel (CP), calculated from the neighbor value. Three different kernels have proposed for better reduction of noise using wavelet transform at different scale. Mark 'x' denotes low value at that position [20]. The neighbor values 1,2,3,4 denote the components of the kernel will be marked as one. This mask is moving in the different sub band of the decomposed image. Then degraded coefficients are enhanced in this technique. These three kernels give better PSNR as compared to all technique at different noise level.

1	X	2
X	CP	X
4	X	3

X	1	X
2	CP	4
X	3	X

1	2	3
4	CP	5
6	7	8

Fig. 6: Kernel at different noise level

3.5 Proposed Threshold 2:

For the image degradation depends on the standard deviation, mean, variance of image. Concentrating on the parameter we have proposed a new technique of threshold algorithm and to get the good results for the different noisy and blurred structures.

$$\lambda = \sqrt{mean \times \log(energy \times std.dev)}$$
(20)

This threshold gives better PSNR performance parameter. For all types of noise image quality is drastically improved. This threshold is applied independently to each sub-band of an image's wavelet coefficients using soft thresholding, including the lowest resolution LL sub-band. This yields a frequency adaptive threshold that is more aggressive in the signal. Use of this threshold optimizes performance by removing the most noise possible while still preserving the original signal.

3.6 Cluster average Technique

This technique applies to the features of the wavelet and is modified on the basis of the best suit by the technique of neighbor of cluster. The best basis determined from wavelet feature is as

$$avg(y) = 1/3 \sum_{1}^{i} X_i \qquad (21)$$

Replace the new value using $store = \min(avg(y_i))$

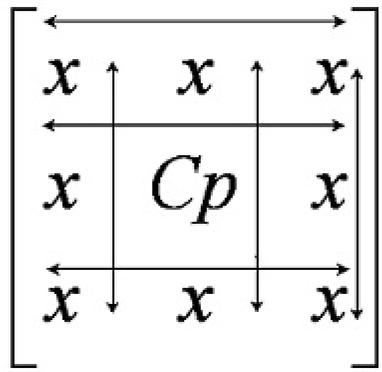

Fig. 7: neighbor pixel

In Fig. (7) the center pixel C_P will be replaced by taking minima of all averages in all dimensions. This 3*3 neighbor varies in the full image and modified the wavelet coefficient at different level. The modified wavelet feature is obtained using the above method applied to the frequency domain. This technique gives a better PSNR of denoised image. This clustering is applied to independently to each sub-band at different level of decomposition. The wavelet coefficients using this neighboring at each level is modified and gives the denoised image by inverse wavelet transform.

4. Implementation and Result

Noise reduction plays a fundamental role in image processing, and wavelet analysis has been demonstrated to be a powerful method for performing image noise reduction. The procedure for noise reduction is applied on the wavelet coefficients achieved using the wavelet decomposition and representing the image at different scales. After noise reduction, the image is reconstructed using the inverse wavelet transform. A degraded noisy image can be approximately described using equation (1); this noisy image is obtained by using all noisy functions. Wavelet transform tool is applied to noisy image to get wavelet coefficient, the steps of algorithm are as follows

i) Read an image.
ii) Apply different types of noise such as Gaussian, Poisson's, speckle and Salt and pepper etc.
iii) Apply wavelet transform to noisy image at a required level, we would get four components namely, Approximation, Horizontal, Vertical, Diagonal coefficients.
iv) Select appropriate coefficients from the above decomposition and find out statistical parameters of coefficients. Based on the statistical parameters and on one of the threshold selection algorithms find out proper threshold value for respective coefficients.

v) In denoising each coefficient is *threshold* by comparing against a threshold; if the coefficient is smaller than the threshold it is set to zero, otherwise, it is kept.
vi) Apply Inverse Wavelet transform to threshold coefficient, it gives a de-noised image.

We have considered here ten images Result of all functions shown in Table (1) for all type of noise. Then we will recover the denoised image. We compared various denoising method on several test images widely used in image processing community. Here, we report the result only for the Lena image. The result shown in Fig. (8) shows graphical representation functions and their PSNR for Gaussian noisy image , Fig. (9) shows graphical representation functions and their PSNR shows the result for Salt and pepper noisy image, Fig. (10) shows the graphical representation functions and their PSNR shows the result for Speckle noise image. Fig. (11) shows graphical representation functions and there PSNR depicts the result of Poisson's noise image.

5. Conclusion

The method describes a new way of denoising the image based on the wavelet transform. Because of some limits of conventional methods in image denoising, several drawbacks such as edge degradation are seen in the conventional methods. Those can be removed by using the new technique which is based on the wavelet transforms. We have analyzed the various techniques of image denoising by using the proposed methods. The proposed method 1 and proposed method 2 has good result at different noise level as compared to the existing methods. The circular kernel and Min Max method gives the better result visually but the PSNR is not good for this method as compared to all methods. This technique preserves the details of the image like edges as compared to the existing technique. The nearest neighbor method has better result as compared to the all existing method as well as all proposed technique. The cluster averaging technique has comparable excellent PSNR values. For Gaussian noise, all functions work better than the existing threshold. In Speckle noise, nearest neighbor methods give a better result. In Poisson's noise, all methods give comparable results. In Salt and Pepper noise, our proposed cluster method has better results. The results would be improved by using various applications of the filter masks. The improvement can be seen with a change in the type of wavelet family function that is used in the image transformation.

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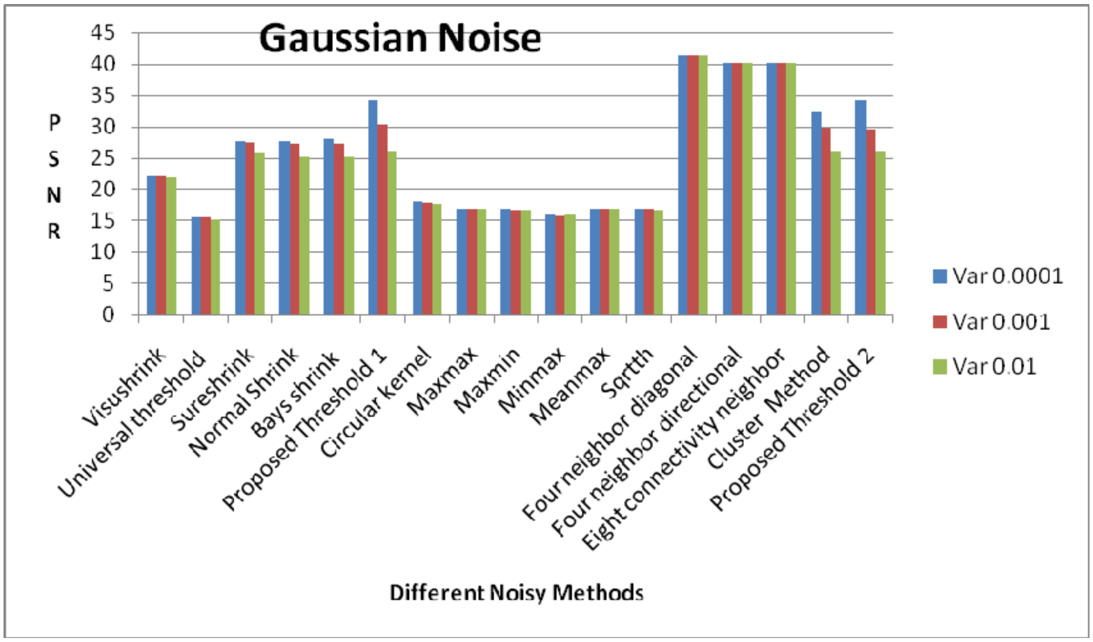

Fig. (8) Different Noisy methods Vs PSNR for Gaussian noisy image

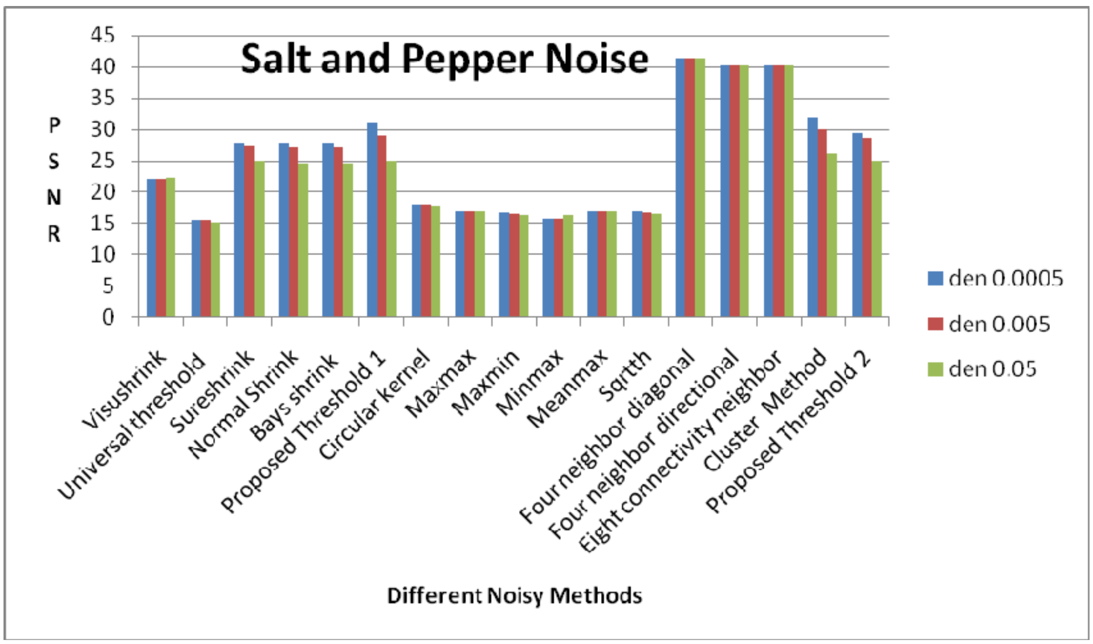

Fig. (9) Different Noisy methods Vs PSNR for Salt and Pepper noisy image

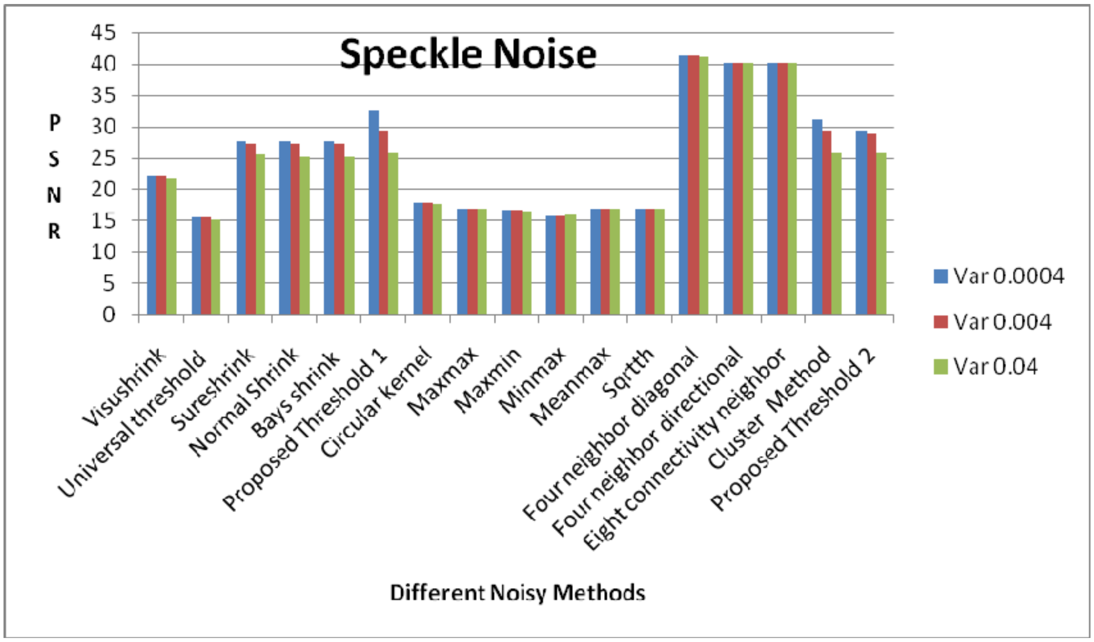

Fig. (10) Different Noisy methods Vs PSNR for Speckle noisy image

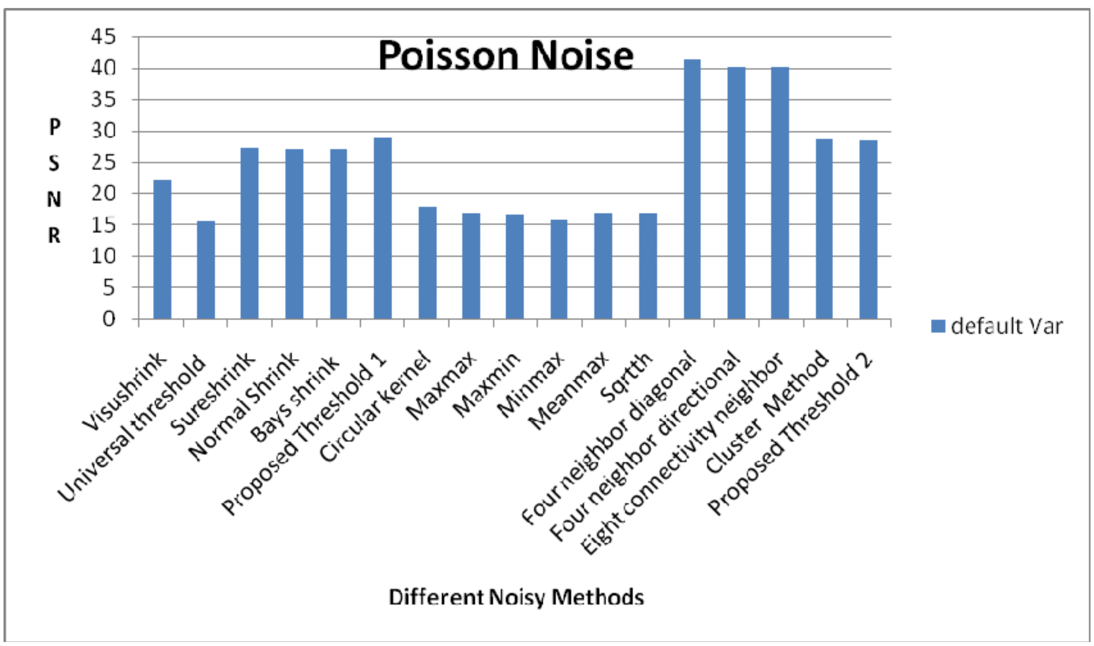

Fig. (11) Different Noisy methods Vs PSNR for Poisson noisy image

Table 1: Result of existing Threshold and proposed threshold for Gray scale image

Average Result Grayscale image PSNR											
Methods / Different Noise with different Variance		Gaussian Noisy Image			Salt and Pepper Noisy Image			Speckle Noisy Image			Poisson's Noisy
		variance 0.0001	variance 0.001	variance 0.01	noise density 0.0005	noise density 0.005	noise density 0.05	variance 0.0004	variance 0.004	variance 0.04	
Visushrink		22.16977	22.14152	21.92842	22.16511	22.18395	22.23792	22.16046	22.12868	21.79722	22.10774
Universal threshold		15.59039	15.55855	15.23345	15.58474	15.5606	15.26962	15.59164	15.54939	15.2169	15.51797
Sure shrink		27.62108	27.41412	25.81209	27.60585	27.28795	24.97738	27.61881	27.38486	25.66125	27.21611
Normal Shrink		27.65737	27.33262	25.27777	27.63827	27.1326	24.34225	27.65563	27.30175	25.13692	27.05406
Bays shrink		28.06855	27.32924	25.28576	27.63227	27.13368	24.3437	27.65462	27.29712	25.12893	27.05518
Proposed Threshold 1		34.24185	30.42973	26.15837	30.94006	29.07881	25.04729	32.74616	29.38359	25.91142	28.77463
Circular kernel		18.01832	17.98021	17.65805	17.86465	17.8454	17.63618	17.86525	17.97706	17.71615	17.82687
Mean Max approximation	Maxmax	16.83597	16.845	16.86032	16.83453	16.84967	16.95953	16.84073	16.86323	16.94244	16.86811
	Maxmin	16.83347	16.74133	16.65495	16.80812	16.63208	16.36204	16.75627	16.74423	16.58488	16.77375
	Minmax	15.8685	15.69424	15.9039	15.73588	15.82213	16.30574	15.68155	15.70579	16.05217	15.73023
	Meanmax	16.90268	16.91443	16.95962	16.90468	16.92339	16.96804	16.90141	16.90642	16.95164	16.91248
	Sqrtth	16.91752	16.91206	16.81611	16.89092	16.81259	16.62759	16.91807	16.91243	16.85461	16.90424
Nearest Neighbor	Four neighbor diagonal	41.28095	41.27857	41.27484	41.28104	41.28083	41.28042	41.27804	41.26721	41.21891	41.2671
	Four neighbor directional	40.22822	40.22491	40.21406	40.22843	40.22753	40.22321	40.22636	40.21482	40.15304	40.21488
	Eight connectivity neighbor	40.23025	40.2276	40.22342	40.2307	40.23078	40.22598	40.22871	40.21883	40.16281	40.21832
Cluster Method		32.28825	29.87745	26.06135	31.89543	30.0085	26.07388	31.14709	29.33233	25.82711	28.71412
Proposed Threshold 2		34.25101	29.56134	26.1228	29.4063	28.70642	24.97664	29.42172	28.87425	25.87373	28.52123